Menoufia University **Faculty of Engineering, Shebin El-Kom,** **Basic Engineering science Department** **Second Semester Examination, 2015-2016** **Date of Exam: 15/6/2016**	MENOUFIYA UNIVERSITY	**Subject: Introduction in Mathematical Physics** **Code : BES 522** **Year : Postgraduate students** **Time Allowed : 3 hours** **Total Marks: 100 marks**

Answer the following questions

1) Evaluate the following integrals:

$$i)\ \int_0^a x^5 J_2(x)\,dx \qquad ii)\ \int_0^\infty \sqrt{y}\, e^{-y^3}\,dy$$

$$iii)\ \int_{-1}^{1} x^2 P_{L+1}(x) P_{L-1}(x)\,dx \qquad iv)\ \int_{-\infty}^{\infty} x e^{-x^2} H_n(x) H_m(x)\,dx$$

2) Show that the following special functions:

$$i)\ \Gamma(n+\frac{1}{2}) = \frac{(2n)!\sqrt{\pi}}{4^n\, n!}\ , n \text{ positive integer}$$

$$ii)\ J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x - x\cos x}{x}\right)$$

$$iii)\ \beta(m,n) = 2\int_0^{\frac{\pi}{2}} (\sin\theta)^{2m-1}(\cos\theta)^{2n-1}\,d\theta$$

3) Expand the function $f(x) = x^4$ in Legendre polynomials.

4) Consider the Hermite Equation of order 5: $y'' - 2ty' + 10y = 0$. Find the solution satisfying the initial conditions $a_0=1$, $a_1=0$.

5) Write the expansion of the hypergeometric function $F(a,b;c;z)$, and then prove that the following expansion: $F(1/2,1/2;3/2;z^2) = \dfrac{\sin^{-1} z}{z}$.

6) Find the general solution of Bessel's differential equation

$$z^2Y'' + zY' + (z^2 - n^2)Y = 0$$

where $n \neq 0, \pm 1, \pm 2, \ldots$

7) Let $u(x,t)$ represent the temperature of a very thin rod of length π, which is placed on the interval $0 \leq x \leq \pi$, at position x and time t. The PDE which governs the heat distribution is given by $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$,

where u, x, t and k are given in proper units. We further assume that both ends are insulated; that is, $u(0,t) = u(\pi,t) = 0$ are impose "boundary condition" for $t \geq 0$. Given an initial temperature distribution of $u(x,0) = 2\sin 4x - 11\sin 7x$, for $0 \leq x \leq \pi$, use the technique of separation of variables to find a (non-trivial) solution, $u(x,t)$.

8) Find the eigen values and eigen functions of the equation:

$$y'' - 4\lambda y' + 4\lambda^2 y = 0 \quad , y(0) = 0, \; y(1) + y'(1) = 0$$