مايو ٢٠١٩
الفرقة الأولى

ختام العام

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University	: Menoufia	*Date*	: 27/05/2019
Faculty	: Electronic Engineering	*Time*	: 3 Hours (10 AM - 1 PM)
Department	: Phys. & Eng. Math.	*No. of pages*	: 1
		Semester	: Second
Academic level	: First year	*Full Mark*	: 50 Marks
Course Name	: Engineering Math. (4)	*Exam*	: Final Exam
		Examiner	: Dr. Ali Kandil

Answer all of the following questions:

Question No. 1: (10 Marks)

(1.a) Find the directional derivative of $\phi = xy^2 - 4x^2y + z^2$ at the point $(1,-1,2)$ in the direction of $6\underline{i} + 2\underline{j} + 3\underline{k}$. Determine the behavior of ϕ in the indicated direction. Then, find the maximum and minimum values of this directional derivative. **(5 Marks)**

(1.b) If $\underline{F} = (x^2y^3 - z^4)\underline{i} + 4x^5y^2z\underline{j} - y^4z^6\underline{k}$, find: **(5 Marks)**

(I) $curl\,\underline{F}$ (II) $div\,\underline{F}$ (III) $div\,curl\,\underline{F}$ (IV) If $\phi = div\,\underline{F}$, then find $curl\,grad\,\phi$

Question No. 2: (20 Marks)

(2.a) If the force field $\underline{F} = (2xy + z^3)\underline{i} + x^2\underline{j} + 3xz^2\underline{k}$, then

(I) Show that it is a conservative force field.

(II) Find its scalar potential ϕ.

(III) Find the work done in moving a body in this field from $(1,-2,1)$ to $(3,1,4)$.

(IV) Evaluate $\oint_C \underline{F} \cdot d\underline{r}$ where C is any closed curve in the domain of the force field. **(5 Marks)**

(2.b) Find the area of the circle $x = a\cos\theta$ and $y = a\sin\theta$ using Green's theorem. **(5 Marks)**

(2.c) Evaluate $\iint_S curl\,\underline{F} \cdot \underline{n}\,dS$ where $\underline{F} = (x^2 + y - 4)\underline{i} + 3xy\underline{j} + (2xz + z^2)\underline{k}$ and S is the surface of the paraboloid $z = 4 - (x^2 + y^2)$ above the xy plane. **(5 Marks)**

(2.d) Evaluate $\oiint_S \underline{r} \cdot \underline{n}\,dS$ where S is the spherical surface of radius 2 and centered at origin. **(5 Marks)**

Question No. 3: (20 Marks)

(3.a) Evaluate each of the following using Gamma function definition: **(6 Marks)**

(I) $\Gamma\left(-\frac{7}{2}\right)$ (II) $\int_0^1 \frac{dx}{\sqrt{-\ln x}}$ (III) $\int_0^\infty e^{-x^3}\,dx$

(3.b) Evaluate each of the following using Beta function definition: **(9 Marks)**

(I) $\int_0^{2\pi} \sin^8\theta\,d\theta$ (II) $\int_0^{\pi/2} \sin^3\theta\cos^2\theta\,d\theta$ (III) $\int_0^a y^4\sqrt{a^2 - y^2}\,dy$

(3.c) Prove that: **(5 Marks)**

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x \quad \text{if you know that} \quad J_p(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(n+p+1)}\left(\frac{x}{2}\right)^{2n+p}$$

End of Questions

Good Luck