ACCURACY OF THE CONVERGENT PHOTOGRAPHY

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ABSTRACT

There is no doubt that the number and variety of the photogrammetric applications are growing. Convergent case photogrammetry is the most commonly used in analytical photogrammetry . The present formulas which are used to estimate the accuracy of convergent case photogrammetry do not give good results, Special at the edge of photograph therefor , the simulation is used for estimating the accuracy of the convergent case of close range photogrammetry . The accuracy obtained in practical experiments closely matches the estimated accuracy from simulation.The disadvantages of the simulation are :

The method needs an experience and skilled photogrammetrist;

The optimal solution is missed and;

The simulation technique is costly and complicated.

This paper gives the mathematical proofs of a newly developed formulas for estimating the accuracy of convergent case.

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INTRODUCTION

The problem which faces the photogrammetrist is in determining the optimum positions of the two camera stations relative to the object in order to achieve the required accuracy. So far , there have been no satisfactory mathematical formulas , in the field of photogrammetry , which allow the photogrammetrist to determine the optimum layout of the two cameras stations. The main objective of this study is to find an ideal solution for the accuracy problem by devloping formulas which make it possible for the photogrammertist to determine the optimum layout of the two cameras stations and also to determine the object accuracy , for any camera set- up , right in the field.The accuracy of the ground coordinates is a function of the elements of the photogrammetric system. The principal parameters influencing acuuracy may be classified into three groups :

A-Geometrical charateristics:

The accuracy depends on the lay out of the two camera stations ; number of stations ; density of control and the foacl length of the camera. The layout of the two camera stations relative to the object is a function of the external orientation parameters. The accuracy of the object - space coordinates obtained from convergent photographs is a function of these parameters Y, B, C, φ,σ_x, σ_z..

These parameters can be classified into two groups:

1. Configuration parameters :

B; Y and φ which change by changing the outer oriention of the two photos.
where
B: is the base between the two photos,
Y: is the mean object distance ,
φ: is the angle of convergence.

2. Camera parameters:

C; σ_x ;σ_z and the formate size.
where
C: is the camera constant ,
σ_x, σ_z : are the standard errors of the image coordinates x and z respectively.

- **B- Physical characteristics of the photogrammetric system**:
- The principal physical characteristics of the photogrammetric system are:
- The quality of the camera objectives and the lens distortions ;The quality of the film ;The definition of the object (natural or artificial) and The accuracy of comparator measurements.

C- Redundance of measurements:

There are three faces for the redundance of measurements :
The number of stations and number of frames at each station ; number of comparator measurements for each point for each photo and number of targets for each object point .

PREDICTION OF ACCURACY

Accuracy predictors are formulas (Abdel Aziz and Karara)[1], simulations and diagrames (Hottier)[6], which give the accuracy as a function of the principal parameters of a photogrammetric system. Accuracy predictors can exist only for simple configurations of the data acquisition system (the symmetric case of the pair) which are the most frequent in practice .
The existing formulas for convergent photogrmmetry case express the accuracy of ground points as a function of the configuration parameters Y, B, φ of the two photos that form the model , This formulas according to Abdel Aziz and Karara [1], which give the errors (σ_X, σ_Y and σ_Z) (X- axis parallel to the base,and Y axis perpendicular to the object Figure 1) reffered to the image plane are:

$$\sigma_X = \frac{Y}{C} \frac{(1 + \tan\alpha \tan\varphi)\,\sigma}{1 - \tan(\alpha - \varphi)\tan\varphi} \tag{1}$$

$$\sigma_Y = \frac{Y}{C} \frac{\sec\varphi\,\sigma}{1 - \tan(\alpha - \varphi)\tan\varphi} \tag{2}$$

$$\sigma_Z = \frac{Y}{C}\frac{Y}{B}\sqrt{2} \frac{(1 + \tan\alpha \tan\varphi)\sigma}{1 - \tan(\alpha - \varphi)\tan\varphi} \tag{3}$$

where:
B: is the base between the two photos,
Y: is the mean object distance ,
σ is the standard deviation of the measurement error,
φ is the angle which the camera axis makes with the direction perpendicular to the base
α is the angle which the line joining the central point O of the object space and the perspective centre makes with the perpendicular direction to the base.
The computation of the accuracy of the object space coordinates is based on the following assumptions :
a- $\sigma_{x1} = \sigma_{x2} = \sigma_{z1} = \sigma_{z2} = \sigma$
b- $\varphi_1 = \varphi_2 = \varphi$, $\alpha_1 = \alpha_2 = \alpha$

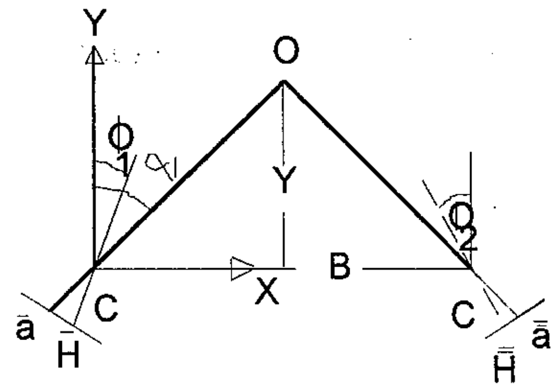

Fig.1 The covergent case of photogrammetry.

Karara and Abdel Aziz [2], have studied the accuracy problem by considering the accuracy only at the centeral object point,where the implicit assumption being that the accuracy at the central object point can reasonably represent the accuracy of the whole object . Hottier [6], has found that the central point gives theoretically a slight lower accuracy . Hottier [6], and Regensburger[8], have concluded that the accuracy at the central point cannot represent in an ideal way the average accuracy over the whole object plane, particularly for small values of the base- to - object - distance ratio ($0.7 < r^1 < 1.0$). They have found that the central point XY accuracy is of course constant , but it is not the same for corner points .

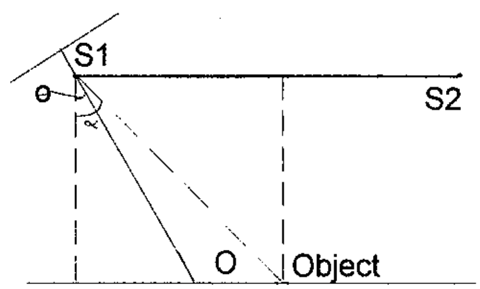

Fig.2. Definition of the central point [6].

This means that these formulas cannot give the real accuracy over the whole object specially at the corner points. Accordingly , Hottier used the simulation technique for estimating the accuracy of the object points. The estimated accuracy by simulations yields results which closely match the accuracy obtained from close-range applications . The simulation technique cannot be recommended for real applications because it is costly and complicated specially for the error model .Moreover , the photogrammetrist cannot estimate the ground accuracy immediately in the field for any camera stations . Accordingly , new formula have been developed in this article expressing the accuracy of the ground points over the whole object as a function of the above mentioned parameters of external oriention . These formulas can be used in the field for determining the positions of the two camera stations and for computing the accuracy of the object points.

[1] r =B/H

THE THEORETICAL ASPECT

Figure 3 shows two coordinate system X,Y and X_1,Y_1 having the same origin O,but the coordinate system X_1,Y_1 is rotated throught an angle φ relative to the fixed coordinate system X,Y. In this figure, the camera axis (CA) coincide to the Y_1 direction and X_1 is perpendicular to the Y_1 direction ,so the camera axis (CA) and its perpendicular axis X_1 represent the coordinate system X_1, Y_1. According to the position of point A with respect to Y_1 direction one have 3 different cases.

Case 1 : Point A lies between camera axis (CA) (Y_1 - direction) and X-axis ,in this case $\alpha>\varphi$,

Case 2: Point A lies between camera axis (CA) (Y_1 - direction) andY-axis,in this case$\varphi>\alpha$,

Case 3: Point A lies on the direction of the camera axis, in this case $\varphi = \alpha$.

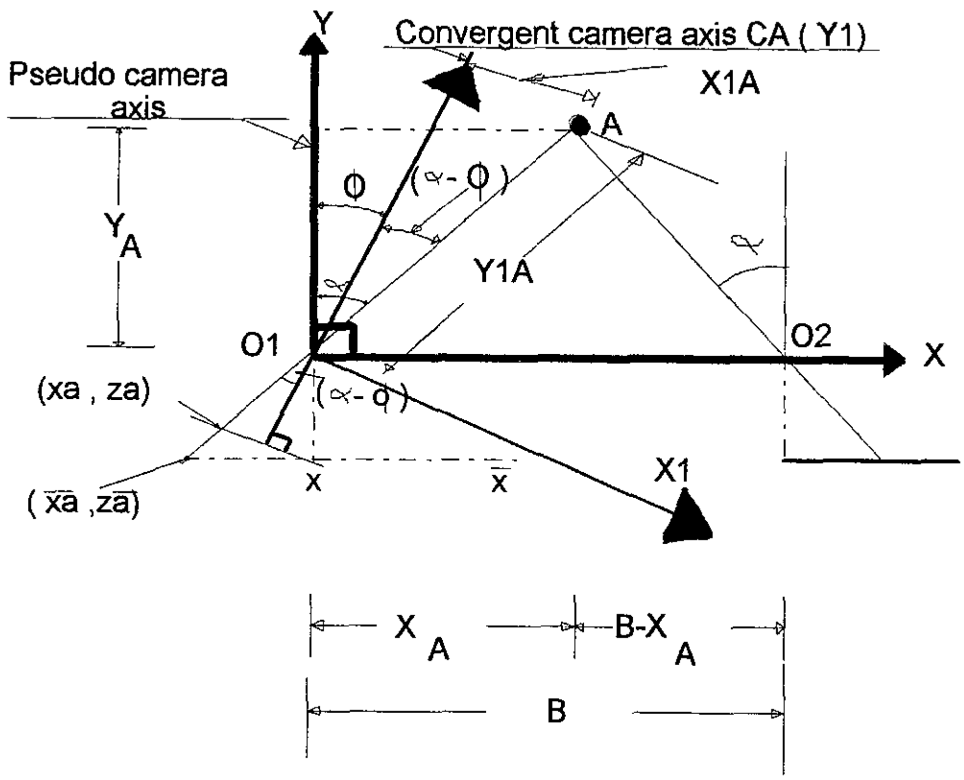

Fig.3 The relationsh between the ground coordinates of the same point A on a convergent photo on two coordinate system (X,Y and Z and X1,Y1 and Z1).

Case 1: Point A lies between camera axis (CA) and the X-axis

According to Figure 3 the relationship between the ground coordinates of the same point A on two different coordinate systems X,Y and X_1, Y_1 can be deduced as follows:

$$X_{1A} = X_A \cos\varphi - Y_A \sin\varphi$$

$$Y_{1A} = X_A \sin\varphi + Y_A \cos\varphi$$

or in matrix form :

$$\begin{pmatrix} X_{1A} \\ Y_{1A} \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} X_A \\ Y_A \end{pmatrix} \qquad (4)$$

From Fig.3 one find :

$$\frac{\bar{x}}{C} = \frac{X_A}{Y_A} = \tan\alpha \qquad (5)$$

$$\frac{x}{C} = \frac{X_{1A}}{Y_{1A}} = \tan(\alpha - \varphi) \qquad (6)$$

by substituting from equation 4, into equation 6 one can gets :

$$\frac{x}{C} = \frac{X_1}{Y_1} = \tan(\alpha - \varphi) = \frac{X \cos\varphi - Y \sin\varphi}{X \sin\varphi + Y \cos\varphi} \qquad (7)$$

According to Abdel Aziz [2]/p. 1344 the accuracy of the image coordinates are :

$$\sigma_x = \frac{\left(1 + \left(\frac{x}{C}\right)\tan\varphi\right)\sigma}{1 - \left(\frac{x}{C}\right)\tan\varphi} \qquad (8)$$

$$\sigma_y = \frac{\sec\varphi \ \sigma}{1 - \left(\frac{x}{C}\right)\tan\varphi} \qquad (9)$$

By substituting from equations 5 and 7 into equations 8 and 9 one finds that, the accuracy of the image coordinates of any point A lies between the camera axis (CA) and X axis are :

$$\sigma_x = \sigma_{x1} = \sigma_{x2} = \frac{(X \sin\varphi + Y \cos\varphi)^2}{Y^2} \qquad (10)$$

$$\sigma_y = \sigma_{y1} = \sigma_{y2} = \frac{(X \sin\varphi + Y \cos\varphi)}{Y} \quad (11)$$

THEORETICAL ACCURACY OF THE OBJECT-SPACE COORDINATES

Karara[7],deduced that the accuracy of the object space coordinates X,Y and Z in the normal case of photogrammetry reffering to Fig (4) are :

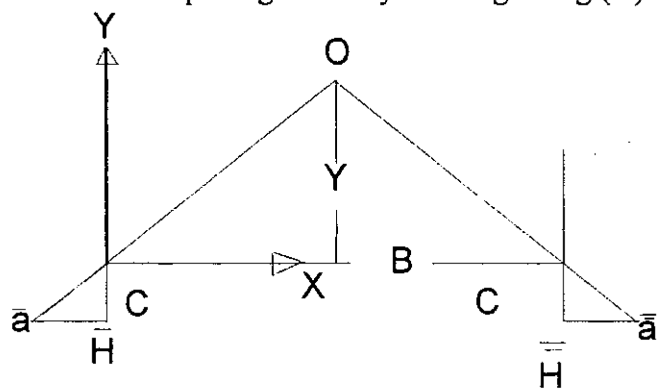

Fig. 4. The normal case of photogrammetry.

$$\sigma_X = \left(\frac{Y}{C}\right) \sigma_x \quad (12)$$

$$\sigma_Z = \left(\frac{Y}{C}\right) \sigma_z \quad (13)$$

$$\sigma_Y = \frac{Y}{C}\frac{Y}{B}\sqrt{2}\,\sigma_x \quad (14)$$

Substituting the values of σ_x, σ_y from equations 10 and 11 into equations 12 and 13 and 14 one gets :

$$\sigma_X = \frac{1}{C}\frac{1}{Y}(X \sin\varphi + Y \cos\varphi)^2_{\sigma} \quad (15)$$

$$\sigma_Z = \frac{1}{C}(X \sin\varphi + Y \cos\varphi)_{\sigma} \quad (16)$$

$$\sigma_Y = \frac{1}{C}\frac{1}{B}\sqrt{2}(X \sin\varphi + Y \cos\varphi)^2_{\sigma} \quad (17)$$

Case 2: Point A lies between camera axis (CA) and Y-axis (in this case $\varphi>\alpha$)

According to Figure 3 the relationship between the ground coordinates of the same point A on two different coordinate system X,Y and X_1,Y_1 can be deduced as follows:

$$X_1 = -X\cos\varphi + Y\sin\varphi$$

$$Y_1 = X\sin\varphi + Y\cos\varphi$$

or in matrix form :

$$\begin{pmatrix} X_1 \\ Y_1 \end{pmatrix} = \begin{pmatrix} -\cos\varphi & \sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} \qquad (18)$$

From Fig.3

$$\frac{\bar{x}}{C} = \frac{X}{Y} = \tan\alpha \qquad (19)$$

$$\frac{x}{C} = \frac{X_1}{Y_1} = \tan(\varphi - \alpha) \qquad (20)$$

by substituting from equation 18, into equation 20 one can gets :

$$\frac{x}{C} = \frac{X_1}{Y_1} = \tan(\varphi - \alpha) = \frac{-X\cos\varphi + Y\sin\varphi}{X\sin\varphi + Y\cos\varphi} \qquad (21)$$

By substituting the values of ($\bar{x}$/C) and (x/C) from equations 19 and 21 into equations 8 and 9 one finds that, the accuracy of the image coordinates of any point A lies between the camera axis (CA) and Y axis are :

$$\sigma_x = \sigma_{x1} = \sigma_{x2} = \frac{1}{Y}\frac{(X\sin\varphi + Y\cos\varphi)^2}{\sin\varphi\cos\varphi(Y+X) + Y(\cos^2\varphi - \sin^2\varphi)} \qquad (22)$$

$$\sigma_y = \sigma_{y1} = \sigma_{y2} = \frac{(X\sin\varphi + Y\cos\varphi)}{\sin\varphi\cos\varphi(Y+X) + Y(\cos^2\varphi - \sin^2\varphi)} \qquad (23)$$

Substituting the values of σ_x and σ_y from equations 22 and 23 into equations 12,13 and 14 one get:

$$\sigma_X = \frac{1}{C}\frac{(X\sin\varphi + Y\cos\varphi)^2\sigma}{\sin\varphi\cos\varphi(Y+X) + Y(\cos^2\varphi - \sin^2\varphi)} \quad (24)$$

$$\sigma_Z = \frac{Y}{C}\frac{(X\sin\varphi + Y\cos\varphi)\sigma}{\sin\varphi\cos\varphi(Y+X) + Y(\cos^2\varphi - \sin^2\varphi)} \quad (25)$$

$$\sigma_Y = \frac{Y}{C}\frac{\sqrt{2}}{B}\frac{(X\sin\varphi + Y\cos\varphi)^2\sigma}{\sin\varphi\cos\varphi(Y+X) + Y(\cos^2\varphi - \sin^2\varphi)} \quad (26)$$

Case 3: Point A lies on the camera axis (CA).

According to Figure 3

$$\frac{x}{C} = 0.0 \quad (27)$$

$$\frac{x^{-}}{C} = \tan\alpha = \tan\varphi \quad (28)$$

Substituting the values of (x^{-}/C) and (x/C) from Equations (27) and (28) into Equations (8) and (9) one finds that, the accuracy of the image coordinates of any point A lies on the camera axis (CA) are :

$$\sigma_x = (1 + \tan^2\varphi) \quad (29)$$

$$\sigma_y = \sec\varphi \quad (30)$$

Substituting the values of σ_x and σ_y from equations 29 and 30 into equations 12;13 and 14 one get:

$$\sigma_X = \frac{Y}{C}(1 + \tan^2\varphi)\ \sigma \quad (31)$$

$$\sigma_Z = \left(\frac{Y}{C}\right)\sec\varphi\,\sigma \quad (32)$$

$$\sigma_Y = \frac{Y}{C}\frac{Y}{B}\sqrt{2}\ (1+\tan^2\varphi)\sigma \quad (33)$$

THE AVERAGE ACCURACIES σ_X ; σ_Y and σ_Z OF THREE-DIMENSIONAL OBJECTS

One can not expect the distribution of the ground points in each plane , Accordingly, the values σ_X ; σ_Y and σ_Z can be calculated in analytical photogrammetry using the numerical integration of σ_{Xi} ; σ_{Yi} and σ_{Zi} and this can take the following form:

Case 1:

$$\sigma_X = \frac{1}{n}\sum_{i=1}^{n} [\frac{1}{C}\frac{1}{Y}(X \sin\varphi + Y \cos\varphi)^2{}_\sigma] \quad (34)$$

$$\sigma_Z = \frac{1}{n}\sum_{i=1}^{n} [\frac{1}{C} (X \sin\varphi + Y \cos\varphi)_\sigma] \quad (35)$$

$$\sigma_Y = \frac{1}{n}\sum_{i=1}^{n} [\frac{1}{C}\frac{1}{B}\sqrt{2} (X \sin\varphi + Y \cos\varphi)^2{}_\sigma] \quad (36)$$

Case 2:

$$\sigma_X = \frac{1}{n}\sum_{i=1}^{n} [\frac{1}{C}\frac{(X \sin\varphi + Y \cos\varphi)^2\sigma}{\sin\varphi\cos\varphi(Y + X) + Y(\cos^2\varphi - \sin^2\varphi)}] \quad (37)$$

$$\sigma_Z = \frac{1}{n}\sum_{i=1}^{n} [\frac{Y}{C}\frac{(X \sin\varphi + Y \cos\varphi)\sigma}{\sin\varphi\cos\varphi(Y + X) + Y(\cos^2\varphi - \sin^2\varphi)}] \quad (38)$$

$$\sigma_Y = \frac{1}{n}\sum_{i=1}^{n} [\frac{Y}{C}\frac{\sqrt{2}}{B}\frac{(X \sin\varphi + Y \cos\varphi)^2\sigma}{\sin\varphi\cos\varphi(Y + X) + Y (\cos^2\varphi - \sin^2\varphi)}] \quad (39)$$

Case 3:

$$\sigma_X = \frac{1}{n}\sum_{i=1}^{n} [\frac{Y}{C} (1 + \tan^2\varphi) \quad \sigma] \quad (40)$$

$$\sigma_Z = \frac{1}{n}\sum_{i=1}^{n} [(\frac{Y}{C}) \sec\varphi \, \sigma] \quad (41)$$

$$\sigma_Y = \frac{1}{n}\sum_{i=1}^{n} [\frac{Y}{C}\frac{Y}{B}\sqrt{2} \quad (1+\tan^2\varphi)\sigma] \quad (42)$$

To determine the positions of points (case 1,case2 and case3) and to compute the average values of σ_{Xi} ; σ_{Yi} and σ_{Zi} from numerical integration of the new Formulas (NEF) and those obtained from the present formulas (PRF), a BASIC computer program was designed in this research work . The computed values of σ_{Xi} ; σ_{Yi} and σ_{Zi} obtained from the numerical integration of the new formulas (NEF) (Equations 34- 42), from present formulas (PRF) and from simulation are given in Table 1

TABLE 1. COMPARIZON BETWEEN σ_X , σ_Y and σ_Z COMPUTED FROM SIMULATION (SIM) ,NEW FORMULAS (NEF) AND PRESENT FORMULAS (PRF)

Pt. No	σ_X			σ_Z			σ_Y		
	SIM	NEF	PRF	SIM	NEF	PRF	SIM	NEF	PRF
1	2.70	3.3	1.8	2.6	3.8	1.8	5.1	5.9	3.2
2	3.2	3.2	1.8	3.9	3.8	1.9	5.6	5.9	3.2
3	3.4	3.4	1.9	4.0	3.9	1.8	6.1	6.3	3.6
4	3.3	3.4	1.6	3.9	3.8	1.7	6.0	6.1	3.4
5	3.6	3.6	2.2	4.0	3.9	2.1	6.5	6.6	3.8
6	3.4	3.6	1.9	4.1	3.9	2.1	6.4	6.5	3.6
7	2.1	3.6	1.8	4.0	3.9	1.9	6.3	6.5	3.6
8	3.8	3.6	2.1	3.8	3.8	2.1	6.3	6.4	3.7
9	3.8	3.8	1.7	4.2	4.0	1.7	7.0	7.1	4.1
10	3.0	3.8	1.8	3.6	3.9	1.9	5.5	6.7	4.1
11	5.5	6.1	2.1	6.0	6.2	2.1	16.5	17.5	8.8
12	3.7	3.9	1.6	3.9	3.9	1.8	7.2	7.2	3.9
13	3.9	3.9	1.5	4.1	4.0	1.7	7.0	7.3	3.8
14	3.8	3.9	1.6	4.7	4.0	1.4	6.6	7.3	3.8
15	5.4	5.5	1.8	5.8	5.6	1.9	11.6	13.9	6.2
16	4.0	4.0	2.1	4.1	4.0	2.1	7.1	7.4	4.2
17	4.0	4.1	1.9	4.2	4.0	1.9	7.6	7.7	4.2
18	1.58	1.57	0.8	1.13	1.12	0.8	0.29	0.49	0.8
19	4.0	4.3	2.1	4.2	4.1	2.1	6.2	6.5	3.3
20	4.1	4.2	2.3	5.1	4.1	2.3	7.6	7.7	4.6
21	5.4	5.8	1.8	5.7	5.6	1.9	14.5	14.8	6.8
22	5.3	5.2	2.3	4.9	4.9	2.3	9.4	9.6	4.2
23	6.1	6.5	1.9	5.6	6.4	1.9	15.8	18.6	8.5
24	5.1	5.2	2.1	5.8	4.9	2.1	9.1	9.5	4.8
25	5.3	5.3	2.1	5.1	4.9	2.1	9.6	9.6	4.9
26	3.9	4.7	2.3	3.9	4.2	2.1	5.9	7.1	3.2
27	5.0	5.4	1.9	5.1	5.0	1.8	9.2	9.9	4.4
28	4.6	4.9	2.1	4.4	4.3	2.1	7.1	7.4	4.1

CONCLUSIONS

From Table 1 one can conclude that :

- The proposed formulas are conform with the simulation technique and give better values of σ_x ; σ_y and σ_z when compared to the present formulas.

-The proposed formulas give good results at the center or at the edge of the photograph.

- One can use the proposed formula to determine the approprite positions right on the field by computing the average values of σ_x ; σ_y and σ_z which can be estimated from the numerical integration of the formulas in the three cases.

-For the simulation technique one need a personal computer but for the proposed formula one need only a small calculator .

For all these reasons one can suggested to use the proposed formulas instead of the exisiting formulas and the simulation method .

REFERENCES

[1]Abdel-Aziz,Y.I.,andKarara,H.M.,1973, "Photogrammetric Potentional of Non-Metric Cameras," Photogrammetry Series No:35, University of Illinoise, Urbana.Illinois.

[2]Abdel-Aziz,Y.I.,1974, Expected Accuracy of Convergent Photography, Photogrammetric Engineering and Remote Sensing , Vol.40, No.11 November 1974.

[3]Abdel-Aziz Y.I.,1982, Accuracy of the normal case of close -range photogrammetry,Photogrammetric Engineering and Remote Sensing,Vol.48,No.2, February 1982,pp. 207-213.

[4]Donald L. Light,1992, The New Camera Calibration System At the U.S. Geological Survey Photogrammetric Engineering and Remote Sensing , Vol.58, No.2 February 1992,pp.185-188.

[5]Franz W.Leberl and Hubert Ranzinger 1997 ,Registration of Digitized Aerial Photography with a Digital Map Data Base, DIBAG-Report No.6, Institute for Image Processing and Computer Graphics, Tecnical University and Graz Research Center.

[6]Hottier,PH.(1976) Accuracy of Close Range Analytical restitutions: practical experiments and prediction.PE, vol.42, No.3 march 1976, pp345-375.

[7]Karara,H.M.,1966,"Stereometric Systems of High Precision," Civil Engineering Studies, Photogrammetry Series No.15, University of Illinoise, Urbana.Illinois.

[8]Regensburger,K.(1990) Photogrammetrie, Anwendungen in Wissenschaft und Technik.1990: Berlin VEB verlag fur Bauwesen.

الدقة المتوقعة فى الصور المتقاربة

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ليس هناك شك فى زيادة واتساع تطبيقات الفوتوجرامترى وتعد الصور المتقاربة الأكثراستخداما وشيوعا فى مجال الفوتوجرامترى التحليلى . وقد لوحظ من خلال هذا البحث أن المعادلات المستخدمة حتى الآن لتقدير دقة الصور المتقاربة لاتعطى نتائج مرضية خاصة عند حواف الصور مما يضطر معة المشتغلون بالفوتوجرامترى لاستخدام طريقة المحاكاة رغم عيوبها الكثيرة لتقدير الدقة .

لهذا فقد كان هدف هذا البحث هو اشتقاق معادلات جديدة يمكن من خلالها ايجاد حل مثالى لمشكلة تعيين الدقة المتوقعة لاحداثيات النقط فى أى موضع على الصورة بالاضافة الى تعيين الوضع الأمثل لمحطات التصوير.وقد تم تقسيم مواضع النقط الى ٣ حالات مختلفة وقد أمكن اشتقاق ٣ معادلات مستقلة وجديدة لكل حالة على حدة وتم عمل مقارنة بين المعادلات المشتقة الجديدة وطريقة المحاكاة والمعادلات المستخدمة حتى الآن .

وقد تبين من هذة المقارنة تطابق النتائج بين المعادلات المشتقة الجديدة وطريقة المحاكاة لهذا يوصى البحث باستخدام المعادلات المشتقة الجديدة بدلا من المعادلات المستخدمة حاليا لما لهذة المعادلات الجديدة من مميزات كثيرة.