Menofia University Faculty of Engineering Shebin El-Kom Dep. of Basic Engineering Sciences First Semester Examination 2015-2016	جامعة المنوفية	Subject: Math. (1-A) Code: BES 011 Time Allowed: 3 hours Total Marks: 100 marks Date of Exam: 12/1/2015

First: Algebra (Answer all the Following Questions)

(Question Number-1) :(25 Marks)

A- Find all the roots of the polynomial:

$$f(x)=x^5-x^4-3x^3+3x^2-4x+4=0$$

B- Resolve into partial fractions:

i) $\dfrac{x^2+15}{x^4+6x^3+12x^2+18x+27}$

ii) $\dfrac{5x-1}{(2-x)(1+x)}$, Then find the coefficient of x^n and the condition of expansion.

(Question Number-2) :(25 Marks)

A- Use the mathematical induction to prove:

i) $3^{2n+1}+2^{n-1}$ is a multiple of 7.

ii) $1^2-2^2+3^2-4^2+\dots-n^2=\left(-\frac{1}{2}\right)(n)(n-1)$ for any even number.

B- Define mathematically the following:

Skew Hermitian matrix, extreme point, convex set, Cayley-Hamilton theorem, rank of matrix.

C- If $B^T=\begin{bmatrix}7 & 5 & 13\end{bmatrix}$ and $A=\begin{bmatrix}9 & 2 & 1\\ 1 & 3 & -1\\ 7 & 1 & 1\end{bmatrix}$; where its columns are a_1, a_2 and a_3:

i) Explain why the system $Ax=B$ is inconsistent; where $x^T=\begin{bmatrix}x_1 & x_2 & x_3\end{bmatrix}$.

ii) Show whether the vectors a_1, a_2 and a_3 are dependent or independent.

Second: Differential calculus(Answer all the Following Questions:)

(Question Number-3) :(50 Marks)

A- Find $\dfrac{dy}{dx}$ of the following functions: (16 Marks)

1) $y=\dfrac{(\log_x \sin x)\sqrt{x^2+\cos x}}{(3x-2)^3\tan^{-1}(x^2+1)}$	2) $y=\tan\left(\dfrac{(\sec x-e^{2x})+\sqrt{3\ln x+5}}{x^3+\sin^{-1}x}\right)$
3) $(\cos x)^y=(\cos y)^x$	4) $y=(\cosh 2x)^{\sin x}$

B- If $u=xe^y+ye^x$, $x=r+2t$ and $y=3r-t$, find $\dfrac{\partial u}{\partial r}$ and $\dfrac{\partial u}{\partial t}$. (5 Marks)

C- Evaluate the following limits: (9 Marks)

1) $\lim\limits_{x\to 0}\left[\dfrac{1}{\sin x}-\dfrac{1}{x}\right]$ 2) $\lim\limits_{x\to 0^+}(\cos x)^{1/x}$ 3) $\lim\limits_{x\to \pi/3}\dfrac{\cos x-1/2}{x-\pi/3}$.

D- Prove that the function $f(x,y)=xy^2+e^{x^2y}$ is continuous. (4 Marks)

E- Prove that $D_n(\cos(ax+b))=a^n\cos\left(ax+b+\frac{n\pi}{2}\right)$, then if $y=\left(x+\sqrt{x^2-1}\right)^m$

prove that $(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$. (8 Marks)

F- Prove that $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)^n=e$, where $2\prec e\prec 3$. (3 Marks)

G- Prove that $\operatorname{csch}^{-1}x=\ln\left(\dfrac{1}{x}+\dfrac{\sqrt{1+x^2}}{|x|}\right)$, $x\neq 0$. (5 Marks)

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