CALCULATION OF THE SEPARATION POINT
FOR RADIAL DIFFUSERS

BY

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Nomenclature:

b	diffusor width m
d	diffusor exit diameter m
$\bar{c}$	velocity at outer edge of the boundary layer where $\partial c_x/\partial y = 0$, m/sec
c_f	friction drag co-efficient
p	pressure bar
r	radius m
R	radius m
x y	coordinates m
δ	boundary layer thickness, resp. the distance from the wall at which the velocity is maxium m
δ^*	displacement thickness m
δ^{**}	momentum thickness m
ν	kinematic viscosity m^2/sec
ρ	density kg/m^3
τ	shear stress N/m^2

Suffixes:

R	pipe
D	diffusor
e	exit
i	entrance
0	entrance of the parallel part of diffusor
w	wall

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Introduction

The phenomenon of flow separation is defined as the departure of streamlines from the surfaces. Separation may occur, for many reasons, in radial diffusers and causes failure in the concept of boundary layer. Separation is defined by the vanishing of the wall shearing stress. It depends entirely on the development of the boundary layer upstream of separation.

It is important in engineering applications to determine how the point of separation is affected by the pressure distribution and how a non-separating flow, can be economically maintained up to pressure increase.

Several authors [1] ; [2] ; [3] ; [4] ; [5] ; [6] ; [7] examined the radial diffusers. They were interested in the relation of efficiency with the diffuser width. However a criterion for flow separation is not considered. This paper deals with this point. A criterion based on integrating the momentum equation of boundary layer is established. The present study contains a solved example to illustrate the new method.

1. Governing Equations And Boundary Layer Conditions

An exact solution for turbulent friction layers is not possible, many approximate methods were therefore developed. All these methods were defined for the first time by TH.Von KÁRMÁN [8],[9] in the well known form of momentum equation. This equation in its general form can be written as:

$$\frac{d\delta^{**}}{dx} + \left(2 + \frac{\delta^{*}}{\delta^{**}}\right)\frac{\delta^{**}}{\bar{c}}\frac{d\bar{c}}{dx} + \frac{\delta^{**}}{r}\frac{dr}{dx} - \frac{\delta^{**}}{\bar{c}}\frac{d\bar{c}}{dx}\bar{M}^2 = \frac{\tau_w}{\rho.\bar{c}^2} \quad (1)$$

It is valid for laminar, turbulent, compressible and incompressible flows, for plane and rotational symmetrical arrangements. The definition of different terms in equation (1) is illustrated in figure 1.

Applying the momentum equation for boundary layers restric- to radial diffusor, with boundary condition, of incompressi- ow ($\bar{M}^2 \rightarrow 0$) near separation, gives:

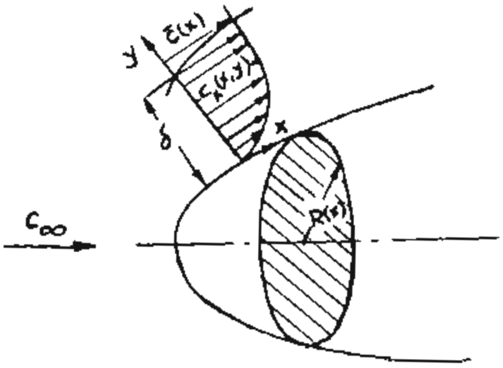

Fig. 1 Rotaional Symmetrical Body

$$\frac{d\delta^{**}}{dx} + \left(2 + \frac{\delta^{*}}{\delta^{**}}\right)\frac{\delta^{**}}{\bar{c}}\frac{d\bar{c}}{dx} + \frac{\delta^{**}}{r}\frac{dr}{dx} = 0 \qquad (1.a).$$

The continuity equation for parallel wall parts, on the basis of Fig.2, has the following form:

$$\frac{\bar{c}}{\bar{c}_o} = \frac{r_o . b_o}{r . b} \qquad (2).$$

The criterion for flow near separation is

$$\lambda = -\frac{1}{\bar{c}}\frac{d\bar{c}}{dx}\delta^{**} \qquad (3).$$

The considerable diffusor width at entrance and at any cross section, considering the displacement thickness are defined as:

$$\left.\begin{aligned} b &= b_w - 2.\delta^{*}, \\ b_o &= b_w - 2.\delta^{*} \end{aligned}\right\} \qquad (4).$$

Substituting Eq.(4) in (2) gives the velocity at any cross section

$$\frac{\bar{c}}{\bar{c}_o} = \frac{r_o}{r}\,\frac{b_w - 2.\delta^{*}_o}{b_w - 2.\delta^{*}} \qquad (5).$$

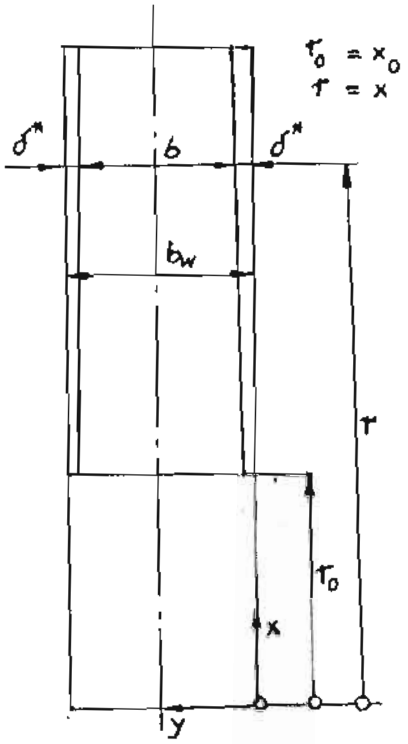

Fig. 2 Coordinates Represntation On Radial Diffusor

A good approximation can be made, in which one may consider that $r \approx x$ for the parallel wall parts and obtain with it $dr/dx = 1$. Equation (1.a) and (5) becomes:

$$\frac{d\delta^{**}}{dx} + \left(2 + \frac{\delta^{*}}{\delta^{**}}\right) \frac{\delta^{**}}{\bar{c}} \frac{d\bar{c}}{dx} + \frac{\delta^{**}}{x} = 0 \qquad (6),$$

and:

$$\frac{\bar{c}}{\bar{c}_o} = \frac{x_o}{x} . \frac{1 - 2.\delta^{*}/b_w}{1 - 2\delta^{*}_o/b_w} \qquad (7).$$

The boundary layer thicknesses are defined here also as in the case of plane flow, so long as they are very small in comparison with the diffusor radius:

$$\delta^{*} = \int_0^{\delta} (1 - c_x/\bar{c}).dy \ ,$$

$$\delta^{**} = \int_0^{\delta} (1 - c_x/\bar{c}).(c_x/\bar{c}).dy.$$

The differential equation (6) has the form:

$$\frac{d\delta^{**}}{dx} = f(x).$$

The solution of this differential equation follows through separating the variables,

$$\frac{d\delta^{**}}{dx} = - \frac{\delta^{**}}{x} + \left(2 + \frac{\delta^{*}}{\delta^{**}}\right) \frac{\delta^{**}}{\bar{c}} \frac{d\bar{c}}{dx} \qquad (8),$$

$$\int_{\delta^{**}_o}^{\delta^{**}} \frac{d\delta^{**}}{\delta^{**}} = - \int_{x_o}^{x} \left[\frac{1}{x} + (2 + H_{12}) \frac{1}{\bar{c}} \frac{d\bar{c}}{dx}\right] dx \qquad (9),$$

where $H_{12} = \delta^{*}/\delta^{**}$ is the form parameter and is considered constant so that the integration of (9) gives by rearranging and solving according to x/x_o,

$$\frac{x}{x_o} = (\delta^{**}/\delta^{**}_o)^{1/B} . \left[\frac{1 - 2.\delta^{*}_o/b_w}{1 - 2.\delta^{*}/b_w}\right]^{A/B} \qquad (10),$$

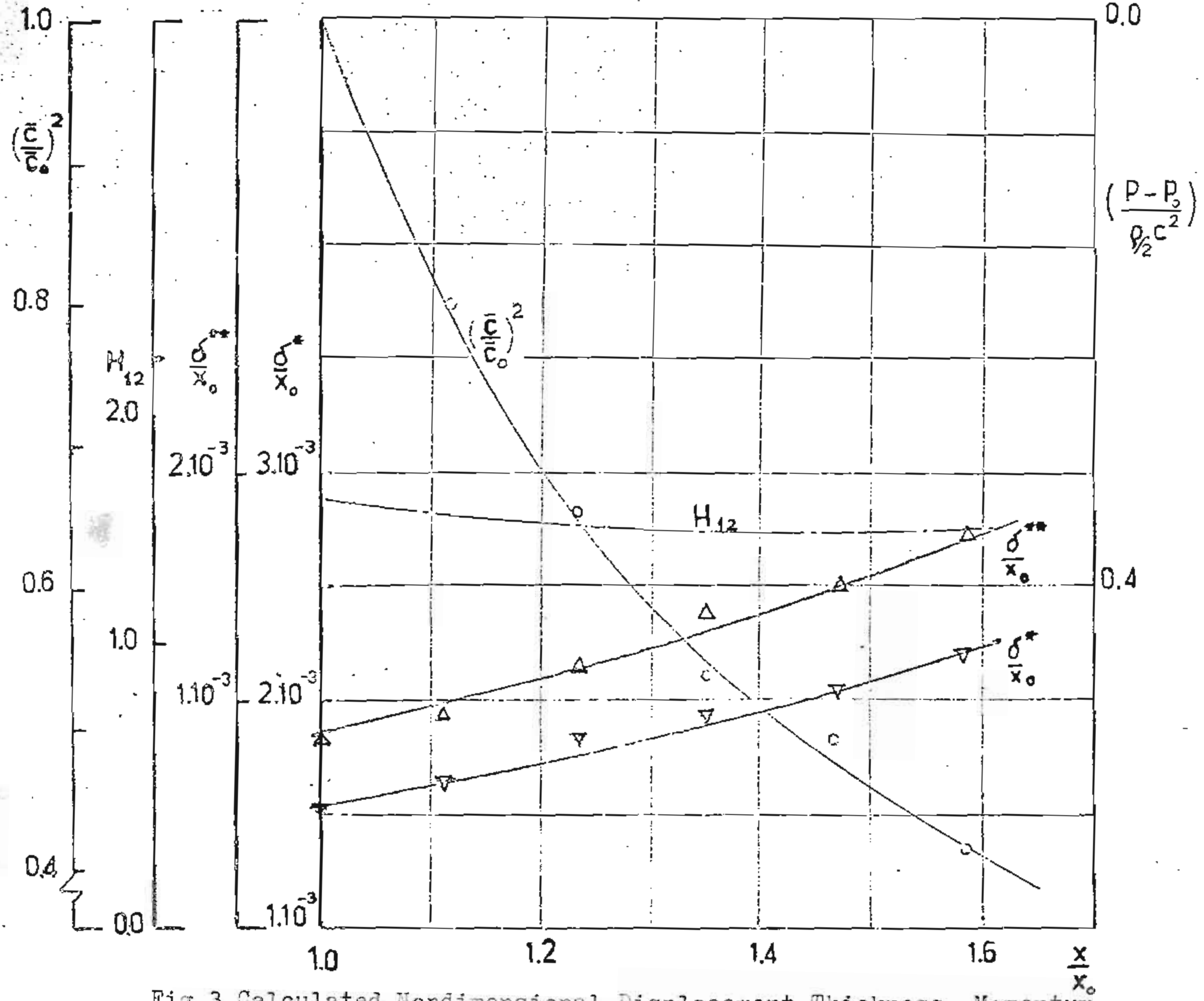

Fig.3 Calculated Nondimensional Displacement Thickness, Momentum Thickness, Form Parameter and Velocity Versus the Nondimensional Distance x/x_o for Radial Diffuser

where $A = (2 + H_{12})$, and
$B = (1 + H_{12})$.

Equation (10) gives the point of separation in radial diffuser if one may estimate the value of the form parameter H_{12}, for a definite diffuser width b_w, from any given velocity profile.

2. Example:

For applying equation (10), the displacement thickness and momentum thickness are calculated and plotted in dimensionless form versus the coordinate x/x_o in Fig. 3. The form parameter H_{12} from these values is also represented in the same figure and it is concluded that its value is approximetely constant.

The constant value of the form parameter emphasizes too that the approximation made when integrating the applied simplified momentum equation (6) has reached value of about 1.42 (for our case) . The Reynolds number for this example was found to have a value of 4.5×10^5. The chosen diffuser width b_w was taken for maximum diffuser efficiency [2] and equal to 20 mm. On this basis, the introduction of these values into equation (10) resulted in the indicies A/B and $1/B$ to be of values 1.41; 0.41 respectively and that separation occurs at $x/x_o \approx 1.48$.

3. Conclusions:

The paper gives the relation between point of separation and the influence of the momentum thickness δ^{**} and the form parameter H_{12} of radial diffusers.

The obtained equation has its validity in determining a criterion for flow separation in radial diffusers.

The applied example shows important point of view, namely that using such a form will avoid a failure in the concept of boundary layer. Thus, the controlling of separation of flow depends upon the relation between form parameter H_{12}; momentum thickness δ^{**}. A great attention is also needed when chosing diffuser width b_w.

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