



**Allowed Tables and Charts: None**

**Answer all the Following Questions**

**Question (1)**

**(50 Marks)**

- A. For 3-dimensional incompressible flow develop the continuity and Navier-Stokes equations in Cartesian coordinates. Reduce the final forms to steady incompressible flow. **(20 Marks)**
- B. Explain how to develop the turbulent Reynolds-stresses tensor matrix in three-dimensional turbulent flow. **(20 Marks)**
- C. The velocity profile in a laminar boundary layer over a smooth flat plate (length L) immersed parallel to the flow stream can be approximated by a fourth degree polynomial velocity distribution as follows:

$$\frac{u}{U_{\infty}} = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2 + d\left(\frac{y}{\delta}\right)^3 + e\left(\frac{y}{\delta}\right)^4$$

- (I) Compute the coefficients a, b, c, d and e. **(5 Marks)**
- (II) Prove the validity of the following relations:  $\delta_1/\delta = 3/10$ ,  $\theta/\delta = 37/315$ ,  $\delta/x = 5.84/\sqrt{\text{Re}_x}$  and  $C_d = 1.37/\sqrt{\text{Re}_L}$ , where  $C_d$  is the drag coefficient.

**(5 Marks)**

**Question (2)**

**(50 Marks)**

- A. Explain with neat sketches the following terms:  
Length scale, Energy cascade, Energy backscatter, Two-point correlation. **(8 Marks)**
- B. For a boundary layer flow, derive the momentum integral equation of von-Kármán, in which the momentum thickness  $\Theta$  and displacement thickness  $\delta_1$  are related to the wall shear stress  $\tau_w$  with the presence of pressure gradient and free-stream velocity  $U_{\infty}$  as the following relation:  $\frac{d\theta}{dx} + \frac{1}{U_{\infty}} \frac{dU_{\infty}}{dx} (2\theta + \delta_1) = \frac{\tau_w}{\rho U_{\infty}^2}$  **(18 Marks)**
- C. Derive the growth of the turbulent boundary layer thickness, the displacement thickness, the momentum thickness and the wall skin friction coefficient of a turbulent flow over a flat plate (without pressure gradient). How do you compute the total drag force of the plate? **(14 Marks)**
- D. Discuss the different regions in the boundary layer. Write the law of the wall, from which how do you derive the Clauser's plot relation? **(10 Marks)**

*Best wishes*

*Assoc. Professor Wageeh El-Askary*