THE CHARACTERISTICS OF LATERAL CRITICAL SPEEDS OF FLEXIBLE ROTORS WITH RIGID SUPPORTS

By

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ABSTRACT:

A simple method of calculation of lateral critical speeds of flexible rotors with rigid supports is presented. The method is mainly based on the former work of Myklestad and Prohl and considers the shaft to be devided into a number of finite elements connected by massless elastic sections. A digital computer program is then prepared to solve the finite equations resulting from the analysis. By means of this program, the critical speeds of flexible rotors can be predicted at different stiffnesses of the rigid supports and the mode shape of the whirling rotor is predicted. An actual case of a centrifugal compressor rotor having different characteristics is considered.

INTRODUCTION:

The high performance of turbomachines, such as turbines and compressors, requires a high speed rotor design. An accurate dynamic analysis is required to determine the lateral critical speeds of such high speed rotor-bearing systems. The standard approach used to evaluate the critical speeds involves the calculation of lateral critical speeds for the undamped rotor-bearing system[1]+. Following this, the evaluation of the mode shape of the whirling rotor is made and the relative deflection of the rotor at the undamped critical speeds is analysed. The next step is the determination of the steady state unbalance response and the stability analysis.

In this stage of study, only the first part of the analysis is considered. The differential equation of motion for flexural vibration of a balanced shaft is analytically solved. The method of solution is a kind of Holzer[2] similar to Mykelstad[3] and Prohl[4] methods. In this analysis, the rotor is considered to be divided into a finite number of sections connected by massless elastic sections. A digital computer program is then prepared for the solution of the differential equation. The input data to the computer program are those of an actual centrifugal compressor having different characteristics.

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+ Number in brackets refer to references at end of paper

THEORETICAL ANALYSIS:

The differential equation of motion for flexural vibrations of a balanced shaft of variable cross-section is

$$\frac{d^2}{dx^2}\left[EI \frac{d^2y}{dx^2} \right] = \rho\, \omega^2 y \qquad (1)$$

where EI is the flexural rigidity of the shaft. The above equation is a fourth order differential equation, the solution of which gives the critical speeds of the rotor. A special computer program is deviced for solving equation(1) based on finite sections of mass points connected by massless shaft sections, each section having a constant moment of inertia.

The common method of analysis used is to express the dynamic configuration in terms of influence numbers resulting in a set of amplitude equations. These equations are then solved by iteration method. This is done by assuming a certain frequency and proceeding successively through the set of equations, four in number, to finally determine an error value. Repeating the same procedure another value of the frequency is used to minimize the error. The method of computation is a kind of Holzer type and similar to Myklestad and Prohl methods.

As the rotor is whirling in a deflected shape, there will be a centrifugal force of each mass equal to $(W\omega^2/g)y$ and a spring force restoring the rotor equal to (ky). The resulting force is then:

$$\left[\frac{W\omega^2}{g} - k \right] y$$

The spring force will be neglected except in bearing stations where k will be the bearing stiffness.

From beam theory,

$$EI \frac{d^2y}{dx^2} = M$$

Thus, equation (1) may be rewritten in the form,

$$\frac{d^2M}{dx^2} = \rho\omega^2 y \qquad (3)$$

Also,

$$\frac{dM}{dx} = V \qquad (4)$$

From the above equations, and based on dynamic equilibrium and geometry, the following relationships can be written,

$$V_n = V_{n-1} + M_{n-1} \quad {}^{2}y_{n-1} \tag{5}$$

$$M_n = M_{n-1} + V_n L_{n-1} \tag{6}$$

$$y_n = \frac{L_{n-1}^2}{3\ EI} M_{n-1} + \frac{L_{n-1}^2}{6\ EI}(M_{n-1} + V_n L_{n-1}) + \theta_{n-1} L_{n-1} + y_{n-1}$$

$$= \frac{L_{n-1}^2}{2\ EI} M_{n-1} + \frac{L_{n-1}^3}{6\ EI} V_n + \theta_{n-1} L_{n-1} + y_{n-1} \tag{7}$$

$$\theta_n = \theta_{n-1} + \frac{L_{n-1}}{EI} M_{n-1} + \frac{L_{n-1}^2}{2\ EI} V_n \tag{8}$$

The above equations, (5) through (8), are derived from the basic beam equations, the quantities represent,

$\frac{L_n^2}{2\ EI}$ Tangential deviation or vertical distance from point on the beam at station (n) to the tangent at station (n-1) due to unit moment at station (n-1).

$\frac{L_n^3}{6\ EI}$ Tangential deviation or vertical distance from point on the beam at station (n) to the tangent at station (n-1) due to unit shear force at station (n-1).

$\frac{L_n}{EI}$ Slope of the beam at station (n) relative to the tangent at station (n-1) due to unit moment at (n-1).

$\frac{L_n^2}{2\ EI}$ Slope of the beam at station (n) relative to the tangent at station (n-1) due to unit shear force at station (n-1)

The solution of equations (5 through 8) gives the critical speed of the whirling shaft provided selection of proper boundary conditions is made. Such boundary conditions depend on the end conditions of the shaft and the locations of the bearings.

METHOD OF SOLUTION:

The critical speed is defined as the speed at which the force and moment at both ends of the shaft are zero. In general, four boundary conditions defining the quantities V, M, Θ and y must be available for the solution of equations (5 through 8). Two of these quantities, V and M, are defined at the left end where they are equal zero (overhanged end). The deflection y and the slope Θ are considered unknown. The solution can be carried out into two parts giving the values of unity and zero to the deflection y and the slope Θ, alternatively, as follows:

Part I	Part II
$M_1 = 0$	$M_1 = 0$
$V_1 = 0$	$V_1 = 0$
$y_1 = 1$	$y_1 = 0$
$\Theta_1 = 0$	$\Theta_1 = 1$

At the other end of the shaft (also overhanged end) the shear force and bending moment should also equal zero, so that,

$$V_{right} = 0 \quad \text{and} \quad M_{right} = 0$$

This will result in the following set of equations:

$$(V_{right}\ \Theta_1)_I + (V_{right} y_1)_{II} = 0 \qquad (9)$$

$$(M_{right}\ \Theta_1)_I + (M_{right} y_1)_{II} = 0 \qquad (10)$$

where the subscripts I and II denote boundary conditions of parts I and II of the solution. A condition existing when V_{right} and M_{right} are both zero is that the determinant,

$$D = (M_{right}\ V_{right})_I - (M_{right}\ V_{right})_{II} = 0 \qquad (11)$$

Plotting a curve between the values of the determinant D and the speed of the rotor, the intersections of the curve with the axis representing the rotor speed give the critical speeds. The computer program is prepared to supply the mode shape at the corresponding critical speeds.

RESULTS:

An example of the shape mode of the rotor at the corresponding critical speeds is shown in figures (1) and (2). The presented example is the case of a centrifugal compressor, the length of its rotor is 2.71 m., the bearing span is 2.34 m. and its weight is 634.4 kg. The detail of the rotor dimensions is presented in Table 1. It should be pointed out that the data which appear in Table 1 are in English system of units similar to most data existing in the industrial field. However, the computer program is prepared to handle data in both English and French systems of units. The actual compressor used in this analysis is a multistages compressor having three impellers and is coupled with the driving unit (steam turbine) by a gear type coupling. In this analysis, the common standard of decoupling the compressor and the driver in the coupling was adopted by adding half the weight of the coupling to the weight of the rotor as an overhanged weight. The influence of the driver on the lateral vibration of the compressor is neglected. A computer run was performed with an arbitrary value for the stiffness of 9920 kg/cm.

DISCUSSION:

The use of digital computers has facilitated the solution of vibration problems. In this analysis, a computer program was written in Fortran IV language to solve the problem of lateral vibrations of flexible rotors as represented by equations (5 - 8). The disadvantage of this method is the need to a relatively high capacity computer. Since the preliminary estimation of the required core was around 112 K, an overlap technique was adopted to handle the solution.

From figure (1), it is clear that the influence of the coupling on the mode shape is major. Consequently, any misalignment or unbalance in the coupling will be a direct reason for the increase in the level of vibration. The rocking mode shape of the rotor at the second critical speed, figure (1), is due to the low magnitude of the stiffness. In practice, the stiffness of the bearing should be determined first and then a value around the actual stiffness is introduced to the program.

The reason of the low magnitude of the critical speed is the exagerated added weight to the coupling to the study of the influence of a possible rotating unbalance in coupling. Also, the shaft contained an undercut next to the thrust bearing to study the influence of the thrust bearing weight. The reduction in shaft diameter due to the undercut reduces the spring force and, consequently, decreasing the critical speeds.

CONCLUSION:

The described method of solution of beam equations presents a simple method of solution. The results provide directly the mode shape at the corresponding critical speeds. In the field of turbomachinery, the method is a practical tool for the designer to proceed with the rotor response study. For the field engineer, the method facilitates the search for the origins of high vibration levels in operation.

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NOMENCLATURE:

Latin Symbols

- E Modulus of elasticity
- g Gravity acceleration
- I Moment of inertia of the cross-section
- L Length of the finite sections
- M Bending moment
- V Shear force
- W Weight of section
- x Coordinate in longitudinal direction
- y Deflection

Greek Symbols

- θ Slope of the elastic line of the deflected shaft
- ρ Weight of the rotor per unit length
- ω Angular velocity

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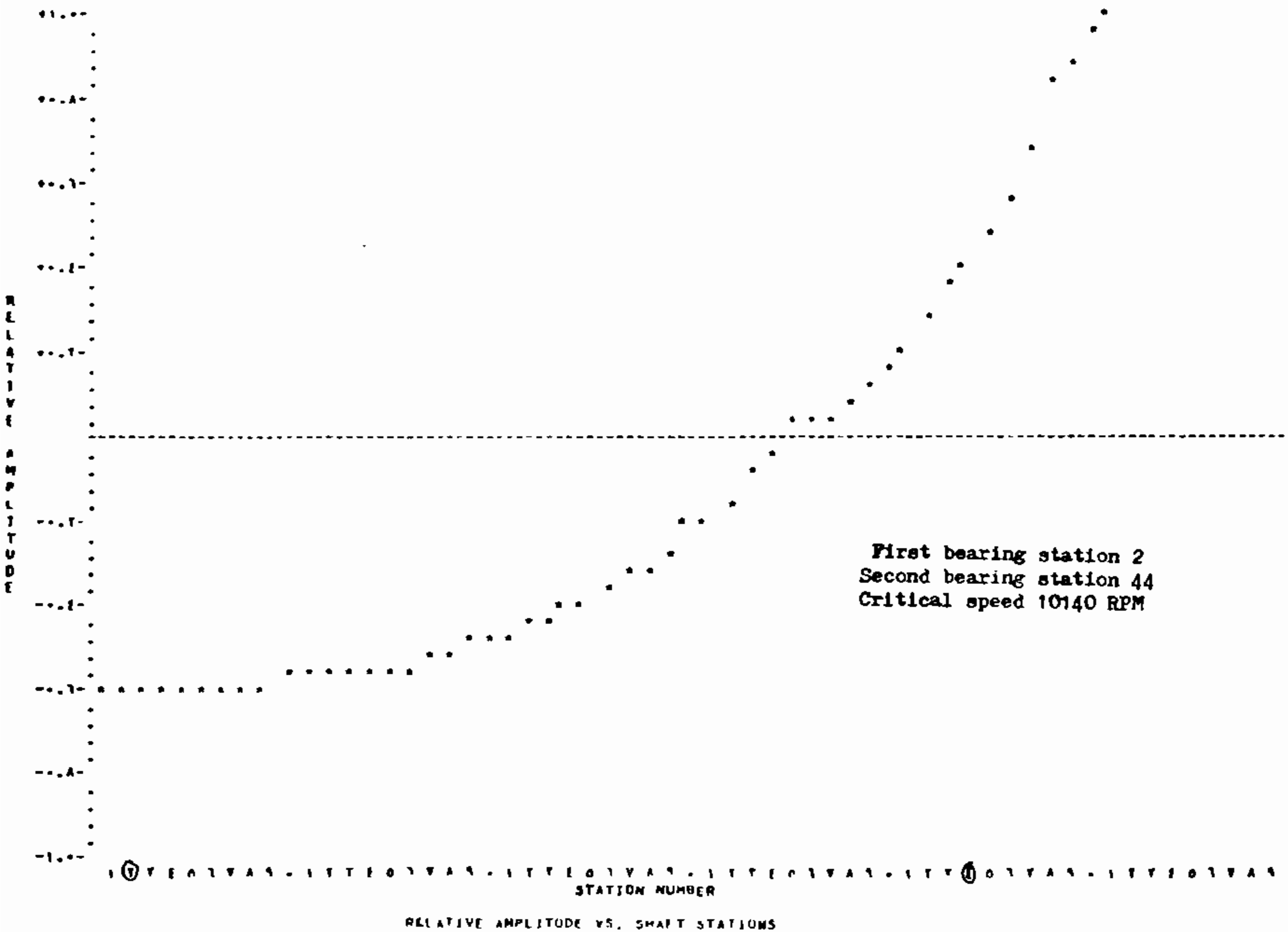

Fig.1. The mode shape of the rotor at the second critical speed(rocking mode).

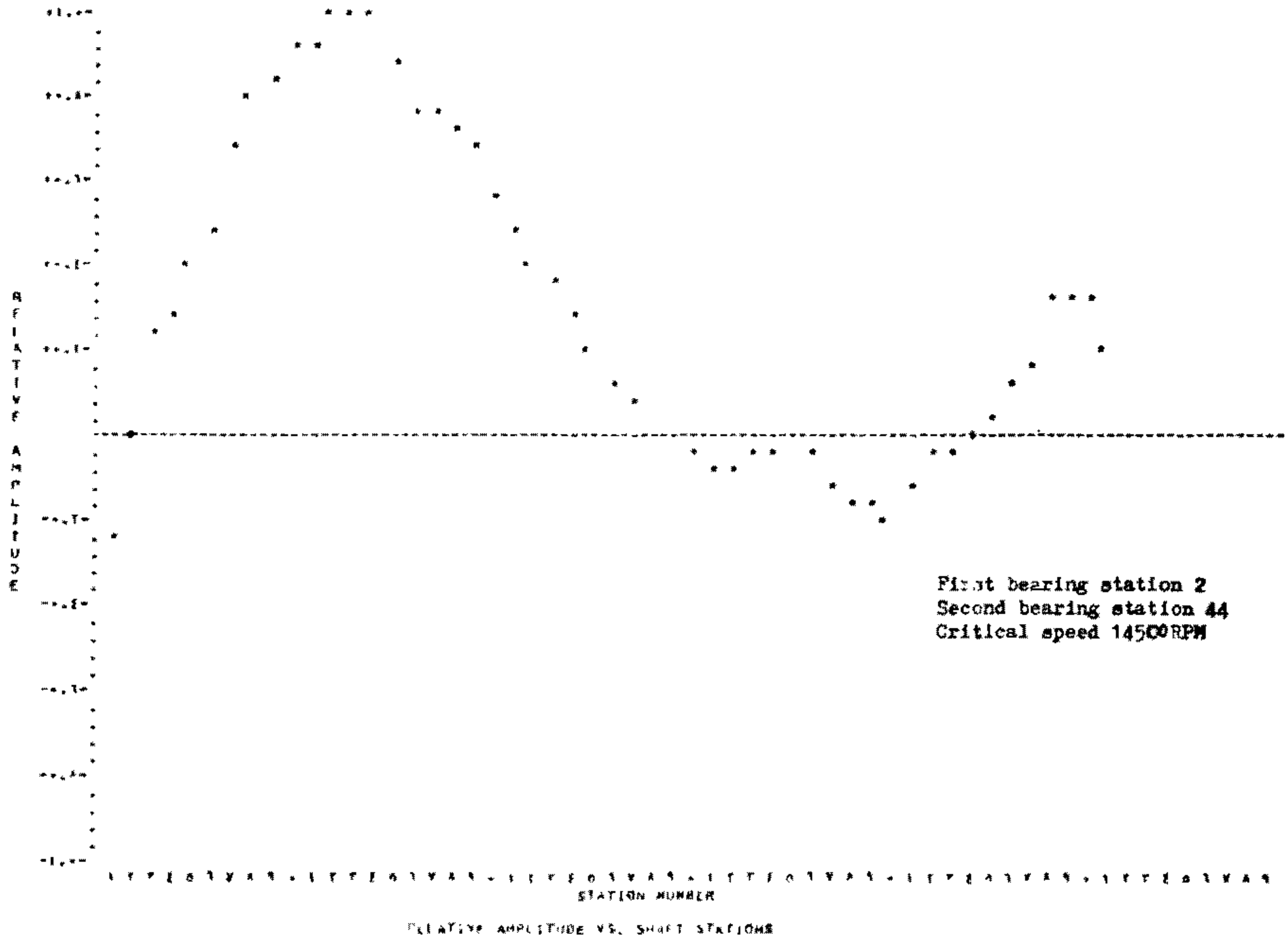

Fig.2. The mode shape of the rotor at the thired critical speed.

Table (٢) The rotor characteristics

	W lb	L in	D_o in	D_i in	I in^4	I^* in^4	E psi	ρ lb/in³	EI lb.in²
١	١.٢٣٠	٢.٥٠٠	٥.٥٠٠	٠.٠٠٠	٤٤.٩٨	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	١٣٤٩.٤٢٥
٢	١٢.١٣٩	١.٢٨٢	٥.٥٠٠	٠.٠٠٠	٤٤.٩٨	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	١٣٤٩.٤٢٥
٣	٧.١٩٤	٠.٢٩٤	٥.٣٥٠	٠.٠٠٠	٣٩.٤٤	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	١١٨٣.٢٦٠
٤	٥.٢٨١	١.٥٦٢	٥.٤٦٠	٠.٠٠٠	٤٦.٩٧	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	١٤٠٩.٢١٧
٥	٥٩.٣٣٤	١.٥٦٢	٥.٥٦٠	٠.٠٠٠	٤٦.٩٧	١١٥٧.٠٠٤	٣٠.٠٠	٠.٢٨٣	١٤٠٩.٢١٧
٦	١٢.٦٨٤	٢.٥٩٤	٥.٩٣٧	٠.٠٠٠	٦١.٠٣	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	١٨٣٢.٤١٣
٧	١١.٩٠١	٢.٥٩٣	٣.٩٣٧	٠.٠٠٠	١١.٨٤	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٣٥٤.٢٢٨
٨	١١.٤٠٩	٢.٠١١	٦.٥٠٠	٠.٠٠٠	٨٧.٦٤	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٢٦٢٩.٢٦٠
٩	١٥.٥٢٨	٢.٠١١	٦.٥٠٠	٠.٠٠٠	٨٧.٦٤	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٢٦٢٩.٢١٠
١٠	١٥.٥٢٨	٢.٠١١	٦.٥٠٠	٠.٠٠٠	٨٧.٦٤	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٢٦٢٩.٢١٠
١١	٢١.٩٤٢	٢.٢٤٠	٦.٨١٠	٠.٠٠٠	١٠٥.٥٣	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٣١٧١.٥٥٤
١٢	٢٤.٣٢٩	١.٢٧٥	٦.٩٤٠	٠.٠٠٠	١١٤.٠٣	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٣٤٢٠.٧٤٨
١٣	٣٠.٩٢٤	١.٢٧٥	٦.٩٣٧	٠.٠٠٠	١١٣.٨٣	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٣٤١٤.٨٤٣
١٤	١٢.٤٨٥	٠.٦٨٧	٦.٩٤٠	٠.٠٠٠	١١٤.٠٦	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٣٤٢٠.٧٤٨
١٥	٤.٦٨٩	٠.٤٠٦	٦.٩٨٧	٠.٠٠٠	٩٨.١٩	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٢٩٤٨.٥٥٣
١٦	٣٥.١١١	٢.٠٠٠	٧.٠٠٠	٠.٠٠٠	١١٨.٠٢	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٣٥٤٠.٥٩٣
١٧	٢١.٩٠١	٢.٠٠٠	٧.٠٠٠	٠.٠٠٠	١١٨.٠٢	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٣٥٤٠.٥٩٣
١٨	٣٦.٩٠١	٢.٠٠٠	٧.٠٠٠	٠.٠٠٠	١١٨.٠٢	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٣٥٤٠.٥٩٣
١٩	٣١.٤١٧	٢.٠٠٠	٧.٠٠٠	٠.٠٠٠	١١٨.٠٢	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٣٥٤٠.٥٩٣
٢٠	١٧.٥١٢	١.٩٠٦	٧.٠٠٠	٠.٠٠٠	١١٨.٠٢	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٣٥٤٠.٥٩٣
٢١	١٩.٨٨٨	٢.٥٢٠	٧.٠٠٠	٠.٠٠٠	١١٨.٠٢	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٣٥٤٠.٥٩٣
٢٢	٣٢.٦٨٦	٢.٥٢٠	٧.٠٠٠	٠.٠٠٠	١١٨.٠٢	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٣٥٤٠.٥٩٣
٢٣	١٢.٦٨٦	٢.٥٣٠	٧.٠٠٠	٠.٠٠٠	١١٨.٠٢	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٣٥٤٠.٥٩٣
٢٤	١١٥.٨٨٩	١.٩٠٦	٧.٠٠٠	٠.٠٠٠	١١٨.٠٢	٢٠٦٦٠.٠١٠	٣٠.٠٠	٠.٢٨٣	٣٥٤٠.٥٩٣
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٤٥	١٣.٤٤٧	٢.٥٠٠	٥.٥٠٠	٠.٠٠٠	٤٤.٩٨	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	١٣٤٩.٤٢٥
٤٦	١٠.٩٧٤	٢.٦٨٧	٤.٥٠٠	٠.٠٠٠	٢٠.١٣	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٦٠٣.٩٩٠
٤٧	٢٤.٢٩٤	٢.٣٦٠	٧.٨٧٠	٠.٠٠٠	١٨٨.٥٦	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٥٦٥٦.٩٢٨
٤٨	٢٨.٣١٤	١.٥٠٠	١٤.٢٥٠	٠.٠٠٠	٢٠٢٦.٨٢	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٦٠٨٠٥.٦٣٠
٤٩	٤٤.٠٠٥	١.٥٠٠	١٢.٧٥٠	١٠.٠٠٠	١٨٢٥.٤٠	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٥٥٠٥٢.٠١٠
٥٠	١٣.٥٧٦	٠.٧٥٠	١١.٠٠٠	[illegible]	٤٢٨.١٢	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٣٨٤٣.٧١٢
٥١	١.٤٤٩	٠.٠٠٠	٠.٠٠٠	٠.٠٠٠	٠.٠٠	٠.٠٠٠	٣٠.٠٠	٠.٢٨٣	٠.٠٠٠
	١١٩٤.١٥٥	١٠٧.٣٤٠							

BEARING STATIONS ٣ ٤٤

BEARING REACTION SUCTION END ٥٤٧.١٢٣٩

BEARING REACTION DISCHARGE END ٧٩٧.٢٢٩١