



Answer the following problems... assume any missing data

**Question 1**

- Define each of the following items: Global solution- Hessian matrix- Positive definite- Positive semi definite
- State some methods for constrained and unconstrained optimization problems.
- Discuss briefly the concepts of Dynamic programming.
- Determine the maximum and minimum values of the function

$$f(x) = 24x^5 - 45x^4 + 10x^3 + 4$$

- Find the extreme points for the following multivariable function

$$f(x_1, x_2) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6$$

- Use dynamic programming to solve the following problem:

$$\text{Min } Z = x_1^2 + x_2^2 + x_3^2$$

$$\text{subject to : } x_1 + x_2 + x_3 = 15, \quad x_1, x_2, x_3 \geq 0.$$

- Use the Lagrange multipliers to find the maximum of the function

$$\text{Max } f(x) = 2x_1 + x_2 + 10$$

$$\text{subject to : } g(x) = x_1 + 2x_2^2 = 3$$

**Question 2**

- For the payoff matrix:  $M = \begin{bmatrix} 5 & -3 & 3 & 4 \\ -4 & 5 & 4 & 5 \\ 4 & -4 & -3 & 3 \end{bmatrix}$ , use the game theory principle to find the

optimal strategies of the two players

- Construct the dual problem for the following problem

$$\text{Max } Z = 3x_1 + 5x_2$$

Sub.to.

$$2x_1 + 6x_2 \leq 50$$

$$3x_1 + 2x_2 \leq 35$$

$$5x_1 - 3x_2 \leq 10$$

$$x_2 \leq 20$$

$$x_1, x_2 \geq 0$$

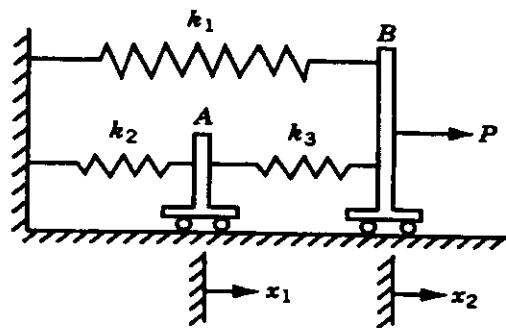
c) Optimize the following LP problem by Big-M method

$$\text{Min } Z = 2x_1 + 3x_2$$

$$\text{Sub to: } \frac{1}{2}x_1 + \frac{1}{4}x_2 \leq 4, \quad x_1 + 3x_2 \geq 20, \quad x_1 + x_2 = 20, \quad x_1, x_2 \geq 0.$$

d) A small machine tool manufacturing company entered into a contract to supply 80 drilling machines at the end of the first month and 120 at the end of the second month. The unit cost of manufacturing a drilling machine in any month is given by  $\$(50x + 0.2x^2)$ , where  $x$  denotes the number of drilling machines manufactured in that month. If the company manufactures more units than needed in the first month, there is an inventory carrying cost of \$8 for each unit carried to the next month. Find the number of drilling machines to be manufactured in each month to minimize the total cost. Assume that the company has enough facilities to manufacture up to 200 drilling machines per month and that there is no initial inventory. Solve the problem as a final value problem.

e) For the following figure, shows two frictionless rigid bodies (carts) A and B connected by three linear elastic springs having spring constants  $k_1$ ,  $k_2$  and  $k_3$ . The springs are at their natural positions when the applied force  $P$  is zero. Find the displacements  $x_1$  and  $x_2$  under the force  $P$  by using the principle of minimum potential energy.



*With my best wishes*

This exam measures the following ILOs						
Question Number	Q1(1-a)	Q1(1-b)	Q1(1-c), Q3(1-g)	Q1(1-e)	Q3(1-f)	Q1(1-h)
	Q2(2-a)	Q4(1-d)	Q2(2-c)	Q2(2-b)	Q4(1-e)	Q2(2-d)
Skills	Knowledge & understanding Skills		Intellectual Skills		Professional Skills	