A REVIEW OF CONVENTIONAL FRICTION FORMULAE

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مراجعة لمعادلات الاحتكاك التقليدية

الخلاصة - تهدف هذه الدراسة الى اختيار أنسب المعادلات التصميمية لتطبيقها فى مجال تصميم شبكات قنوات الرى فى ظل الظروف المحلية لجمهورية مصر العربية . وقد تمت دراسة هذا الموضوع بعمل نموذج رياضى ومعايرته لترعة المنصورية شرق الدلتا وذلك بغرض عمل دراسة مقارنة بين معادلات الاحتكاك واختيار أنسبها . وقد خلص البحث الى امكانية استخدام معامل ماننج بدرجة عالية من الدقة عند الاخذ فى الاعتبار أن معامل ماننج يتراوح ما بين ٢٤٠ر الى ٢٦٠ر كما أن السريان فى ترعة المنصورية من النوع المضطرب الناعم وأن معادلة كوليبروك هى أنسب معادلة يمكن تطبيقها فى الظروف المحلية بعد تعديل معاملها .

ABSTRACT:

The purpose of the present research is to choose a suitable resistance formula for the local conditions of unlined open channels. This was achieved by using a mathematical model for El-Mansoura canal, east of delta,Fig.(1). To construct the numerical model field data were collected and analysed.

Manning's equation could be used for design purposes on the condition that Manning's coefficient varies between 0.024 and 0.026. Cole-brook's formula for smooth turbulent flow gave the best accurate simulated depth and discharge values by using a modified factor.

INTRODUCTION:

The continued efforts by engineers and scientists have produced hundred of empirical flow resistance formulae. The formula of Chezy with coefficient derived from kutter's and Powell's formulae is currently in use in many countries.

Manning's formula is also equally widely used in other countries. Canal design practices of proven success vary from country to country depending on the conditions particularly soil formation, sediment transport characteristics, operational needs and desired standard of maintenance. It would therefore, not to be advisable for any one to follow the design practices of another country regardless the change required to meet the local conditions.

A mathematical model was costructed to choose the most convenient formula to be applied for irrigation canals under new local conditions specially after the erection of Aswan High Dam.

Two reaches of El Mansoura canal, each of 18.0 Kms in length, were selected to collect the necessary data for the mathematical model. The first reach starts from Meit Ghamr to Sanayt regulator and the second reach from Sanayt regulator to Bahr Tanah canal, Fig.(2).

FRICTION FORMULAE:

Chezy's formula (1768)

$$V = C\, R^{1/2}\, S^{1/2} \qquad (1)$$

in which:

V = mean velocity;
R = hydraulic radius;

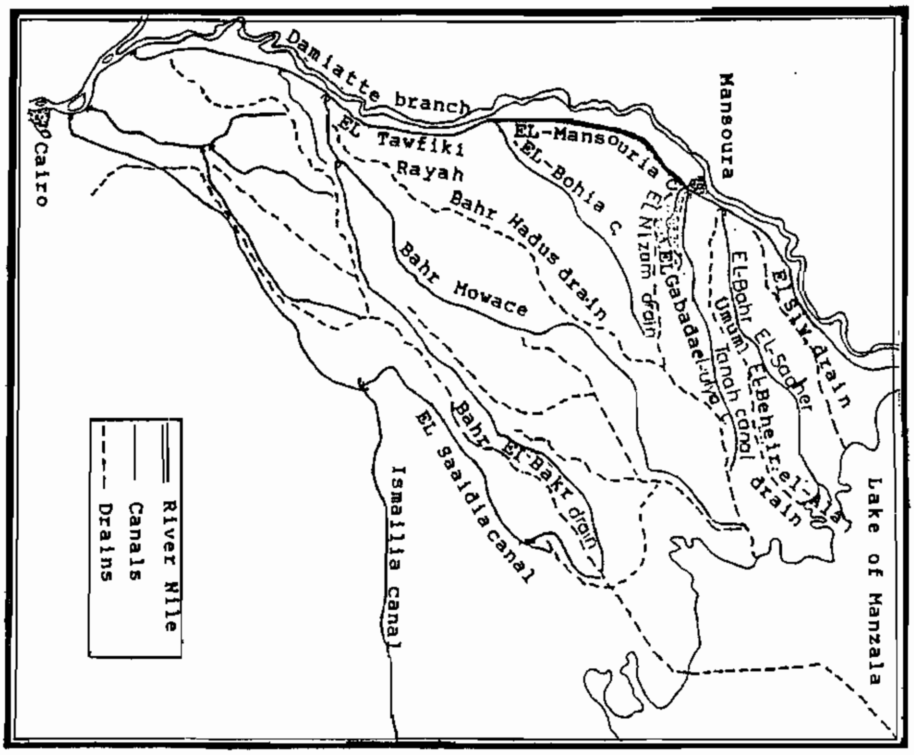


Fig. (1) GENERAL LAYOUT

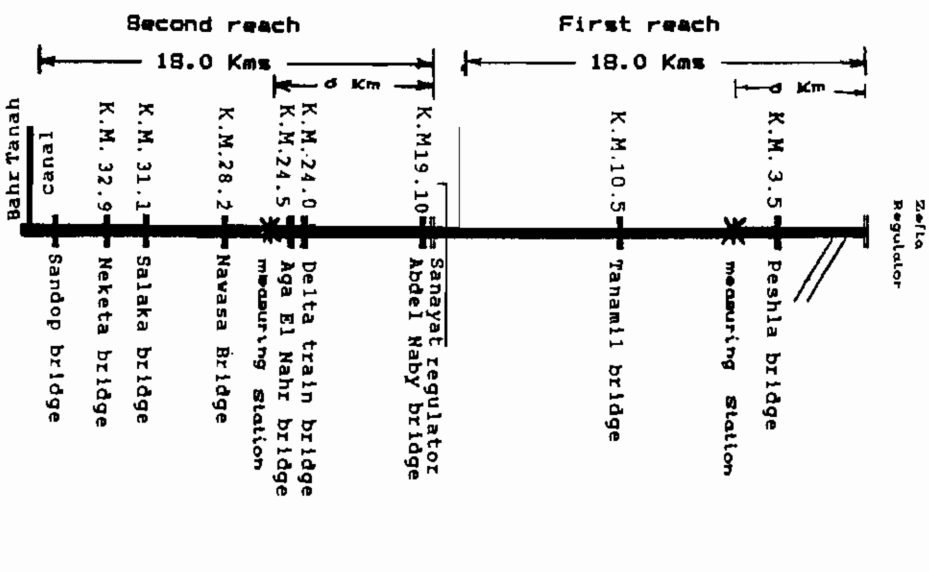


Fig. (2) The Two Reaches Under Study

C = Chezy's coefficient ; and
S = slope of the channel.

Ganguillet and Kutter (1869) gave the following formula for Chazy's coefficient (C):

$$C = \frac{23 + \frac{1}{N} + \frac{0.00155}{S}}{1 + \left(23 + \frac{0.00155}{S}\right)\frac{N}{\sqrt{R}}} \qquad (2)$$

where, N = Kutter's coefficient

Bazin (1897) proposed the following formula for (C)

$$C = \frac{157.6}{1.81 + \frac{m}{\sqrt{R}}} \qquad (3)$$

where, m = Bazin's coefficient

Manning's equation (1889)

$$V = \frac{1}{n} R^{2/3} S^{1/2} \qquad (4)$$

where, n = Manning's coefficient

The relationship between Chezy's coefficient (C) and Manning's coefficient (n) is given by:

$$C = \frac{1}{n} R^{1/6} \qquad (5)$$

The relationship between the average sediment diameter (d) in ft. and Manning's coefficient (n) for sand streams is given by (11):

$$n = 0.034\ d^{1/6} \qquad (6)$$

Cole-brook and White (1938) gave the familiar transition formula (17):

$$\frac{1}{\sqrt{f}} = 2.0 \log\left(\frac{K_s}{14.82\ R} + \frac{2.52}{R_e\sqrt{f}}\right) \qquad (7)$$

in which:

f = Darcy-Weisbach friction factor;
K_s average height of roughness;
R_e = Reynold's number; and
R = hydraulic radius.

Zegzhda (1989) made one of the first attempts to apply this concept to open channels, since then extensive studies have been performed by many investigators.

The resistance to flow in open channel with fully developed roughness:

$$\frac{1}{\sqrt{f}} = c \log\left(\frac{a\ R}{K_s}\right) \qquad (8)$$

for $R_* = \frac{K_s\ V_*}{\nu} > 70$

where:

R_* = friction Reynolds number;

V_* = shear velocity; and

ν = kinematic viscosity

The resistance to smooth turbulent flow is given by:

$$\frac{1}{\sqrt{f}} = c \log \left(\frac{R_* \sqrt{f}}{b} \right) \qquad (9)$$

for

$$R_* = \frac{K_s V_*}{\nu} < 5$$

For transition turbulent flow (16):

$$\frac{1}{\sqrt{f}} = c \log \left(\frac{a R}{K_s} + \frac{R_* \sqrt{f}}{b} \right) \qquad (10)$$

where a,b and c are constants.

In general (f) is afunction of relative roughness. Reynolds number. Shape of cross section and the parameters a,b and c.

For sand bed channels, $K_s = d_{65}$ as suggested by Einstein (1950), $K_s = d_{90}$ as given by Meyer peter (1948) and $K_s = d_{85}$ as suggested by Simon and Richardson (1966).

In 1950 Powell published his logarithmic formula to calculate Chezy's coefficient for artificial channels in the form (12) :

$$C = -42 \log \left(\frac{C}{4 R_e} + \frac{\varepsilon}{R} \right) \qquad (11)$$

in which C is a measure of the channel roughness.

Powell's formula for fully rough flow is given by:

$$C = 42 \log \left(\frac{R}{\varepsilon} \right) \qquad (12)$$

and for hydraulically smooth flow is:

$$C = 42 \log \left(\frac{4 R_e}{\varepsilon} \right) \qquad (13)$$

The relationshilp between Darcy Weisbach coefficient (f) and Chezy's (C) is given by:

$$C = \sqrt{\frac{8 g}{f}} \qquad (14)$$

where: g = acceleration due to gravity.

Composite Roughness:

In most natural channels the roughness changes along the channel perimeter due to the difference of flow duration, therefore it is necessary to calculate an equivalent roughness coefficient (n_e) for the entire wetted perimeter.

Different methods for obtaining the equivalent roughness coefficient have been proposed (8).

Lotter (1933) assumed that the total discharge of the section is equal to the sum of subsection discharge, he derived the following formula:

$$n_e = \frac{P R^{5/3}}{\sum_{i=1}^{N} \frac{P_i R_i^{5/3}}{n_i}} \qquad (15)$$

in which;

n_e = equivalent Manning's coefficient;
P = wetted perimeter of the complete section;
R = hydraulic radius of the complete section;
R_i, P_i = mean hydraulic radius and wetted perimeter of the ith subsection respectively; and
N = number of subsections.

Horton and Einstein assumed that each of the subdivision of the flow area has the same average velocity of the total section:

$$n_e = \left[\frac{\sum_{i=1}^{N} P_i \ n_i^{3/2}}{P} \right]^{2/3} \qquad (16)$$

Pavlovski, Einstein and Banks assumed that the total force resisting the flow is equal to the forces developed in the subdivided area:

$$n_e = \left[\frac{\sum_{i=1}^{N} P_i \ n_i^{2}}{P} \right]^{1/2} \qquad (17)$$

For alluvial channels the resistance due to bed forms must be added to the. resistance due to grain roughness. The evaluation of the resistance due to bed forms is complicated. Mostafa and McDermid expressed Manning's equation for alluvial channels, in dimensionless form as follows (6) :

$$V = \frac{\sqrt{g}}{C_M \ \varepsilon^{1/6}} R^{2/3} \ S^{1/2} \qquad (18)$$

in which;

C_M= dimensionless Manning coefficient which is given as a function of froude number and the ratio d_{50}/δ , where δ is the thickness of sublayer (6).

ε = d_{65} as suggested by Einstein, or d_{90} by Meyer peter or d_{85} as given by Simon and Richardson.

Based on the analysis of flume data Liu and Hwang introduced the following equation to calculate the mean velocity in straight alluvial channels:

$$V = C_a \ R^x \ S^y \qquad (19)$$

in which C , x and y are given by charts as a function of bed forms and sediment size (13).

MATHEMATICAL MODEL

Mathematical model for an open channel is the simulation of the flow conditions depending on the formulation and solution of mathematical relationships expressing the known principles of hydraulics. The implicit numerical scheme was chosen, among the other numerical techniques, for solving the governing equations of motion. Several investigators have devised logarithms for the solution of the partial differential equations of unsteady flow (1,3,4,5,14) by using the implicit method. This method is found to be stable for large time steps. Also it is foundto be fast and suitable for long duration and long reaches with complex geometry.

The most popular implicit schemes are Priessmann scheme Abbott-Ionecue scheme and Verwey's variant of the Preissmann scheme. In the Preissmann scheme the newton iteration method is used for the solution of the system of the nonlinear equations by reducing them to a system of linear equations. The convergence of this method depends mainly on the choice of the intial values of the unknowns at the higher time level. The closer the trial values to true values, the faster is the convergence between the measured and calculated values, but if the trial values are chosen in an arbitrary manner the system may fail to converge.

In the Abbott-Ionescue scheme, the discharge or velocity and the water depth are computed at different grid points which causes practical difficulties for applying the boundary conditions. Also it is necessary to interpolate between different computational points, since the discharge and water depth are computed at different grid points which causes practical difficulties for applying the boundary conditions. Also it is necessary to interpolate between different computational points, since the discharge and water depth are not known at the same points, which affect the accuracy of this scheme.

In the Verwey's variant of the Priessmann scheme, the discharge and water depth are computed directly at the same grid points and for this reason there is no need to interpolate between points as in the above mentioned scheme. Also there is no problem of convergence as in the Priessmann scheme (9).

For the previous reasons, the Verwey's variant of the Priessmann implicit scheme was chosen for the simulation of unsteady flow in the present research.

Preissmann Scheme:

Consider the governing equations:

$$\frac{\partial y}{\partial t} + \frac{1}{b}\frac{\partial Q}{\partial X} = 0 \qquad (20)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial X}\left(\frac{Q^2}{A}\right) + g\,A\,\frac{\partial y}{\partial X} + g\,A\,\frac{Q\,|Q|}{K^2} - g\,A\,S_o = 0 \qquad (21)$$

The application of the Preissmann scheme to the derivatives in equation (20) and equation (21) yields,

$$\frac{\partial y}{\partial t} = \frac{y_{j+1}^{n+1} - y_{j+1}^{n}}{2\,\Delta t} + \frac{y_j^{n+1} - y_j^{n}}{2\,\Delta t} \qquad (22)$$

$$\frac{\partial Q}{\partial X} = \theta\,\frac{Q_{j+1}^{n+1} - Q_j^{n+1}}{\Delta X} + (1-\theta)\,\frac{Q_{j+1}^{n} - Q_j^{n}}{\Delta X} \qquad (23)$$

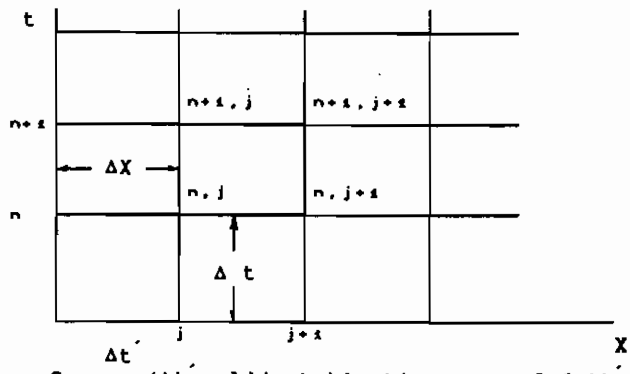
$$\frac{\partial (Q^2/A)}{\partial X} = \frac{\theta}{\Delta X}\left[\frac{(Q_{j+1}^{n+1})^2}{A_{j+1}^{n+1}} - \frac{(Q_j^{n+1})^2}{A_j^{n+1}}\right] + \frac{(1-\theta)}{\Delta X}\left[\frac{(Q_{j+1}^{n})^2}{A_{j+1}^{n}} - \frac{(Q_j^{n})^2}{A_j^{n}}\right] \qquad (24)$$

$$\frac{\partial y}{\partial X} = \theta\,\frac{y_{j+1}^{n+1} - y_j^{n+1}}{\Delta X} + (1-\theta)\,\frac{y_{j+1}^{n} - y_j^{n}}{\Delta X} \qquad (25)$$

where: $\Delta X = X_{j+1} - X_j$

The coefficients of eqn.(20) and eqn.(21)are represented according to Priessman as:

$$f(X,t) = \frac{\theta}{2}\left(f_{j+1}^{n+1} + f_j^{n+1}\right) + \frac{(1-\theta)}{2}\left(f_{j+1}^{n} + f_j^{n}\right)$$



$\theta = \frac{\Delta t'}{\Delta t}$ ($\Delta t'$ = Adjustable time step, $0 \le \Delta t' \le \Delta t$
And Δt = Time step)

Fig. (3) Computational grid for Preissman scheme.

In the above expression, all the variable with superscripts (j) are known and all the variables with superscripts (j+1) are the unknowns. Equations (3.7) constitute a system of two nonlinear algebric equations in four unknowns. As there are (N) points on the row (j+1), there are (N-1) rectangular grids and (N-1) cells in the channel. Thus there are 2(N-1) equations for the evaluation of (2N) unknowns. Two boundary conditions provide the necessary two additional equations to close the system. Different approaches are used to obtain the solution of 2(N -1) nonlinear equations. To find the value of unknowns at the time level (n+1), Newton Raphson iteration method may be used to provide a set of linear equations to be solved simultaneously.

Verwwoy's Variant of the Preissmann scheme:

A scheme of the Preissmann type was derived by Verwey (13), who used a different approximation for some terms, as follows:

$$\frac{\partial}{\partial X}\left(\frac{Q^2}{A}\right) = \frac{1}{\Delta X}\left[\frac{Q^n_{j+1}\,Q^{n+1}_{j+1}}{A^{n+1/2}_{j+1}} - \frac{Q^n_j\,Q^{n+1}_j}{A^{n+1/2}_j}\right] \qquad (26)$$

$$\frac{Q\,|Q|}{K^2} = \frac{1}{2}\left[\frac{|Q^n_j|\,Q^{n+1}_j}{(K^{n+1/2}_j)^2} - \frac{|Q^n_{j+1}|\,Q^{n+1}_{j+1}}{(K^{n+1/2}_{j+1})^2}\right] \qquad (27)$$

$$b = \frac{b^{n+1/2}_{j+1} + b^{n+1/2}_j}{2} \;;\; A = \frac{A^{n+1/2}_{j+1} + A^{n+1/2}_j}{2} \qquad (28)$$

The superscripts (n+1/2) means that the function is computed between two time levels (n) Δt and (n+1) Δt. Substituting these approximations into equations (20) and (21) leads to:

$$\left[\frac{b_{j+1}^{n+1/2}+b_j^{n+1/2}}{2}\right]\left[\frac{Y_{j+1}^{n+1}-Y_{j+1}^{n}}{2\,\Delta t}+\frac{Y_j^{n+1}-Y_j^{n}}{2\,\Delta t}\right]+$$

$$\frac{1}{2}\left[\frac{Q_{j+1}^{n+1}-Q_j^{n+1}}{\Delta X}+\frac{Q_{j+1}^{n}-Q_j^{n}}{\Delta X}\right]=0 \qquad (29)$$

$$\left[\frac{Q_{j+1}^{n+1}-Q_{j+1}^{n}}{2\,\Delta t}+\frac{Q_j^{n+1}-Q_j^{n}}{2\,\Delta t}\right]+\frac{1}{\Delta X}\left[\frac{Q_{j+1}^{n}\,Q_{j+1}^{n+1}}{A_{j+1}^{n+1/2}}-\frac{Q_j^{n}\,Q_j^{n+1}}{A_j^{n+1/2}}\right]+$$

$$g\left[\frac{A_{j+1}^{n+1/2}+A_j^{n+1/2}}{2}\right]\left[\frac{Y_{j+1}^{n+1}-Y_j^{n+1}}{\Delta X}+\frac{Y_{j+1}^{n}-Y_j^{n}}{\Delta X}\right]+$$

$$g\left[\frac{A_{j+1}^{n+1/2}+A_j^{n+1/2}}{2}\right]\frac{1}{2}\left[\frac{|Q_j^{n}|\,Q_j^{n+1}}{(K_j^{n+1/2})^2}-\frac{|Q_{j+1}^{n}|\,Q_{j+1}^{n+1}}{(K_{j+1}^{n+1/2})^2}\right]-$$

$$g\left[\frac{A_{j+1}^{n+1/2}+A_j^{n+1/2}}{2}\right]S_o=0 \qquad (30)$$

If the coefficients of eqn. (29) and eqn. (30) with superscript (n+1) are considered as known functions of flow variables computed at time level (n) Δt, these equations may be rewritten under the following form:

$$A\,Y_{j+1}^{n+1}+B\,Q_{j+1}^{n+1}+C\,Y_j^{n+1}+D\,Q_j^{n+1}+G=0 \qquad (31)$$

$$A'\,Y_{j+1}^{n+1}+B'\,Q_{j+1}^{n+1}+C'\,Y_j^{n+1}+D'\,Q_j^{n+1}+G'=0 \qquad (32)$$

The computation begings by setting $b_j^{n+1/2}=b_j^n$; $A_j^{n+1/2}=A_j^n$; $K_j^{n+1/2}=K_j^n$ .The resulting system of linear equations in Y_j^{n+1}, Q_j^{n+1}, j = 1,2,. . . . . , N is solved to give a first approximation to these values Y_j^{n+1}, Q_j^{n+1} ; and a second approximation to the coefficients,

$$b_j^{n+1/2} = \frac{1}{2}\left[b_j\,(\,Y_j^{n+1}) + b_j\ \ (\ \ Y_j^n) \right]$$

$$A_j^{n+1/2} = \frac{1}{2}\left[A_j\,(\,Y_j^{n+1}) + A_j\ \ (\ \ Y_j^n) \right]$$

$$K_j^{n+1/2} = \frac{1}{2}\left[K_j\,(\,Y_j^{n+1}) + K_j\ \ (\ \ Y_j^n) \right]$$

The second resolution of the linear system leads to the approximation of the unknowns and so on.

Flow chart of the main computer program is given in Appendix (1).

Subroutines Asolv, Psolv and Tsolv were prepared to calculate the area, the wetted perimeter and top width of every channel cross section, at different water levels, respectively. Also subroutine NSOLV was prepared to compute the eqivalent Manning coefficient (15).

FIELD AND EXPERIMENTAL WORK:

Sectional profiles across the channels were made from Meit Ghamr to El Mansoura city (Bahr Tanah Canal) at equal distances of 2.0 Km. For each cross section, the level of the channel bed and its sides were measured every 2.0 ms along the channel section.

To measure the levels of each channel section, the nearest datum point made by the ministry of Irrigation was used. Section levels include backsights, intermediate sights and foresights.

The levels of the channel bed and its sides for the first reach were measured within the winter closure. For the second reach, the channel bed and its sides were measured after the end of winter closure. Figs.(4) through (6) show surveyed cross sections at the indicated locations.

Least square method was used to get the longitudinal bed slopes of the first and second reaches which are 5.3 cm/km and 4.5 cm/km respectively.

The discharge and water depths for each reach were measured daily for a period of 25 days at fixed stations shown in Fig.(2).

The cross section of flow was divided into a number of subareas using a marked wire, width of each subarea was taken equal to 5.0 ms, then the mean velocity of each subarea was measured by using the currentmeter. For shallow depths, the mean velocity was considered equal to the velocity at 0.6 of the depth below the water surface. For other depths the mean velocity was obtained as the average of the two velocities at o.2 and 0.8 of depth below the water surface. The total discharge was obtained by summing the products of each subsection area and its mean velocity (10).

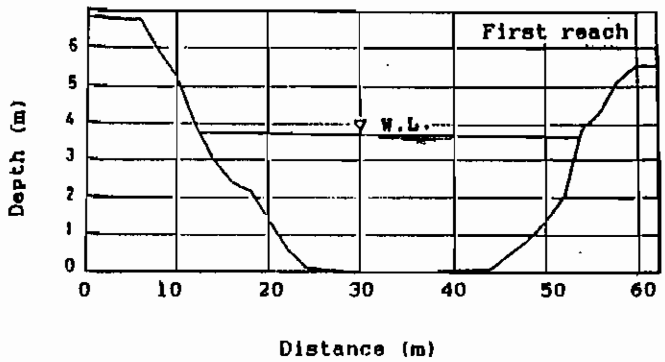


Fig.(4) Cross section at K.M.(0.00)

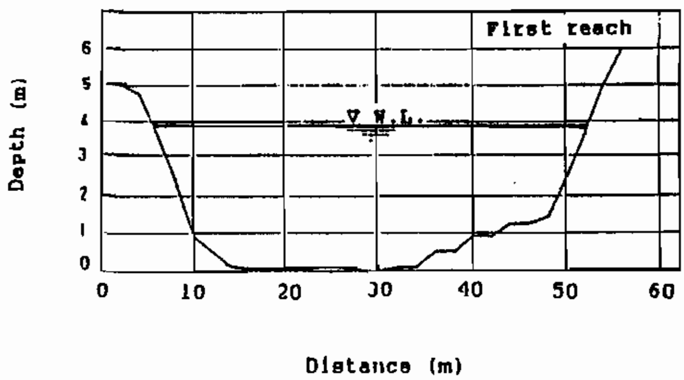


Fig.(5) Cross section at K.M.(18.00)

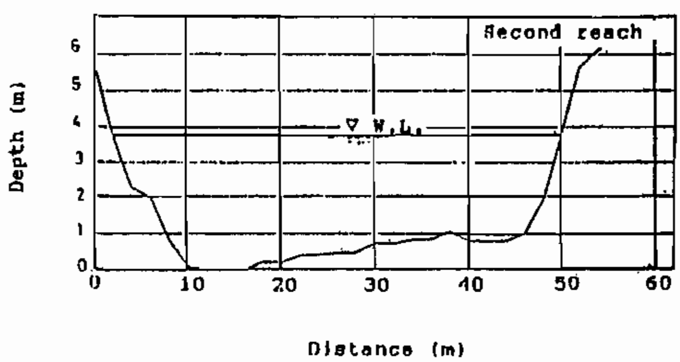


Fig.(6) Cross section at K.M.(32.50)

Samples of soil for every surveyed cross section of the channel were taken from the surface of the channel bed and sides. Every sample was sieved by using a set of sieves started from 2.5 mm to 0.6 mm. The soil particles passing sieve No. 63 were analysed using the fixed position pipette method according to the British Standard No. 3406 (2).

Fig. (7) through (9) give the particle size distribution at some locations.

MODEL CALIBRATION:

The main objective of model calibration process is to get simulated results very close to the corresponding measured values. The necessary data required for calibration are hydrographs and depth time relationships. When a high level of calibration can be achieved and verified, then it may be possible to extend the application of the model beyond the limits of the data used in the calibration process.

Values of time increment (Δt) of 11 hours, 22 hours and 44 hours were examined. The space increment (ΔX) was kept constant at 2.0 kms. Cole-brook's equation was used to represent the friction term in the mathematical model. The least square method was used to measure the degree of accuracy between the measured and simulated values. It was found that Δt = 22 hours is more suitable for the model.

Values of space increment (ΔX) of 1 Km. 2 Kms and 6 Kms were tested. The time increment used was 22 hours. Also Cole-brook formula was used to represent the boundary resistance. It was concluded that the space increment of 2 Kms is more convenient.

The effect of applying lotter formula, Horton and Einstein formula, and Pavlovski, Einstein and Banks formula to the model, for the calculation of equivalent roughness coefficient for every watted perimeter was examined. The time increment and space increment were kept constant at 22 hours and 2 Kms respectively. Strickler's formula was used to represent Manning's coefficient as a function of the median particle size. The results obtained by Horton Einstein formula were found more accurate than those obtained by the other two formula.

Statistical analyses for model calibration are given in Appendix (2)

ANALYSES OF RESULTS:

Manning's Equation:

The frication term required for the mathemtical model using Manning's equation was represented by:

$$S_f = \frac{n^2 \, P^{4/3} \, Q \mid Q \mid}{A^{10/3}}$$

Using Strickler's formula and then Horton-Einstein equation, eqn (16), was applied to get an equivalent roughness of each channel section.

The computed restults using Manning's formula are illustrated in figures(10) through (13). It is clear from the figures that the values of simulated water depths are less than the corresponding measured depths. To improve the simulated values of water depths, Manning's coefficient was investigated using the following two hypotheses:

a) Constant Manning's coefficient along the channel reach, for the first reach three trail values were examined, n=0.024, n=0.025 and n=0.026, and for the second reach, n=0.0245, n=0.025 and n=0.0255. Using the least squares method, the value of Manning's coefficient which gave accurate results was found to be 0.025 for both the first and second reaches. However the computed results by using the above mentioned values of Manning's coefficient are accepted at 5% level of significance using the F test.

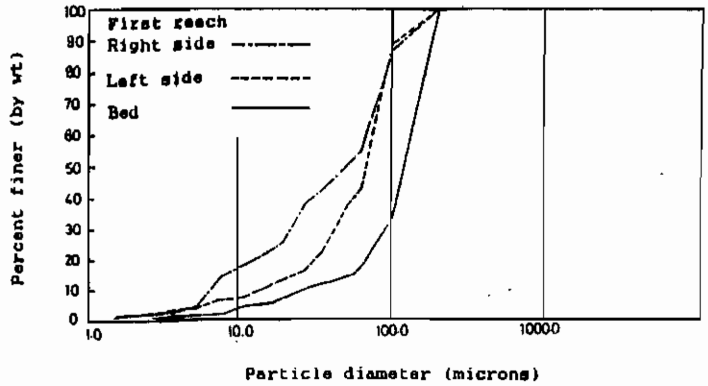


Fig.(7) The particle size distribution at K.M.(0.2,4)

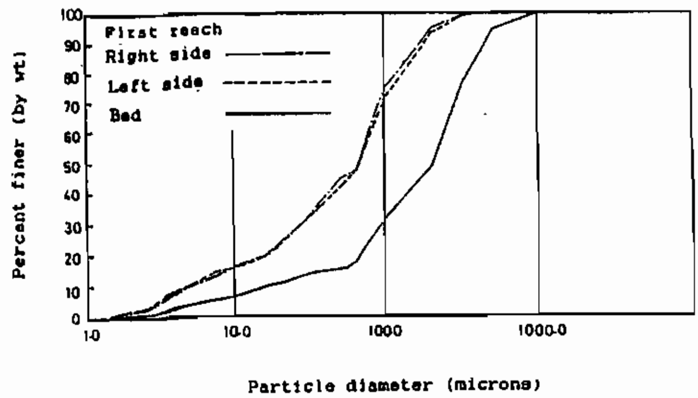


Fig.(8) The particle size distribution at K.M.(16,18)

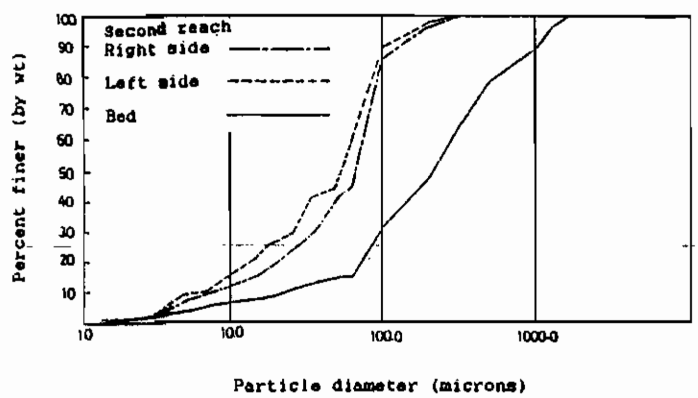


Fig.(9) The particle size distribution at K.M.(32.5,34.5)

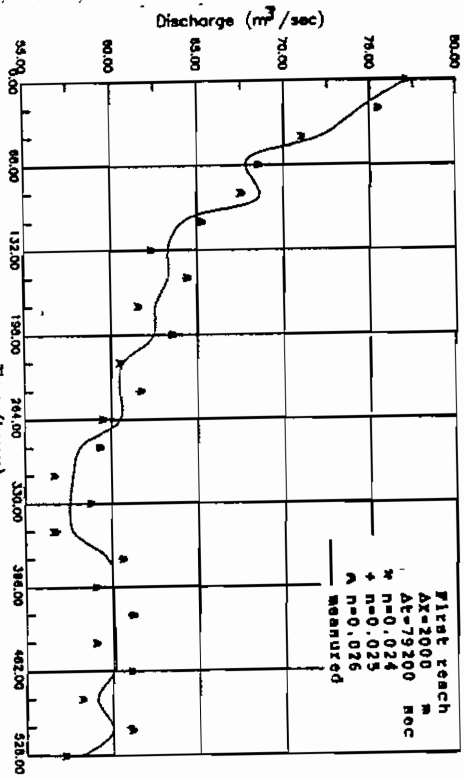


Fig.(10) Comparison between simulated and measured hydrographs

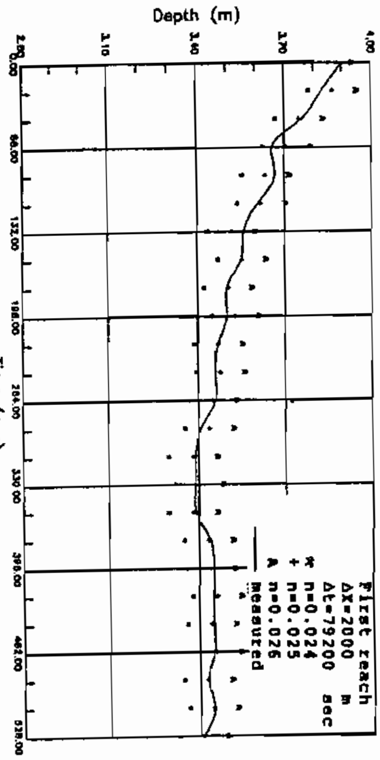


Fig.(11) Comparison between simulated and measured water depths

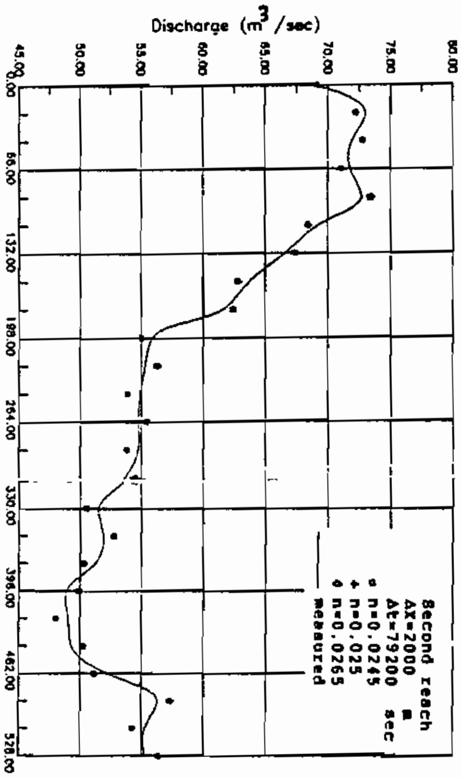


Fig.(12) Comparison between simulated and measured hydrographs

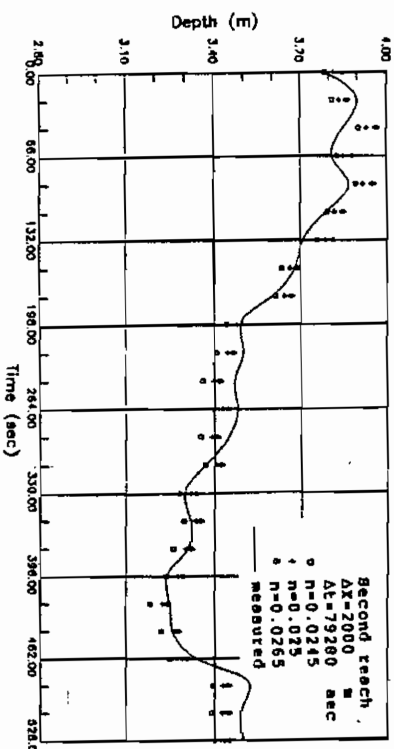


Fig.(13) Comparison between simulated and measured water depths

b) Varying Manning's coefficient along the channel reach, Strickler formula has to be modified to $n=0.095d^{1/6}$ for the first reach and $n=0.096d^{1/6}$ for the second reach to get the best between the simulated hydrographs and water depths and the corresponding measured values, Figures (14) through (17) values of Manning's coefficient are given in Appendix (3) however there is no significant difference between the two modified factors at 5% level.

Chezy's Equation:

The friction term required for the mathematical model by using Chezy's equation is given by:

$$S_f = \frac{P\,Q\,|\,Q\,|}{C^2\,A}$$

Many attempts were made to get Chezy's coefficient (C)

1) Ganguillet and Kutter formula:

The simulated results are given in Figures (18) through (21)

2) Powell formula:

The flow in El Mansoura canal was found to be smooth turbulent flow as $R_* = V_* \cdot K_s / \nu < 5$. The application of Powell formula showed less accuracy in the simulated water depths and hydrographs, Figures (18) through (21)

Appendix (4) gives the values of R_* along the channel.

3) Cole-brook Formula:

The friction term (f) was obtained for smooth turbulent flow from the equation:

$$\frac{1}{\sqrt{f}} = C \log \left(\frac{R_* \sqrt{f}}{2.5} \right)$$

The factor (C) was taken equal to 2.0 and it was changed to other values for the first and second reaches.

The least squares method showed that the value of factor (C) Which gave more accurate simulated values was found to be 1.2 for the first reach and equal to 1.1 for the second reach. There is any significant ,difference between the two values at 5% level.

4) Channel with movable boundaries:

Two equations were selected for this study. The first equation by Mostafa and McDermid. eqn. (18), and the second equation by Liu and Hwang, eqn. (19). The two equations showed less accuracy in the simulated hydrographs and depths than the corresponding values by using either Manning's equation or Chezy's formula. Simulated hydrographs and depths are shown in Figures (26) through (33).

In Figures 14, 21,23,27,29,31 and 33 water depths exhibited lower values than the corresponding measured ones, this is due to small values of friction coefficient which give bigger values of velocities. It is noticed that the calculated depths versus time are similar in shapes to the corresponding measured depths, this is mainly due to regular shapes of cross sections of the channel under study.

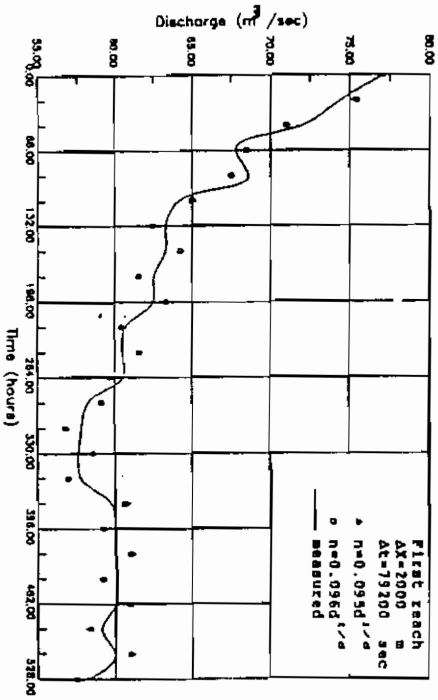


Fig.(14) Comparison between simulated and measured hydrographs

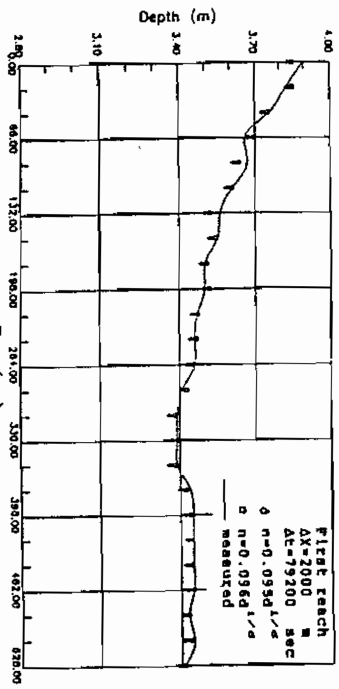


Fig.(15) Comparison between simulated and measured water depths

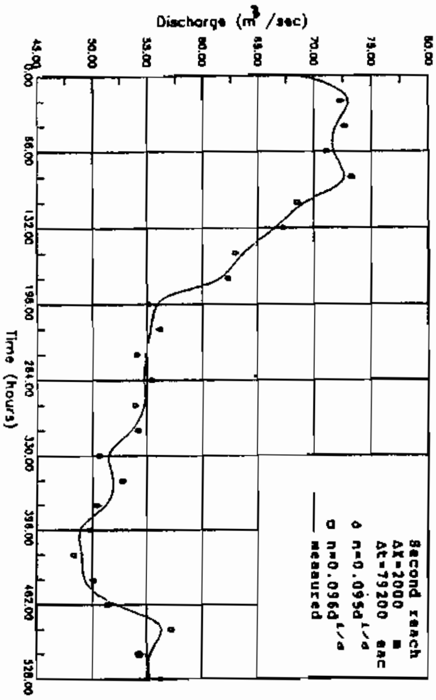


Fig.(16) Comparison between simulated and measured hydrographs

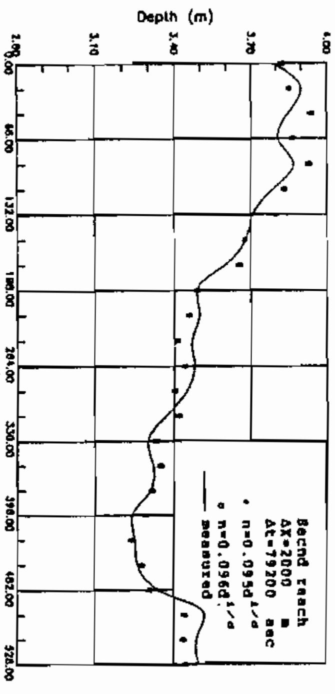


Fig.(17) Comparison between simulated and measured water depths

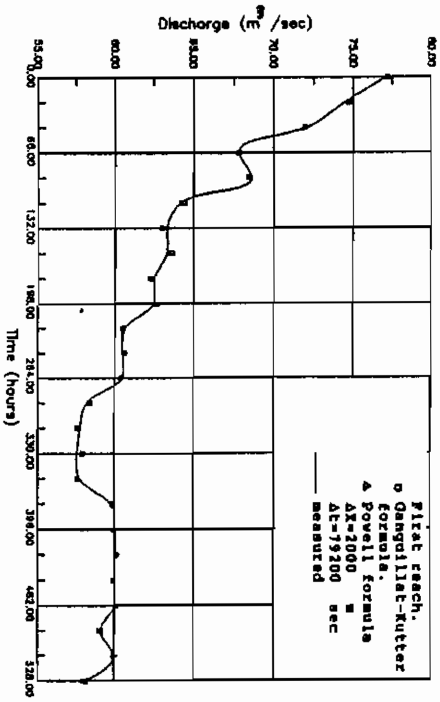


Fig.(18) Comparison between simulated and measured hydrographs

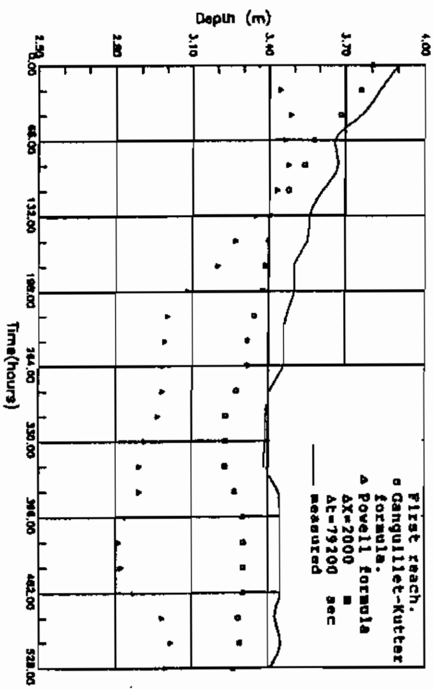


Fig.(19) Comparison between simulated and measured water depths

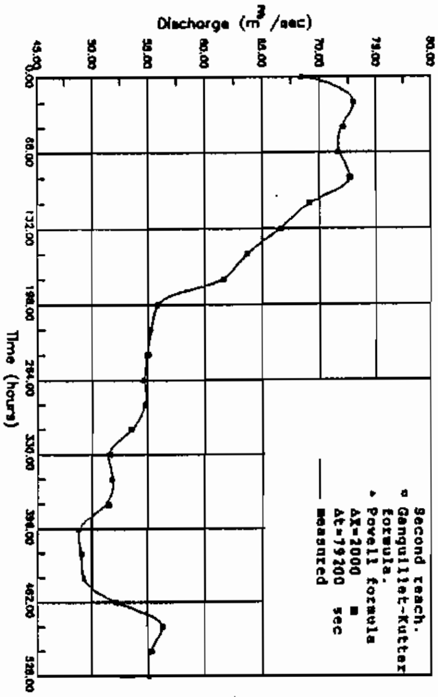


Fig.(20) Comparison between simulated and measured hydrographs

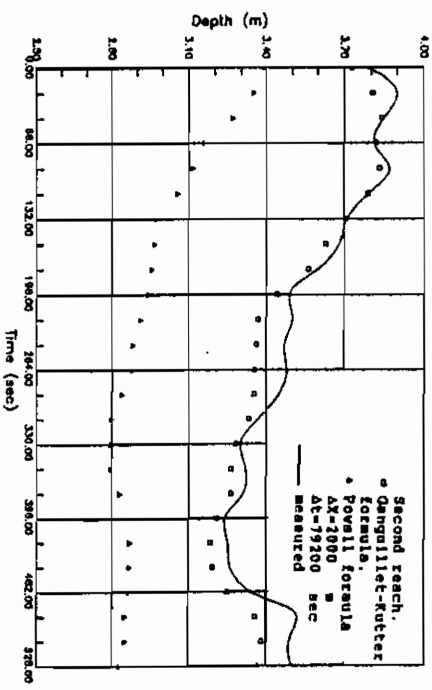


Fig.(21) Comparison between simulated and measured water depths

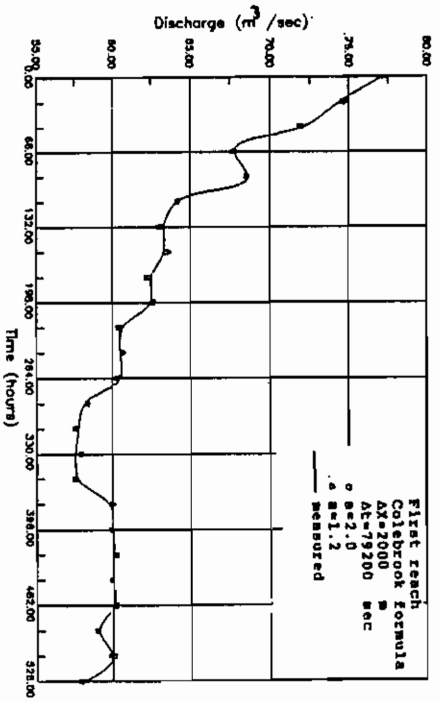


Fig.(22) Comparison between simulated and measured hydrographs

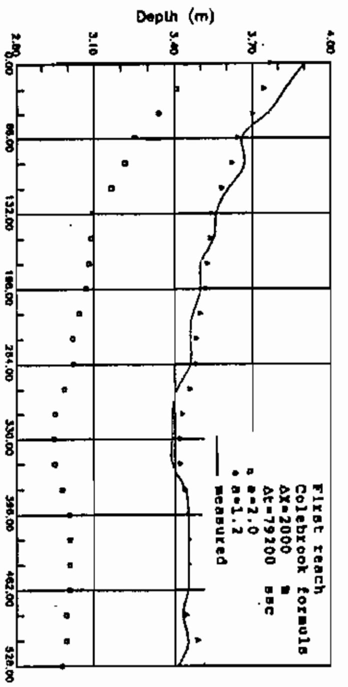


Fig.(23) Comparison between simulated and measured water depths

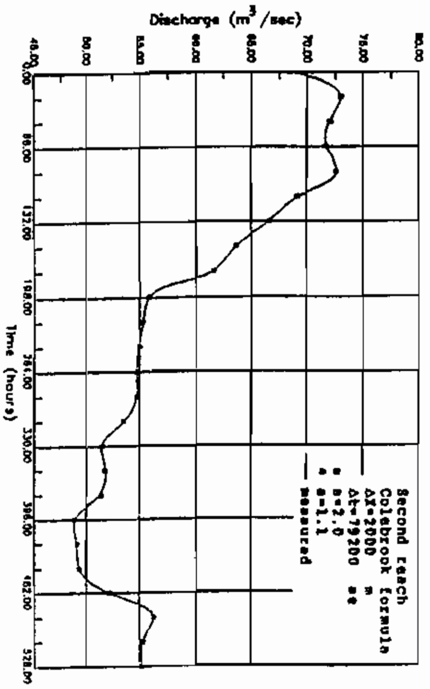


Fig.(24) Comparison between simulated and measured hydrographs

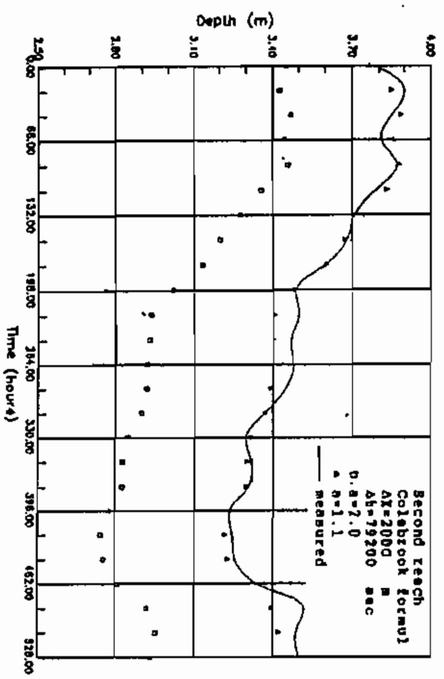


Fig.(25) Comparison between simulated and measured water depths

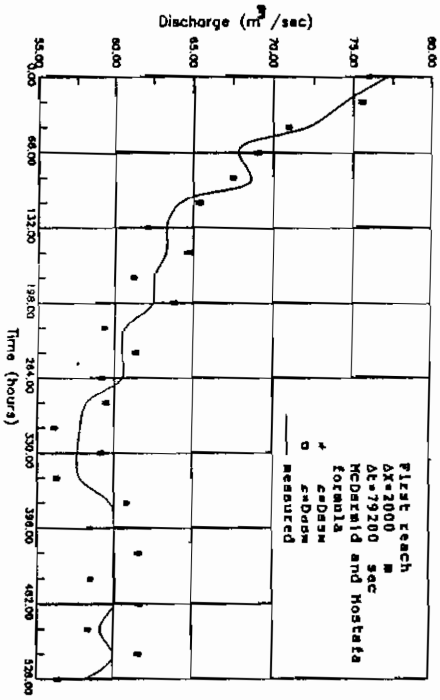


Fig.(26) Comparison between simulated and measured hydrographs

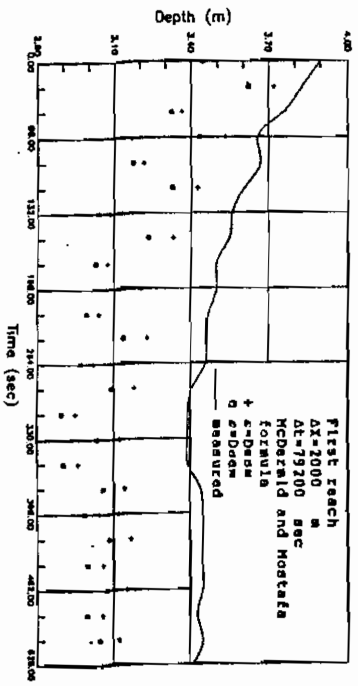


Fig.(27) Comparison between simulated and measured water depths

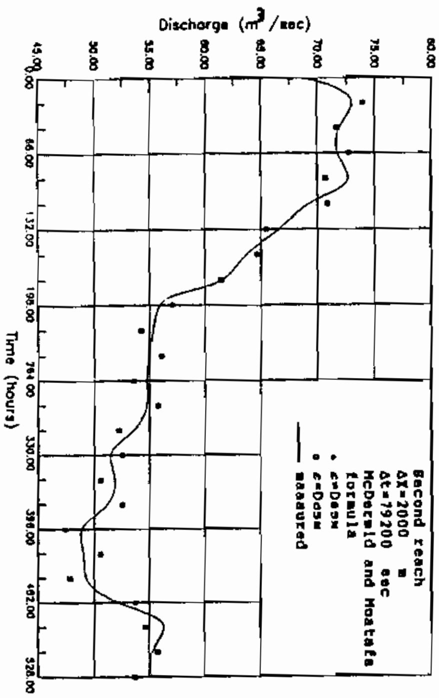


Fig.(28) Comparison between simulated and measured hydrographs

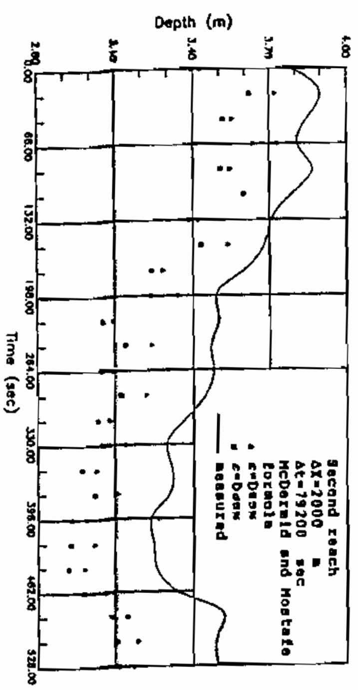


Fig.(29) Comparison between simulated and measured water depths

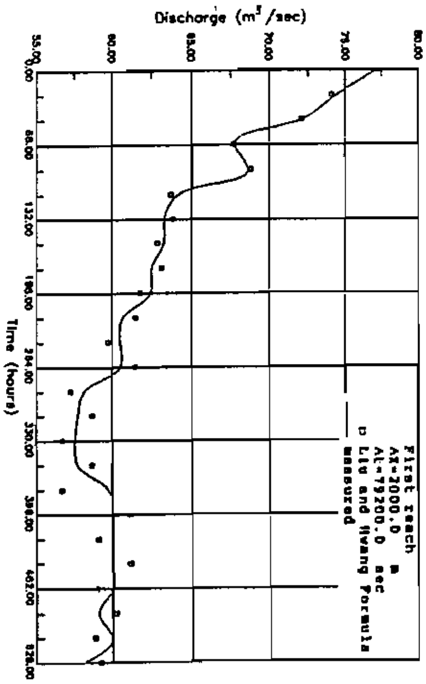


Fig.(30) Comparison between simulated and measured hydrographs

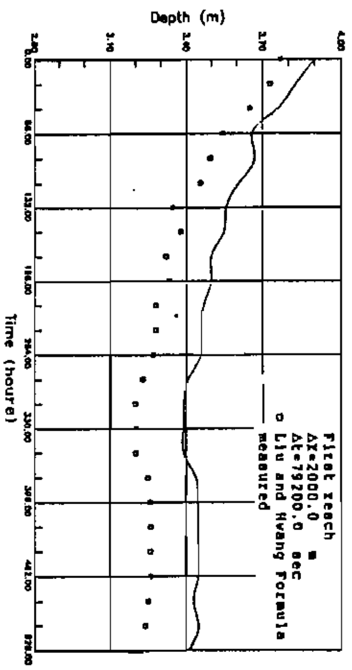


Fig.(31) Comparison between simulated and measured water depths

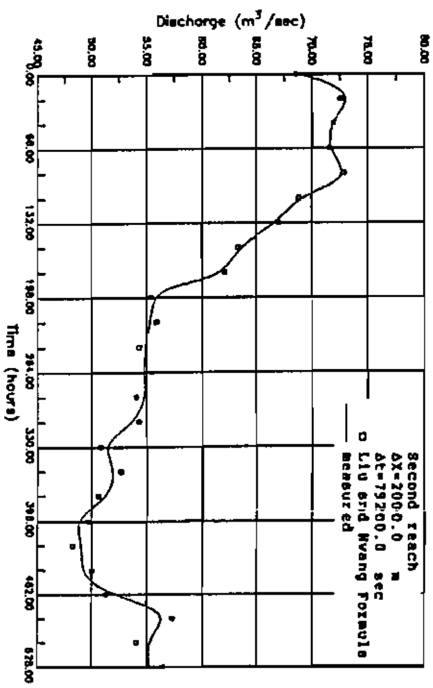


Fig.(32) Comparison between simulated and measured hydrographs

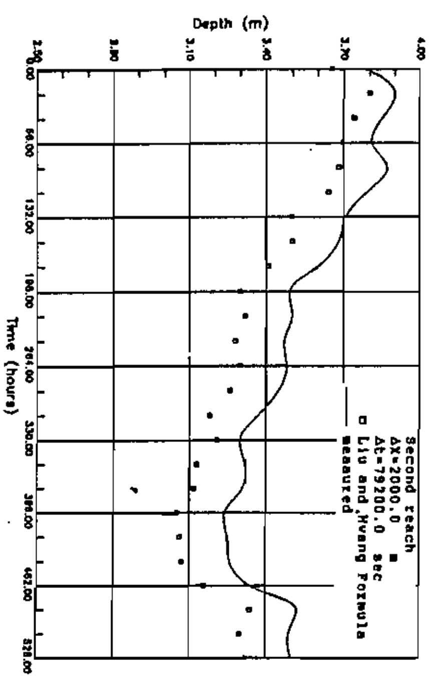


Fig.(33) Comparison between simulated and measured water depths

CONCLUSIONS :

At the outset of this investigation to choose the most convenient friction formula to be applied for Egyptian irrigation canals, a decision was made in favour of mathematical model. The numerical model was constructed on Verwey's variant of the Priessmann scheme.

The time increment of 22 hours and space increment of 2.0 kms are suitable for the model calibration regarding the computer time and the high degree of accuracy.

The simulated reults by using Horton-Einstein equation for the computation of the equivalent Manning's coefficient are more accurate than the corresponding results by using either Pavlovski-Einstein formula or lotter formula.

Manning's equation could be used for design purposes with values of Manning's coefficient varies between 0.024 and 0.026.

Ganguillet and Kutter formula gave good accuracy for depth values but less than that obtained by using either Manning's n=0.025 or $n=0.095d^{1/6}$. On the other hand, optimum accuracy for discharge values were given by Ganguillet and Kutter formula than those obtained by using constant or varying Manning's coefficient.

Powell formula showed big differences between measured and simulated water depths and discharges.

The flow in El mansouria canal is smooth turbulent, Cole-brook's formula with modified coefficient gave more accurate simulated hydrographs and depths. Also, the simulated results by varying Manning coefficient along the reach as $n=0.095d^{1/6}$ gave the optimum accuracy for depth values.

Further investigations are recommended in this respect, to get an adjustable relationship between Manning's coefficient (n) and the median particle size (d) and modified coefficients for cole-brook white formula and the effect of movable boundaries on friction formulae. This could be achieved by using more extensive data of field measurements.

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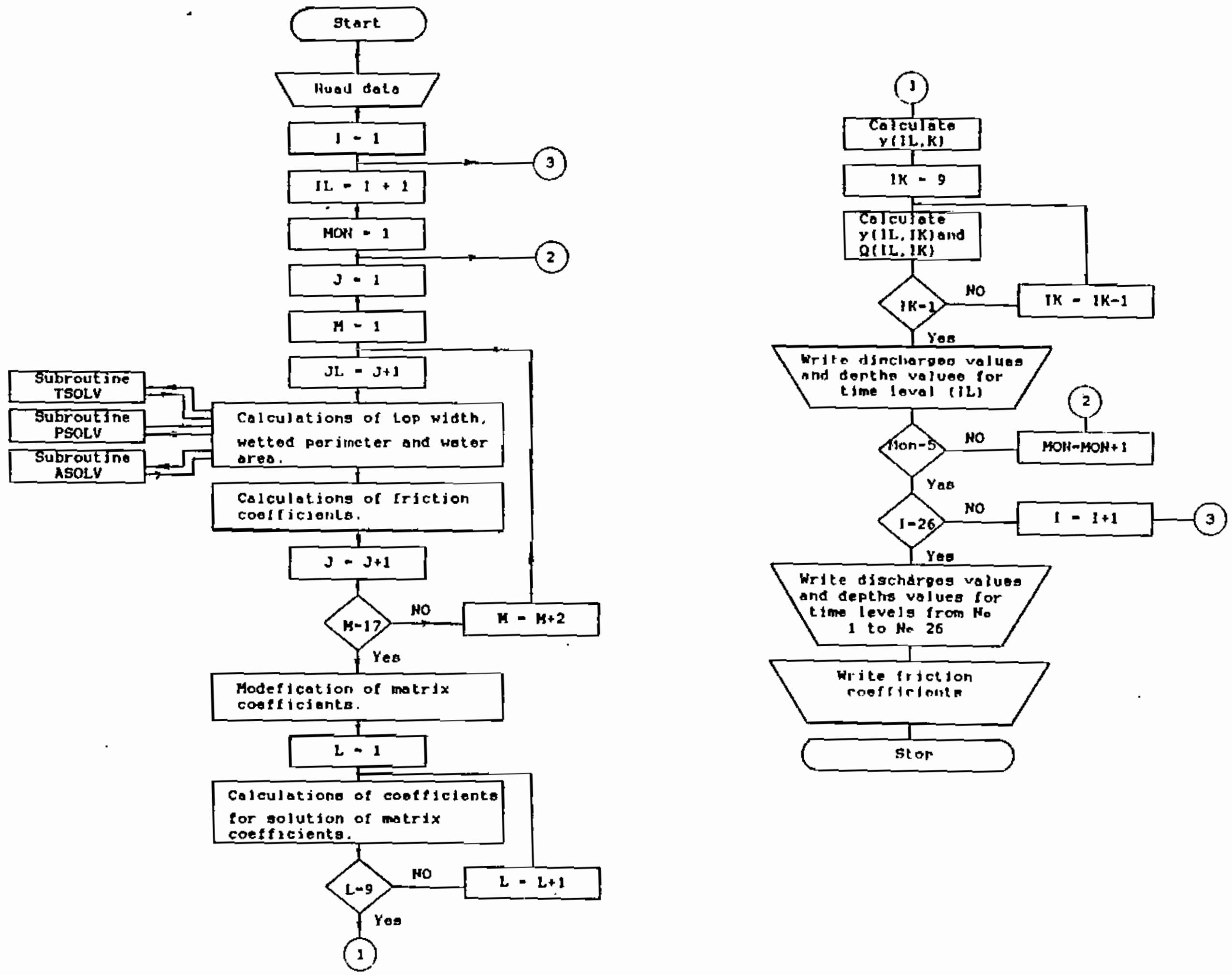
NOTATION

A = cross sectional area;
b = width of cross section;
C = Chezy coefficient;
c = factor;
Ca = coefficient for Liu and Hwang equation;
C_M = dimensionless Manning coefficient;
d = diameter of pipe;
$d50$ = median grain size;
f = friction factor;
g = acceleration due to gravity;
j = computational point index;
K = channel conveyance factor;
K_s = average height of roughness;
m = Bazin coefficient;
n = Manning coefficient;
n_e = equivalent Manning coefficient;
P = wetted perimeter;
Q = volumetric water discharge;
R = hydraulic radius;
R_e = Reynolds number;
R_* = Reynolds number of friction;
S_o = bed slope in the X-direction;
S_f = friction slope in the X-direction;
T = top width of channel;
t = time;
t_n = time interval;
V = steady flow velocity;
V_* = shear velocity;
X = longitudinal space co-ordinate in horizontal plane;
x = coefficient for Liu and Hwang formula

y = depth of water;
y = coefficient for Liu and Hwang formula;
δ = thickness of the viscous sublayer;
ε = d_{50}, d_{65} or d_{90};
Δt = time between two computational intervals;
ΔX = distance between two computational points; and
ν = Kinematic viscosity.

Appendix (1)

Flow Chart for Main Program :



Appendix (2)

Statistical Analysis of Model Calibration

$F_{24,1} = 7.82$ First Reach

Time Increment

	Statistical results for hydrographs		
	Δt=22hours	Δt=11hours	Δt=44hours
Least square method	0.6199386	0.55196	0 662277
F-Test	1.001042812	1.047541377	1.2722467

	Statistical results for water depths		
	Δt=22hours	Δt=11hours	Δt=44hours
Least square method	4.854543	4.14389	10.03246
F-Test	1.940131966	1.031795593	2.0156011

Space Increment

	Statistical results for hydrographs		
	ΔX=2 Kms	ΔX=1 Km	ΔX=6 Kms
Least square method	0.6199386	29.77739	0.7400827
F-Test	1.001042812	1.04927474	1.001511328

	Statistical results for water depths		
	ΔX=2 Kms	ΔX=1 Km	ΔX=6 Kms
Least square method	4.8545430	5.2425990	4.7311680
F-Test	1.035577	1.0357306	1.0562763

Friction Formulae

	Statistical results for hydrographs		
	Einstein	Pavlovski	Lotter
Least square method	21.562763	24.86314	26.277781
F-Test	1.0299	1.0303232	1.035987

	Statistical results for water depths		
	Einstein	Pavlovski	Lotter
Least square method	44.89001	46.5961	47.9712
F-Test	1.250936	1.113315842	1.1133158

Statistical Analysis of Model Calibration

$F_{24,1} = 7.82$

Second Reach

Time Increment

	Statistical results for hydrographs		
	Δt=22hours	Δt=11hours	Δt=44hours
Least square method	0.1161864	0.114208	0.11946
F-Test	1.00288196	1.008882494	1.05441497

	Statistical results for water depths		
	Δt=22hours	Δt=11hours	Δt=44hours
Least square method	6.010584	5.991889	11.64167
F-Test	1.39632001	1.36704958	1.9151522

space Increment

	Statistical results for hydrographs		
	ΔX=2 Kms	ΔX=1 Km	ΔX=6 Kms
Least square method	0.1161864	23.78912	0.1159129
F-Test	1.0028881964	1.01343231	1.00288196

	Statistical results for water depths		
	ΔX=2 Kms	ΔX=1 Km	ΔX=6 Kms
Least square method	6.010584	6.55725	5.79392
F-Test	1.39632001	1.39631978	1.39632

Friction Formulae

	Statistical results for hydrographs		
	Einstein	Pavlovski	Lotter
Least square method	19.92823	21.407643	25.99378
F-Test	1.019594	1.00288107	1.0325824

	Statistical results for water depths		
	Einstein	Pavlovski	Lotter
Least square method	42.6222	44.40984	46.42775
F-Test	2.2799068	2.21722	2.21232

Appendix (3)

Values of Manning's coefficient (N)

First Reach				Second Reach			
Y= 3.67799800	N= .024292450	5	1	Y=3.89917400	N= .025381350	5	11
Y= 3.68724100	N= .024467560	5	2	Y=3.88721000	N= .025177490	5	12
Y= 3.69077400	N= .024232110	5	3	Y=3.87935300	N= .024522110	5	13
Y= 3.69418200	N= .025420460	5	4	Y=3.86298200	N= .024564460	5	14
Y= 3.70941000	N= .025391440	5	5	Y=3.85054700	N= .024303950	5	15
Y= 3.69567600	N= .024870820	5	6	Y=3.86482800	N= .025264800	5	16
Y= 3.68758000	N= .024636510	5	7	Y=3.88609700	N= .025161700	5	17
Y= 3.70248400	N= .024763410	5	8	Y=3.87879900	N= .024819610	5	18
Y= 3.73126600	N= .024728160	5	9	Y=3.88123200	N= .024170660	5	19
Y= 3.74332600	N= .025674800	5	10	Y=3.88723900	N= .025146810	5	20
Y= 3.50093300	N= .024310600	10	1	Y=3.53874800	N= .025441650	10	11
Y= 3.48595100	N= .024499360	10	2	Y=3.56554300	N= .025217610	10	12
Y= 3.51030900	N= .024254680	10	3	Y=3.59186800	N= .024587750	10	13
Y= 3.50557300	N= .025460450	10	4	Y=3.64998700	N= .024601190	10	14
Y= 3.49813400	N= .025451270	10	5	Y=3.68914000	N= .024324270	10	15
Y= 3.56239800	N= .024893160	10	6	Y=3.64239000	N= .025317980	10	16
Y= 3.60673600	N= .024649060	10	7	Y=3.59987300	N= .025222070	10	17
Y= 3.50046500	N= .024709530	10	8	Y=3.63128100	N= .024848860	10	18
Y= 3.54250200	N= .024790740	10	9	Y=3.61289100	N= .024198350	10	19
Y= 3.55301400	N= .025706890	10	10	Y=3.62827800	N= .025188670	10	20
Y= 3.39823000	N= .024321410	15	1	Y=3.40981800	N= .025463690	15	11
Y= 3.40891300	N= .024512170	15	2	Y=3.40399200	N= .025241060	15	12
Y= 3.41636300	N= .024263340	15	3	Y=3.40562400	N= .024635720	15	13
Y= 3.42611400	N= .025477730	15	4	Y=3.39882300	N= .024647830	15	14
Y= 3.44975400	N= .025466890	15	5	Y=3.39143100	N= .024364660	15	15
Y= 3.44668800	N= .024912880	15	6	Y=3.40340700	N= .025377820	15	16
Y= 3.44590700	N= .024674470	15	7	Y=3.42017500	N= .025261530	15	17
Y= 3.46387300	N= .024813770	15	8	Y=3.41220600	N= .024876340	15	18
Y= 3.49419600	N= .024810000	15	9	Y=3.40827900	N= .024220220	15	19
Y= 3.51037800	N= .025714320	15	10	Y=3.40772200	N= .025225650	15	20
Y= 3.44095300	N= .024316890	20	1	Y=3.19071400	N= .025502210	20	11
Y= 3.42573000	N= .024509350	20	2	Y=3.21688800	N= .025268950	20	12
Y= 3.44851800	N= .024260430	20	3	Y=3.23740000	N= .024680220	20	13
Y= 3.44009600	N= .025474670	20	4	Y=3.29003500	N= .024668550	20	14
Y= 3.42671300	N= .025474410	20	5	Y=3.32857400	N= .024375240	20	15
Y= 3.48468100	N= .024906370	20	6	Y=3.28891000	N= .025410400	20	16
Y= 3.52462200	N= .024661960	20	7	Y=3.25459800	N= .025322520	20	17
Y= 3.50667000	N= [illegible]	20	8	Y=3.29243100	N= .024891690	20	18
Y= 3.46317200	N= [illegible]	20	9	Y=3.27507700	N= .024235220	20	19
Y= 3.47405700	N= [illegible]	20	10	Y=3.28071700	N= .025246070	20	20
Y= 3.41029700	N= [illegible]	25	1	Y=3.42357300	N= .025461320	25	11
Y= 3.42402600	N= .024509640	25	2	Y=3.42208900	N= .025238410	25	12
Y= 3.42829400	N= .024262260	25	3	Y=3.43199700	N= .024628860	25	13
Y= 3.43505600	N= .025475770	25	4	Y=3.43326600	N= .024641330	25	14
Y= 3.45823300	N= .025464140	25	5	Y=3.43003400	N= .024359230	25	15
Y= 3.45719800	N= .024911070	25	6	Y=3.44101300	N= .025368210	25	16
Y= 3.45577900	N= .024672890	25	7	Y=3.45589100	N= .025253590	25	17
Y= 3.46703400	N= .024818020	25	8	Y=3.44765300	N= .024871850	25	18
Y= 3.48816300	N= .024814960	25	9	Y=3.44605000	N= .024216130	25	19
Y= 3.49857800	N= .025716350	25	10	Y=3.44849500	N= .025218720	25	20

Appendix (4)

Values of $R_* = \dfrac{V_* \, K_*}{\nu}$ along El Mansouria Canal

Y= water depth K= section No.

L = Time level

First Reach

Y- 3.5856630	R*-2.72837000	L- 5	K- 1
Y- 3.5969990	R*-2.00745900	L- 5	K- 2
Y- 3.6244630	R*-2.43914100	L- 5	K- 3
Y- 3.6442860	R*-3.90272500	L- 5	K- 4
Y- 3.6747190	R*-3.31475200	L- 5	K- 5
Y- 3.7184970	R*-2.88599500	L- 5	K- 6
Y- 3.7556140	R*-3.11072000	L- 5	K- 7
Y- 3.7793530	R*-3.34505100	L- 5	K- 8
Y- 3.8018520	R*-3.25030500	L- 5	K- 9
Y- 3.8425100	R*-3.22052000	L- 5	K- 10
Y- 3.4632380	R*-2.66056400	L- 10	K- 1
Y- 3.4758500	R*-2.72957300	L- 10	K- 2
Y- 3.5051270	R*-2.35597600	L- 10	K- 3
Y- 3.5275570	R*-3.77599800	L- 10	K- 4
Y- 3.5603420	R*-3.19029300	L- 10	K- 5
Y- 3.6068620	R*-2.76681000	L- 10	K- 6
Y- 3.6469410	R*-2.98551200	L- 10	K- 7
Y- 3.6743520	R*-3.20629000	L- 10	K- 8
Y- 3.7004050	R*-3.12017600	L- 10	K- 9
Y- 3.7437200	R*-3.08395500	L- 10	K- 10
Y- 3.3798420	R*-2.56073700	L- 15	K- 1
Y- 3.3980820	R*-2.63518000	L- 15	K- 2
Y- 3.4316740	R*-2.27443100	L- 15	K- 3
Y- 3.4504560	R*-3.65869400	L- 15	K- 4
Y- 3.4949640	R*-3.08604200	L- 15	K- 5
Y- 3.5450170	R*-2.67234000	L- 15	K- 6
Y- 3.5891370	R*-2.88305000	L- 15	K- 7
Y- 3.6220350	R*-3.08531800	L- 15	K- 8
Y- 3.6547110	R*-2.98880900	L- 15	K- 9
Y- 3.7044240	R*-2.92832300	L- 15	K- 10
Y- 3.3878640	R*-2.63734300	L- 20	K- 1
Y- 3.4000770	R*-2.70906800	L- 20	K- 2
Y- 3.4290360	R*-2.33538100	L- 20	K- 3
Y- 3.4508810	R*-3.75653700	L- 20	K- 4
Y- 3.4831310	R*-3.16799300	L- 20	K- 5
Y- 3.5296710	R*-2.74266500	L- 20	K- 6
Y- 3.5695940	R*-2.96281400	L- 20	K- 7
Y- 3.5969260	R*-3.17743500	L- 20	K- 8
Y- 3.6229810	R*-3.09139200	L- 20	K- 9
Y- 3.6665160	R*-3.04177900	L- 20	K- 10
Y- 3.4137840	R*-2.61213800	L- 25	K- 1
Y- 3.4292830	R*-2.68192800	L- 25	K- 2
Y- 3.4609550	R*-2.31081600	L- 25	K- 3
Y- 3.4861950	R*-3.70373800	L- 25	K- 4
Y- 3.5216390	R*-3.11859300	L- 25	K- 5
Y- 3.5709410	R*-2.69509300	L- 25	K- 6
Y- 3.6145930	R*-2.89961000	L- 25	K- 7
Y- 3.6471880	R*-3.09782800	L- 25	K- 8
Y- 3.6797820	R*-2.99476600	L- 25	K- 9
Y- 3.7295890	R*-2.93135800	L- 25	K- 10

Second Reach

Y- 3.8191500	R*-3.69293000	L- 5	K- 11
Y- 3.0302860	R*-3.81017000	L- 5	K- 12
Y- 3.8424000	R*-3.34455800	L- 5	K- 13
Y- 3.8633370	R*-3.15823300	L- 5	K- 14
Y- 3.8757650	R*-3.72969800	L- 5	K- 15
Y- 3.8633750	R*-4.24446800	L- 5	K- 16
Y- 3.8600440	R*-3.80012200	L- 5	K- 17
Y- 3.8748430	R*-3.31043300	L- 5	K- 18
Y- 3.8695630	R*-4.07210400	L- 5	K- 19
Y- 3.8036120	R*-3.44715700	L- 5	K- 20
Y- 3.5530070	R*-3.49407800	L- 10	K- 11
Y- 3.5669180	R*-3.63045800	L- 10	K- 12
Y- 3.5811420	R*-3.18280100	L- 10	K- 13
Y- 3.6041360	R*-2.99983800	L- 10	K- 14
Y- 3.6182840	R*-3.58021600	L- 10	K- 15
Y- 3.6076520	R*-4.07463200	L- 10	K- 16
Y- 3.6057070	R*-3.63114500	L- 10	K- 17
Y- 3.6224420	R*-3.34201700	L- 10	K- 18
Y- 3.6171220	R*-3.94761900	L- 10	K- 19
Y- 3.6305130	R*-3.29309900	L- 10	K- 20
Y- 3.3427450	R*-3.39787800	L- 15	K- 11
Y- 3.3562600	R*-3.53650300	L- 15	K- 12
Y- 3.3698800	R*-3.09810900	L- 15	K- 13
Y- 3.3927310	R*-2.91180800	L- 15	K- 14
Y- 3.4065320	R*-3.50023200	L- 15	K- 15
Y- 3.3948620	R*-3.98390600	L- 15	K- 16
Y- 3.3916590	R*-3.53151400	L- 15	K- 17
Y- 3.4081440	R*-3.23614600	L- 15	K- 18
Y- 3.3991270	R*-3.89137800	L- 15	K- 19
Y- 3.4085760	R*-3.19630200	L- 15	K- 20
Y- 3.1072210	R*-3.22625000	L- 20	K- 11
Y- 3.2043970	R*-3.36538100	L- 20	K- 12
Y- 3.2217330	R*-2.94209000	L- 20	K- 13
Y- 3.2481700	R*-2.75056200	L- 20	K- 14
Y- 3.2662970	R*-3.33030600	L- 20	K- 15
Y- 3.2603170	R*-3.79142100	L- 20	K- 16
Y- 3.2624000	R*-3.34097300	L- 20	K- 17
Y- 3.2836960	R*-3.06816500	L- 20	K- 18
Y- 3.2794520	R*-3.73653200	L- 20	K- 19
Y- 3.2924290	R*-3.04746800	L- 20	K- 20
Y- 3.3693490	R*-3.37943400	L- 25	K- 11
Y- 3.3842170	R*-3.52203000	L- 25	K- 12
Y- 3.3991130	R*-3.08650400	L- 25	K- 13
Y- 3.4229480	R*-2.90281900	L- 25	K- 14
Y- 3.4378340	R*-3.40654900	L- 25	K- 15
Y- 3.4277130	R*-3.96813700	L- 25	K- 16
Y- 3.4260070	R*-3.52092100	L- 25	K- 17
Y- 3.4436020	R*-3.22963900	L- 25	K- 18
Y- 3.4368780	R*-3.07151900	L- 25	K- 19
Y- 3.4483970	R*-3.19107800	L- 25	K- 20