sea water stream in each stage. A portion of the flashing stream is evaporated in each stage and condensed to produce fresh water.
Performance ratio of a Multi-Stage Flash evaporator is important in calculating energy cost. A high performance ratio plant would be specified for a location where fuel is expensive while the low performance ratio plant would be used where fuel is very cheap. This fact illustrates and insures the relation between the capital cost for the plant and energy cost.

2-THEORETICAL ANALYSIS

In MSF evaporator shown in Fig. (1) sea water is heated in the brine heater to just below the saturation temperature ,T_{max} . Then it enters the recovery and rejection sections respectively. Flashing of sea water occurs due to the reduction in pressure in each stage in these two sections. Distilled water is obtained in each stage due to vapor condensation over condenser tubes, which are placed at the top of each chamber. The interstage temperature difference is taken as a constant in each stage of heat recovery and rejection sections (ΔT_r and ΔT_j). The ratio between the number of stages in the recovery to rejection sections is taken as the same value as that for the performance ratio of the plant[1].

2.1 Mass and energy balance:

It is convenient to start with the equations of mass and energy balance for the MSF evaporator.

The mass balance equation is,

$$m^{\cdot}_{sw} = m^{\cdot}_{so} + m^{\cdot}_{si} \qquad (1)$$

where :

$$m^{\cdot}_{si} = m_{bd} + m_{d}$$

Also , salt concentration balance is given by,

$$m^{\cdot}_{si} \; X_{sw} = m^{\cdot}_{bd} \; X_{bd} + m_{d} \; X_{d} \qquad (2)$$

where,

$$X_d = 0.0$$

Therefore, the concentration ratio (CR) which is defined as (X_{bd} /X_{sw}) takes the following form,

$$CR = m^{\cdot}_{si} / m^{\cdot}_{bd}$$

Substituting for CR, equation (2) takes the following form,

$$m^{\cdot}_{si} = m^{\cdot}_{d} \, (CR / (CR - 1)) \qquad (3)$$

The equation (1) can be rearranged as follows,

$$m_{sw} = m^{\cdot}_{so} + m^{\cdot}_{d} \, (CR / (CR - 1)) \qquad (4)$$

Recirculation ratio (R) is defined as,

$$R = \dot{m}_{b,0} / \dot{m}_d \qquad (5)$$

Inlet brine water mass flow rate to the first stage of recovery section is obtained as follows,

$$\dot{m}_{b,0} = \dot{m}_{si} + \dot{m}_r \qquad (6)$$

Also, salt concentration is given by,

$$\dot{m}_{b,0}\ X_{b,0} = \dot{m}_{si}\ X_{si} + \dot{m}_r\ X_{bd} \qquad (7)$$

Combining equations 3,5,6 and 7 one obtains the following form,

$$m_{si} / m_{b,0} = CR / R\ (CR - 1) \qquad (8)$$

Total input heat to MSF evaporator plant is given by,

$$\begin{aligned} Q &= \dot{m}_{so}\ i_{so} + m_d\ i_d + \dot{m}_{bd}\ i_{bd} - \dot{m}_{sw}\ i_{sw} \\ &= \dot{m}_{b,0}\ Cp_{b,0}\ (T_{max} - t_{in}) \end{aligned} \qquad (9)$$

The heat balance for recovery section takes the following form, as it is clear on Fig.(1),

$$\begin{aligned} m_{b,0}\ Cp_{b,0}\ T_{max} + \dot{m}_{b,0}\ Cp_r\ t_r &= \dot{m}_{b,0}\ Cp_{in}\ t_{in} \\ &+ \dot{m}_{d,N}\ Cp_{d,N}\ T_{d,N} + m_{b,N}\ Cp_{b,N}\ T_{b,N} \end{aligned} \qquad (10)$$

In addition, application of the heat balance for rejection section gives the following relation,

$$\begin{aligned} \dot{m}_{sw}\ Cp_{sw}\ t_{sw} + \dot{m}_{b,N}\ Cp_{b,N}\ T_{b,N} + \dot{m}_{d,N}\ Cp_{d,N}\ T_{d,N} &= \dot{m}_{sw}\ Cp_{so}\ t_{so} \\ &+ (m_{bd} + \dot{m}_r)Cp_{bd}\ T_{bd} + \dot{m}_d\ Cp_d\ T_{d,M} \end{aligned} \qquad (11)$$

2.2 Brine temperatures for different stages of MSF plant:

Heat is transferred from the flashing vapor to the sea water which is flowing inside the condenser tubes in each stage of MSF plant. The latent heat of evaporation is thus recovered and re-used to increase the sea water temperature. Energy balance is performed for the i^{th} stage according to the following equation, (Fig.(2)),

$$Q_i = m^{\cdot}_{b,0}\ Cp_i\ (t_i - t_{i+1}) = U_i\ A_i\ (LMTD)_i \qquad (12)$$

where,

$$(LMTD)_i = [(T_{d,i} - t_{i+1}) - (T_{d,i} - t_i)] / [Ln[(T_{d,i} - t_{i+1}) / (T_{d,i} - t_i)]$$

$$= [t_i - t_{i+1}] / [Ln[(T_{d,i} - t_{i+1}) / (T_{d,i} - t_i)]$$

From equation (12), t_i is obtained as,

$$t_i = t_{i+1} + [U_i\ A_i\ (LMTD)_i\ / (m^{\cdot}_{b,0}\ Cp_i)]$$

The dimensionless parameter $(U_i\ A_i\ / m_{b,0}\ Cp_i)$ includes the condenser heat transfer area, brine flow rate and the overall heat transfer coefficient. Therefore, this parameter can be called the dimensionless energy parameter, denoted by Z_i . In this case t_i can be estimated from the following relation,

$$t_i = T_{d,i} - [(T_{d,i} - t_{i+1})\ Exp(-Z_i)] \qquad (13)$$

To obtain the temperature of the brine at the outlet of the rejection section it is necessary to apply equation (13) on stage number M , hence one gets,

$$t_M = T_{d,M} - [(T_{d,M} - t_{sw})\ Exp(-Z_M)]$$

Referring to Fig.(2) , let $a_J = T_{d,M} - t_{sw}$

Then,

$$t_M = T_{d,M} - [a_J\ Exp(-Z_M)] \qquad (14)$$

Also , for stage number (M-1) the outlet temperature is obtained as follows,

$$t_{M-1} = T_{d,M-1} - [(T_{d,M-1} - t_M)\ Exp(-Z_{M-1})] \qquad (15)$$

Combining equations 14 and 15, t_{M-1} can be obtained as,

$$t_{M-1} = T_{d,M-1} - [T_{d,M-1} - T_{d,M} + a_J\ Exp(-Z_M)\ ]\ Exp(-Z_{M-1})]$$

$$t_{M-1} = T_{d,M-1} - (T_{d,M-1} - T_{d,M})\ Exp(-Z_{M-1}) - a_J\ Exp(-Z_M - Z_{M-1}) \qquad (16)$$

The distilled water temperature and brine flow temperature is obtained from the following equation,

$$T_{d,M} = T_{min} - L_j$$

where , L_j is the sum of the boiling point elevation (Bpe) of the brine and temperature losses (TL) which corresponds to pressure losses suffered by flashed vapor in rejection stage[2].

$$L_j = (Bpe)_j + (TL)_j$$

where, $(Bpe) = -0.4628 + 0.185\,X \qquad \text{for} \quad T < 70\ ^\circ C$

$$= -0.1543 + 0.165\,X \qquad \text{for} \quad 70\ ^\circ C < T < 90\ ^\circ C$$

$$(TL) = 1.15 - 0.01\,T \qquad \text{for} \quad 40\ ^\circ C < T < 90\ ^\circ C$$

The stage temperature drop in the heat rejection stages, ΔT_J is expressed as follows,

$$\Delta T_j = T_{d,M-1} - T_{d,M} = (T_{b,M-1} - L_j) - (T_{b,M} - L_j)$$

$$= (T_{b,M-1} - T_{b,M})$$

Accordingly, the outlet temperature from stage number (M-1) can be obtained from,

$$t_{M-1} = T_{d,M-1} - \Delta T_j\ Exp(-Z_{M-1}) - a_J\ Exp(-Z_M - Z_{M-1}) \qquad (17)$$

Repeating the above procedure until stage number (N+1) in the heat rejection section, the inlet brine temperature can be expressed by,

$$t_{N+1} = T_{d,N+1} - \Delta T_j \sum_{Y=N+1}^{M} \prod_{j=N+1}^{Y} Exp(-Z_j) - a_J \prod_{j=N+1}^{M} Exp(-Z_j) \qquad (18)$$

Distilled temperature at outlet from the first stage of heat rejection section is expressed as follows,

$$T_{d,N+1} = T_{d,N} - \Delta T_J$$

$$= T_{b,N} - L_j - \Delta T_j$$

where , $T_{b,N}$ is the temperature of the brine at inlet to the heat rejection section.

The brine temperature at outlet from the heat rejection section, $t_{s\,o} = t_{N+1}$ is expressed by,

$$t_{s\,o} = T_{b,N} - L_j - \Delta T_j \left[1 + \sum_{Y=N+1}^{M} \prod_{j=N+1}^{Y} Exp(-Z_j) \right] - a_j \prod_{j=N+1}^{M} Exp(-Z_j) \,] \tag{19}$$

Generally for the j^{th} stage in the heat rejection section, the outlet temperature t_j is given by,

$$t_j = T_{b,j-1} - L_j - \Delta T_j \left[1 + \sum_{Y=J}^{M} \prod_{j=J}^{Y} Exp(-Z_j) \right] - a_j \prod_{j=J}^{M} Exp(-Z_j) \,] \tag{20}$$

For the different heat rejection stages, the value of Z_j is substituted by an average value denoted by Z_{ja} .

Then, equation (19) takes the following form,

$$t_{so} = T_{b,N} - L_j - a_j Exp(-MZ_{ja}) - \Delta T_j \; \{[1 - Exp(-MZ_{ja})]/[1 - Exp(-Z_{ja})]\} \tag{21}$$

Inlet brine temperature to the heat recovery section, t_r is obtained from the following relation,

$$t_r = [\dot{m}_{si} Cp_b t_{s\,o} + (\dot{m}_{b,0} - \dot{m}_{si}) Cp_b T_{mm}] / (\dot{m}_{b,0} \; Cp_b) \tag{22}$$

Inserting the expression for ($\dot{m}_{si} / \dot{m}_{b,0}$) from equation (8), then equation (22) becomes,

$$t_r = T_{min} + (t_{s\,o} - T_{min}) \{ CR / [R(CR - 1)] \} \tag{23}$$

Inlet brine temperature to the brine heater can be expressed by,

$$t_{in} = T_{max} - L_r - a_r \prod_{i=1}^{N} Exp(-Z_i) - \Delta T_r \left[1 + \sum_{Y=1}^{N} \prod_{i=1}^{Y} Exp(-Z_i) \right] \tag{24}$$

where , $a_r = T_{d,N} - t_r$

Outlet brine temperature in the condenser tubes for the i^{th} stage in the heat recovery stages can be obtained as follows,

$$t_i = T_{b,i-1} - L_r - a_r \prod_{Y=i}^{N} Exp(-Z_Y) - \Delta T_r \left[1 + \sum_{L=i}^{N} \prod_{Y=i}^{L} Exp(-Z_Y) \right]$$

Similarly, one gets,

$$t_{i+1} = T_{b,i} - L_r - a_r \prod_{Y=i+1}^{N} Exp(-Z_Y) - \Delta T_r \left[1 + \sum_{L=i+1}^{N} \prod_{Y=i+1}^{L} Exp(-Z_Y) \right]$$

$$= T_{b,i} - L_r - a_r Exp(-Z_Y) \prod_{Y=i}^{N} Exp(-Z_Y) - \Delta T_r$$

$$- \Delta T_r \; Exp(-Z_Y) \left[1 + \sum_{L=i}^{N} \prod_{Y=i}^{L} Exp(-Z_Y) \right]$$

The brine temperature rise inside the condenser tubes in the i^{th} stage is given by,

$$\Delta t_i = t_i - t_{i+1}$$

$$\Delta t_i = [Exp(-Z_i) - 1] \left[a_r \prod_{Y=i}^{N} Exp(-Z_Y) + \Delta T_r \left(1 + \sum_{L=i}^{N} \prod_{Y=i}^{L} Exp(-Z_Y) \right)\right] \qquad (25)$$

Temperature rise in the brine heater is expressed by,

$$\Delta T_{DH} = T_{max} - t_{in}$$

$$= L_r + a_r \prod_{Y=i}^{N} Exp(-Z_Y) + \Delta T_r \sum_{L=i}^{N} \prod_{Y=i}^{L} Exp(-Z_Y) \qquad (26)$$

For different heat recovery stages the value of Z_i is substituted by an average value denoted by Z_a, hence one obtains,

$$\Delta T_{DH} = L_r + a_r \; Exp(-NZ_a) + \Delta T_r \{ [1 - Exp(-NZ_a)] / [1 - Exp(-Z_a)] \}$$

For complete condensation of all flashed vapor over condenser tubes in the heat recovery section, the value of a_r is given by ,

$$a_r = \Delta T_{BH} - L_r$$

Accordingly,

$$\Delta T_{BH} = L_r + \Delta T_r / [1-Exp(-Z_a)] \tag{27}$$

Equation (12) shows that the Logarithmic Mean Temperature Difference can be expressed by,

$$LMTD = \Delta t_i / Z_a \tag{28}$$

2.3 Produced mass flow rate:

The fraction (f_1) of the production rate of fresh water, in the first stage of heat recovery section, ($m^{\cdot}_{d,1}$) is expressed as follows,

$$f_1 = m^{\cdot}_{d,1} / m_{b,0}$$

$$= Cp_1 \ (\Delta T_r)_1 / (i_{fg})_1$$

The residual brine is then, $m^{\cdot}_{b,0} (1 - f_1)$

Similarly , for stage number, N the flashed fraction is expressed by ,

$$f_N = Cp_N \ (\Delta T_r)_N / (i_{fg})_N$$

The residual brine from stage number N is the product of each residual brine in each stage, $[m^{\cdot}_{b,0} (1 - f_1) (1-f_2) (1-f_3) ....(1-f_N)]$

In fact, the variation of i_{fg} and Cp with temperature for water can be ignored in the low temperature level. In this case the ratio (Cp/i_{fg}) can be considered constant, that is,

$$Cp_1 / (i_{fg})_1 = Cp_2 / (i_{fg})_2 = \; .... = Cp_N / (i_{fg})_N = Cp_a / (i_{fg})_a$$

where ,

Cp_a = average specific heat for the brine in every stage

$(i_{fg})_a$ = average latent heat of evaporation for the brine in every stage

Assuming equal interstage temperature drop (which is a reasonable assumption),

$$(\Delta T_r)_1 = (\Delta T_r)_2 = \; .... = (\Delta T_r)_N = (\Delta T_r) = FTR/N$$

Substitution with the foregoing average values yields the brine residual from stage N given by , $m^{\cdot}_{b,0}(1-f)^N$

where ,

$$f = (Cp_a\ (\Delta t_r)_a)/(i_{fg})_a$$
$$= (Cp_a\ FTR)/N\ (i_{fg})_a$$

Also, the production rate is obtained from the following relation,

$$m^{\cdot}_{d,N} = m^{\cdot}_{b,0}\ [1 - (1-f)^N]$$

For high values of N, the above equation can be written in the following form (based on the definition of the exponential function),

$$m^{\cdot}_{d,N} = m_{b,0}\ [1 - Exp\ (-Cp_a\ FTR/(i_{fg})_a\ )] \qquad (29)$$

Then , the recirculation ratio can be expressed by ,

$$R = [1 - Exp\ (-\ Cp_a\ FTR/(i_{fg})_a)]^{-1} \qquad (30)$$

2.4 Total input heat per unit distilled mass flow rate:

The required total input heat for MSF evaporator plant is given by ,

$$Q = m_{b,0}\ Cp_{BH}\ \Delta T_{BH}$$

Substituting for ΔT_{BH} , from equation (27) , the total input heat per unit distilled mass flow rate can be obtained as,

$$Q / m^{\cdot}_d = R\ Cp_{BH}\ \{ L_r + FTR / [N\ (1 - Exp(-Z_a)]\ \} \qquad (31)$$

2.5 Performance ratio:

It is known that, plant thermal performance ratio, PR is defined as [1],

$$PR = (2.23*10^6\ \ m_d\ ) / (m_{b,0}\ Cp_{BH}\ \Delta T_{BH}\ )$$

The total amount of heat transferred from the flashing vapor to the sea water flowing inside condenser tubes is given by,

$$m_d\ (i_{fg})_a = m^{\cdot}_{b,0}\ Cp_a\ \ (FTR)$$

Then, PR can be expressed as,

$$PR=[2.23*10^6 \ (FTR)\ Cp_a\ ] / [(i_{fg})_a\ Cp_{BH}\ \Delta T_{BH}\ ] \qquad (32)$$

For simplicity the value of Cp in the above equation can be considered constant. Under this assumption the expression for PR can be reduced to the following form,

$$PR= (FTR) / \Delta T_{BH}$$

Substituting for ΔT_{BH} from equation (27), then the foregoing equation takes the following form,

$$PR= (FTR) /\{ L_r + [(FTR /N) / (1 - EXP(-Z_a))]\} \qquad (33)$$

2.6 The required heat transfer area per unit distilled flow rate:

The total required heat transfer surface area is obtained from,

$$A = m^{\cdot}_{b.0}\ Cp_a\ (FTR) / U\ (LMTD)$$

Using the dimensionless energy parameter (Z_i) then,

$$A / m_d = N\ R\ Cp_a\ (Z_a\ ) / \ U \qquad (34)$$

The following simplified form of surface area per unit mass flow rate of distilled water is derived from the forgoing equations in terms of performance ratio, number of stages and flash temperature range,

$$A/m^{\cdot}_d=[N(i_{fg})_a/U(FTR)]\{Ln\{[(FTR/PR) - L_r\ ]/[\ (FTR/PR)-L_r - (FTR/N)\ ]\}\} \qquad (35)$$

2.7 Operating conditions

The calculations are performed for the following operating conditions,
Inlet brine sea water temperature, t_{sw} =20 °C.
Maximum brine temperature, T_{max} =90 °C.
Blow down temperature, T_{min} =30 °C.
Flash temperature range, FTR =60 °C.
Concentration ratio, CR=1.5.
Sea water concentration, X_{sw} = 4 %.
Distilled water mass flow rate, m_d =100 Ton/hr.

In addition the value of the overall heat transfer coefficient in condenser is taken as an average value of 3000 W/ m^2 °C [1].

3- RESULTS AND DISCUSSION

A computer code is designed to predict the performance of MSF plant for different operating conditions. The behavior of the performance ratio PR, versus dimensionless energy parameter Z, for different number of stages N, is shown in Fig.(3) for the specified value of fresh water flow rate (100 Ton/hr) and flash temperature range (60 °C). The obtained results show that the PR increases rapidly with increasing Z , up to the value, Z = 4. After this value of Z, the increase of PR, with Z, is marginal. This means that, there is an economic value of the parameter Z (Z indicates the heat transfer area), beyond which there is a very small increase in the PR . From the same figure, it is clear that the PR, increases with increasing number of stages N, for a certain value of Z.
To evaluate the effect of increasing the number of stages on the performance ratio, the percentage increase of PR, must be estimated for the same Z. The performance ratio increases by 59.1% as N, increases from 20 to 40 stages, and PR, increases by 23.6% as N, increases from 40 to 60 stages. On the other hand PR, increases by 13.7% as N, increases from 60 to 80 stages. This indicates that the rate of increase in PR, decreases as N, increases for the same Z. This indicates that there is a value of N after which the gain in the performance ratio can be considered unimportant. This number of stages lies between 40 to 60 stage, where the gain in the performance ratio after this range is less than 13.7% (as shown in Fig.(3)).

Similar results can be obtained from Fig.(4) on which the total surface condenser area A, is plotted against the performance ratio PR, for different number of stages N. From this figure, it is clear that the total condenser surface area increases rapidly and sharply beyond a certain values of PR according to the number of stages.

Figure (5) illustrates the behavior of the total input heat Q, in MW with the dimensionless energy parameter, Z . It is clear that the required total input heat decreases sharply with increasing value of Z up to Z=4. After this value of Z. the required total input heat remains approximately constant.

Examining figures (3) and (5), it is clear that the value of Z=4 is an economic operating value for the pre-specified operating condition. At this value, the total input heat is minimum and the performance ratio is maximum for all values of number of stages. For this value of Z, it is also important to note that for all values of N, the minimum total input heat is less than 5 MW.

Figure (6) shows a comparison between the present results and the results obtained by K.Minnich et al [4], which indicates a fair agreement.

4- CONCLUSIONS

A theoretical analysis of MSF evaporator plant is developed to predict the plant performance for different operating parameters. From the above discussion one can conclude that there is a recommended number of stages (N=40-60) to produce a pre-determined fresh water flow rate of 100 Ton/hr for a flash temperature range of 60 °C. In addition, it is found that there is an economic value for the dimensionless energy parameter, which is Z=4.

NOMENCLATURE

A	Heat transfer surface area	(m^2)
Cp	Specific heat	(J / kg ° C)
CR	Concentration ratio. ($CR = X_{bd} / X_{sw}$)	___
FTR	Flash temperature range, ($T_{max} - T_{min}$)	(°C)
i	Enthalpy.	(J / kg)
i_{fg}	Latent heat of evaporation.	(J / kg)
$\dot{m}_b$	Brine water mass flow rate.	(kg/s)
$\dot{m}_{b.0}$	Inlet brine water to the first stage of recovery section.	(kg/s)
$\dot{m}_{bd}$	Brine blow down mass flow rate.	(kg/s)
$\dot{m}_d$	Distilled or product mass flow rate.	(kg/s)
$\dot{m}_r$	Brine recirculation mass flow rate.	(kg/s)
$\dot{m}_{si}$	Inlet sea water mass flow rate.	(kg/s)
$\dot{m}_{so}$	Outlet sea water mass flow rate.	(kg/s)
$\dot{m}_{sw}$	Sea water mass flow rate.	(kg/s)
M	Number of stages in heat recovery section.	___
N	Number of stages in heat rejection section.	___
PR	Performance ratio	___
Q	Total input heat	(W)
R	Recirculation ratio ($R = \dot{m}_{b.0} / \dot{m}_d$)	___
T	Temperature	(°C)
t	Brine temperature inside condenser tubes	(°C)
t_r	Brine temperature inlet to the recovery section	(°C)
t_{in}	Inlet brine temperature to the brine heater.	(°C)
T_{max}	Maximum brine temperature.	(°C)
T_{min}	Minimum brine temperature.	(°C)

Δt_j	Brine temperature rise inside condenser tubes in stage i	(°C)
ΔT_{BH}	Brine heater temperature rise .	(°C)
ΔT_j	Temperature drop in each stage of heat rejection stages	(°C)
ΔT_r	Temperature drop in each stage of heat recovery stages	(°C)
X	Salt concentration.	%
Z	Dimensionless energy parameter.	—

SUBSCRIPTS

$_{bd}$	Blow down
$_{b,0}$	Inlet to first stage of the recovery section
$_{d}$	Distilled
$_{r}$	Recovery stage
$_{si}$	Inlet sea water
$_{so}$	Outlet sea water
$_{sw}$	Sea water

REFERENCES

1- Andrew Porteous "Saline water distillation processes" Longman book company , 1975

2- N.H.. ALY "Simulation of MSF desalination plant" Al-Azhar Engineering Second International Conference pp. 108 ,1991

3- M.A. Darwish and H. El-Dessouky "Desalination by distillation processes: A technical comparison" IDA world congress on desalination and water sciences, pp.219-232, Vol.II, Abu Dhabi, Nov. 18-24, 1995

4- Keith Minnich, John Tonner, and Dorothy Neu "A comparison of heat transfer requirement and evaporator cost for MED-TC and MSF" IDA world congress on desalination and water sciences, pp.233-257, Vol.II, Abu Dhabi, Nov. 18-24, 1995

5- K.S. Spiegler "Principles of desalination" New York, 1966

6- A.W. Veenman "A review of new developments in desalination by distillation processes" Desalination Vol. 27, pp. 21-39, 1978

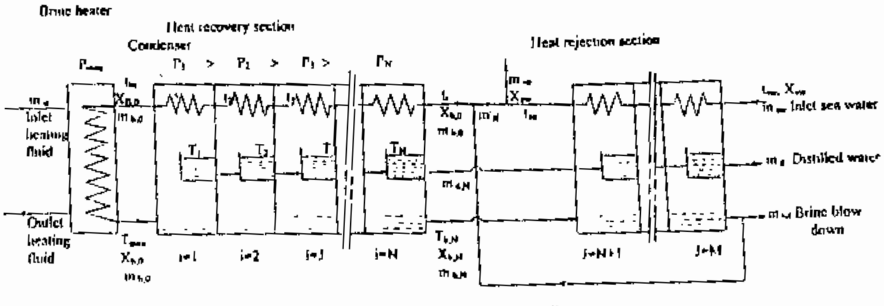
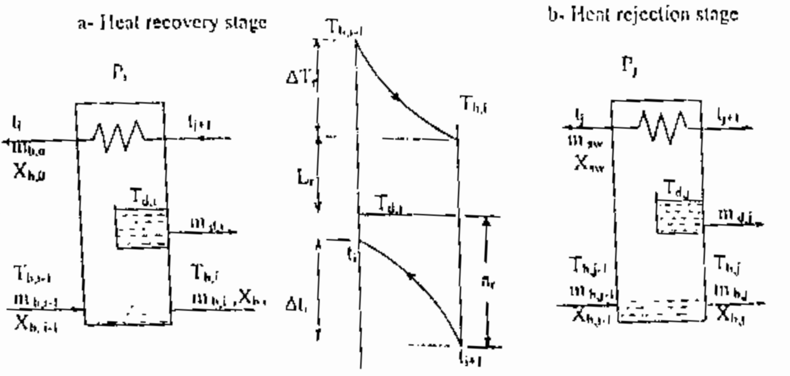


Fig. (1) Schematic representation for MSF plant



a- Heat recovery stage

b- Heat rejection stage

Fig. (2) Temperature, concentration and mass flow rate for different stages of MSF plant

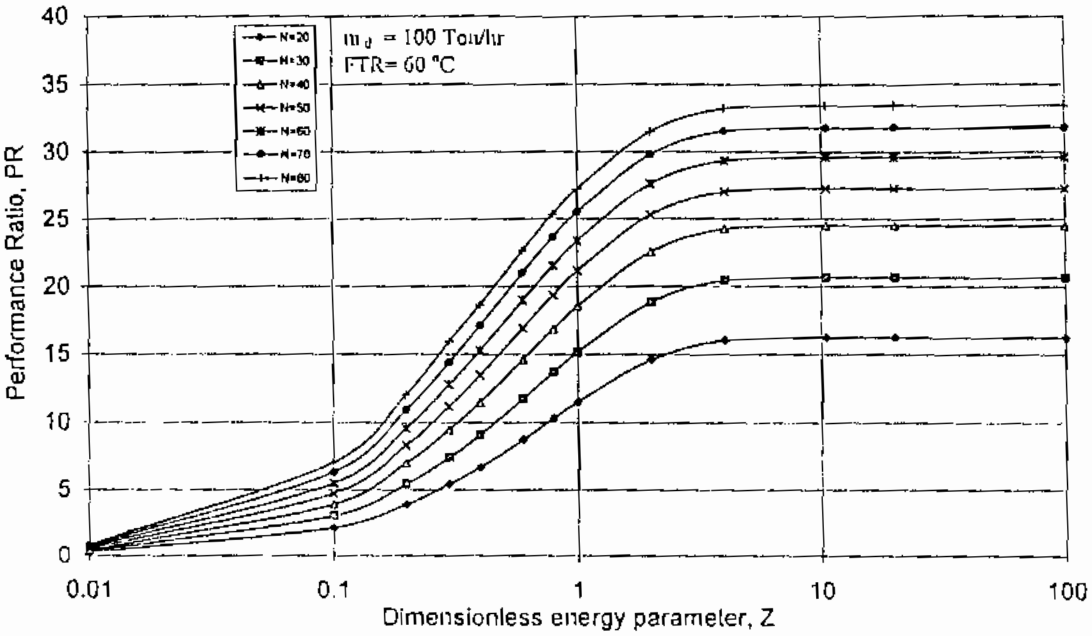


Fig.(3) Performance Ratio versus dimensionless energy parameter for different number of stages

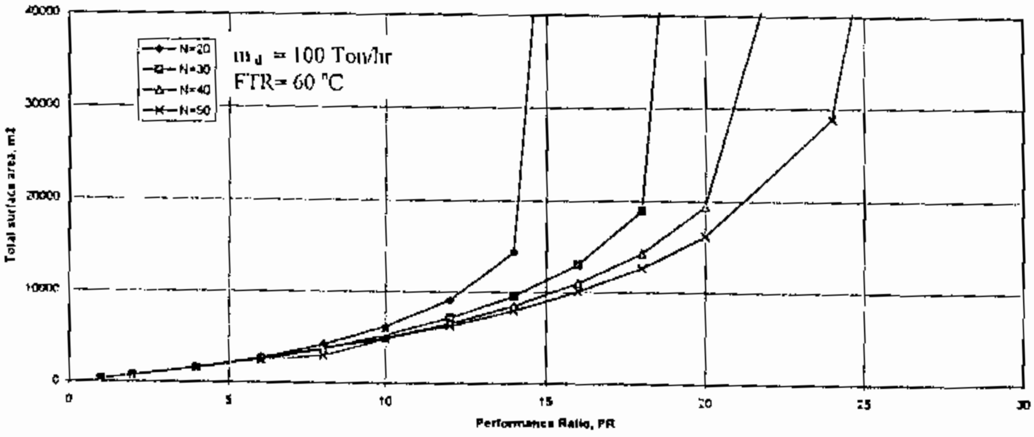


Fig.(4) The required total surface area versus Performance Ratio for different number of stages

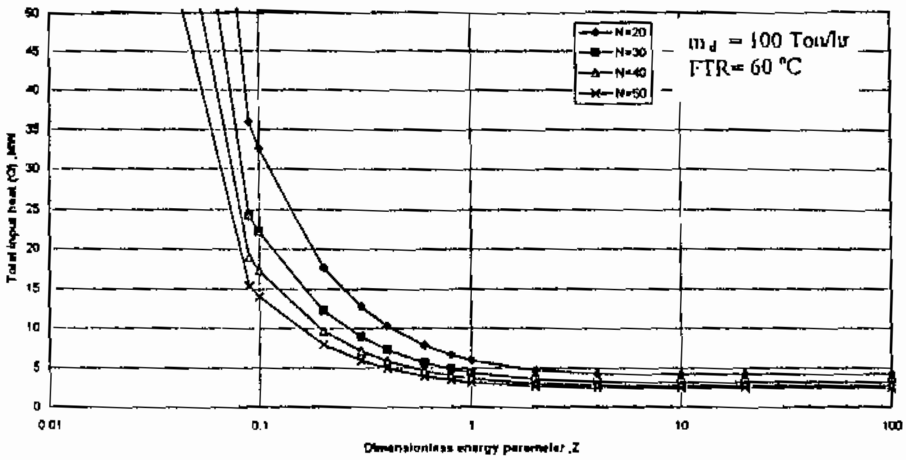


Fig. (5) The required input heat versus dimensionless energy parameter for different number of stages

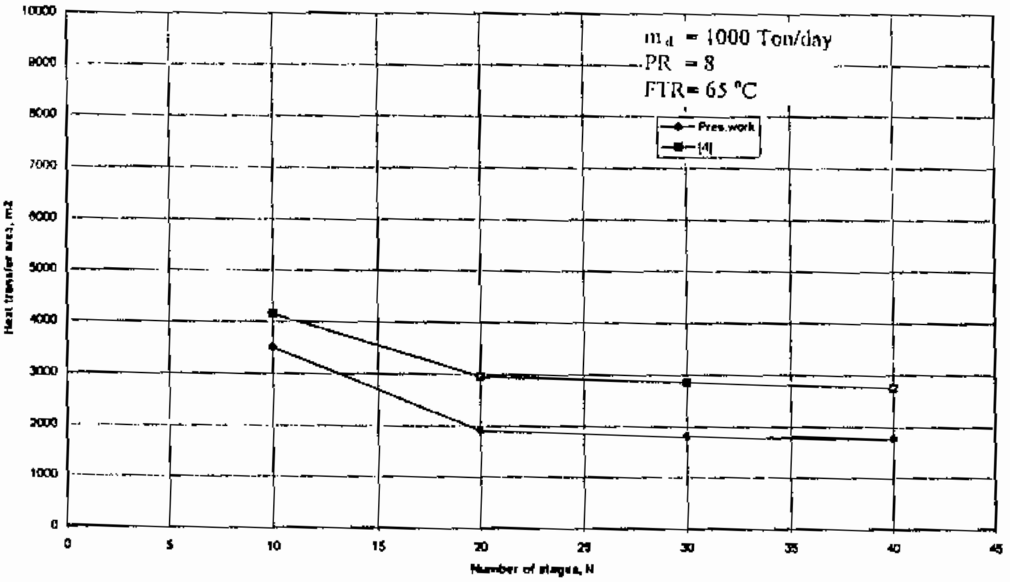


Fig.(6) Comparison between the present work and the model proposed by K.Minnich et al [4]