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DESIGNING MINIMUM-INERTIA SPUR GEAR TRAINS

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ABSTRACT

The problem of determining the optimum dimensions of all the gears in a multi-stage reduction unit and the optimum reduction ratio of each gear mesh has been investigated. One objective, namely, the minimization of the equivalent moment of inertia for all the gears is considered in this work. In addition to the design constraints according to the AGMA method, some coupling constraints have been included in the formulation. The problem is formulated as a standard mathematical programming problem, and nonlinear programming technique is used to solve the problem. The optimization is determined by geometrical programming (GP) combined with linear programming (LP). This mathematical programming approach can handle as many constraints as required, and it is an effective approach, since for a problem with many constraints the "degree of difficulty" is large and the optimization procedure becomes more complex with higher degrees of difficulty. A numerical example is given to illustrate the effectiveness of the GP-LP approach, and the results are compared with those available in the literature.

INTRODUCTION

Servodrives provided with dc servomotors are designed to follow velocity, acceleration, and position command instantaneously and with great accuracy. Many instruments and servomechanism drives must also function intermittently, or be capable of changing speed or starting and stopping quickly. As a result, their mechanical and electrical characteristics are quite different from those of other motors. Whereas ordinary motors are specified by current, voltage, torque and speed ratings, servomotors must be selected on the basis of these factors plus the mechanical time constant, rotor inertia, and heating characteristics as well. Generally, servodrives loads are of two types, inertial and frictional. These two types are considered separately because they affect motor operation in different ways. An ineritial load must be accelerated and deccelerated, a frictional load tends to deccelerate the motor by itself. However, detailed discussion on frictional load is beyond the scope of this investigation.

The servomotor is chosen for its time constant, including its rotor inertia, the inertia of all gears in the reduction unit, and the load inertia which to be accelerated. In servodrives theinertia load is usually predetermined, and the inertia of all gears in the reduction unit is the only factor that must be considered to minimize the mechanical time constant of the servomotor that affect the dynamic of the servodrive. This means that, to minimize the mechanical time constant of servodrive, the key design factor is the inertia. Reducing the inertia of all gears in the reduction unit of servodrive to a minimum value results in a faster response and lower drivemotion torque requirements.

Many previous studies [1-4] have dealt with the design of reduction units for a minimum inertia and or minimum weight. In these studies, the pinions were considered identical and all gears are taken with an equal thickness. Attempts have also been made for a prespecified number of meshes (two). However, the dimensions of the gears (such as pitch diameter, number of teeth, face width and diameteral pitch), and the design constraints (such as strength equation, and fundamental wear equation of the gear tooth) have not been taken in consideration. It is believed that the techniques used in those investigations [1 and 2] would lead to a gear size larger than the optimum one.

The proposed design synthesis presented here pin-points the dimensions of all gears and the reduction ratio of each gear mesh that will give a minimum moment of inertia of all gears in the reduction unit. It requires only the formulation of the objective function from the geometrical arrangement of the reduction unit and selection of a suitable design and side constraints.

The fact that the number of teeth and the diametral pitch of any gear are integer and discrete variables respectively can pose formidable problems for the optimizer, because most of the available methods apply to the problems where all variables are continuous. Probably the simplest approach that can be used to solve such problem is by rounding off. First treat the problem as a problem of continuous variables, ignoring all integer and discrete requirements, and solve it by the available continuous methods. Next, round of the optimal solution to the nearst integer and discrete points [5] . For the sake of conversience, we shall consider first the case of nonlinear continuous problem.

In this paper the optimum design variables for each mesh in the reduction unit are determined by geometrical programming (GP) combined with linear programming (LP). This approach can handle as many constraints as required.

An example is given to illustrate the advantage using the GP-LP optimization technique.

PROBLEM FORMULATION

- Objective Function

The selection of minimization for the moment of inertia of all gears in the reduction unit as a basic objective function is practical and also satisfies indirectly one or more of the following conditions:

1. the total size of the reduction unit is minimum;
2. the wight of all gears are kept to a minimum; and
3. the total cost of the reduction unit is then minimized.

The basic objective function and the design constraints are based on the following:

a) All diameters used are pitch diameter.
b) The surface durability factor of each gear mesh is constant.
c) For simplicity, all gears are solid disks and are made from material of the same density. The fact that the gears usually are not solid disks, but may contain holes and webbed cross-sections, is taken into consideration after determining the optimum parameters of the reduction unit. This will not affect the computed reduction gear relationship.

   For any gear, the moment of inertia can be written as:

$$J = \frac{\pi \cdot \rho . b . d^4}{32} \tag{1}$$

The inertia of any gear in the multi-stage reduction unit shown in Fig. (1) will be calculated as an equivalent inertia referred to the driving pinion. For any pinion (J) this is given by

$$J_{pj} = \frac{C . b_j . d_{pj}^4}{\prod_{k=1}^{j-1} m_k^2} \tag{2}$$

Where $\left(\prod_{k=1}^{j-1} m_k^2\right)$ is the reduction ratio from the driving pinion to pinion (j), and

$j = 1, 2, \ldots\ldots\ldots n.$

The equivalent moment of inertia for any gear (j) is given by

$$J_{gj} = J_{pj} \cdot m_j^2 \qquad (3)$$

Using Eqns. (2) and (3), the total equivalent moment of inertia of all gears in the reduction unit can be computed as:

$$J_t = C \cdot \sum_{j=1}^{n} \frac{b_j \cdot d_{pj}^4 \cdot (1 + m_j^2)}{\prod_{k=1}^{j-1} m_k^2} \qquad (4)$$

Equation (4) can be rewritten as a function of the discrete nature of the pinion (J) (number of teeth N_{pj}, face width b_j, and diametral pitch P_j), the number of gear mesh m_j, i.e.

$$J_t = C \sum_{j=1}^{n} \frac{b_j \cdot N_{pj}^4 \cdot (1 + m_j^2)}{P_j^4 \prod_{k=1}^{j-1} m_k^2} \qquad (5)$$

Now the objective function of this design synthesis, namly Eqn. (5), would be used to find the design variables of each mesh in the reduction unit (b_j, N_{pj}, P_j, m_j) that will give the minimum equivalent moment of inertia for all gears. In order to accomplish this objective, the design synthesis can be treated as an optimization problem so that the design and side constraints will be stated below, are fulfield.

- Design and Side Constraints

Designing gears presents an extremely difficult problem because it is primarily a trial and error procedure. However, there are several methods that can be used to develop the design procedure. The AGMA method [6] may be used. It is particularly useful because it applies correction factors to the original design equations that compensate for some of erroneous assumptions made in the derivation as well as for important factors not originally included. Furthermore, since most of the factors are obtained empirically. The AGMA equations can be kept up to date by merely changing the values of the factori as more information about gear behavior is obtained.

Gears generally fail because the actual loads applied to the teeth are greater than the allowable loads based upon either the beam strength of the tooth (tooth fracture) or its wear strength (surface failure). The final strength equation to be used is the AGMA modification of the Lewis equation. The strength equation for any pinion is written as follows [6]:

$$\sigma_t = \frac{F_t \cdot P \cdot K_o \cdot K_s \cdot K_n}{b \cdot K_v \cdot G} \qquad (6)$$

It is necessary to compare the calculated stress at the root of tooth with the maximum allowable design stress. The AGMA equation for the maximum allowable design stress (S_{ad}) is

$$S_{ad} = \frac{S_{at} \cdot K_l}{K_t \cdot K_R} \qquad (7)$$

To sum up the AGMA method for designing spur gears for strength, the calculated stress of equation (6) must be less than or equal to the maximum allowable design stress as determined by equation (7), i.e.,

$$\sigma_t \leq S_{ad} \qquad (8)$$

Normalizing inequality (8) with respect to the face width the number of teeth, diametral pitch and the reduction ratio from the driving pinion to the jth pinion, the first design constraint is expressed by

$$\frac{2\, T_{p1} \cdot P_j^2 \cdot \prod_{k=1}^{j-1} m_k}{b_j \cdot N_{pj}} \leq Q \qquad (9)$$

Where (Q) is equal to $\left(\frac{S_{at} \cdot K_l \cdot K_v \cdot G}{K_o \cdot K_s \cdot K_l \cdot K_t \cdot K_R}\right)$.

Having discussed the first of the general causes of tooth failure, it is now appropriate to consider the second category, namely; surface destruction. The fundamental wear equation for any pinion is given by:

$$\sigma_c = C_p \cdot \sqrt{\frac{F_t}{d_p . b} \cdot \frac{C_p \cdot C_s \cdot C_m \cdot C_f}{C_v \cdot I}} \qquad (10)$$

The AGMA specifies that the calculated stress number must be less than or equal to an allowable contact stress number (S_{ac}) which has been modified by several correction factors, i.e.,

$$\sigma_c \leq S_{ac}\left(\frac{C_l \cdot C_H}{C_T \cdot C_R}\right) \qquad (11)$$

Normalizing inequality (11) with respect to the face width, the number of teeth, diametrial pitch, and the reduction ratio from the driving pinion to the jth pinion, the second design constraints can be expressed as:

$$\frac{2T_{p_1} \cdot P_j^2 \cdot \prod_{k=1}^{j-1} m_k}{b_j \cdot N_{pj}^2} \leq U \qquad (12)$$

Where (U) equals to $\left(\frac{S_{ac} \cdot C_l \cdot CH}{C_T \cdot C_R \cdot C_p}\right)^2 \cdot \left(\frac{C_v \cdot I}{C_o \cdot C_s \cdot C_m \cdot C_f}\right)$.

It is obvious that, the values of the factors (K_l, K_v, G, K_o, K_s, K_n, K_t, K_R, S_{at}, C_l, C_H, C_T, C_R, C_p, C_V, I, C_o, C_s, C_m, C_f and S_{ac}) are chosen, baded either on the values suggested by the AGMA or upon the personal experience the designer.

At this stage of the problem formulation, the optimum design of a gear train consists of the objective function the design constraints (9) and (12). However, at this point we might not be certain as to what other constraints should be included in the formulation of the optimization problem to properly complete tying the optimization problem together mathematically. The basic key for completion of the formulation of the optimization problem lies in a recognization of the side constraints which we consider to be significant for the problem. Hence in review of our machine setting including manufacturing and practical considerations, suppose we recognize the significance of the following additional side constraints:

$$N_{pj} \geqslant N_{min} \tag{13},$$

$$\frac{b_j \cdot P_j}{N_{pj}} \leq \beta \tag{14}$$

$$(m - \Delta m) \leqslant \prod_{j=1}^{n} m_j \leqslant (m + \Delta m) \tag{15}$$

SOLUTION PROCEDURE

The minimum inertia design problem formulated in the previous section can be converted to the standard form of a nonlinear mathematical programming problem without much difficulty. This section gives the method of solution.

As the objective function, Eqn. (5) and the constraints, Eqns. (9, 12, 13, 14 and 15), are posynomials in b_j, P_j, N_{pj} and m_j the problem is a geometric programming in nature. It is usually formulated as follows[7]:

-Primal Program:

Find the design vector X = $\begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \\ \hline P_1 \\ P_2 \\ \vdots \\ P_n \\ \hline N_{p1} \\ \vdots \end{Bmatrix}$

$$\begin{bmatrix} \vdots \\ N_{pn} \\ \hline m_1 \\ m_2 \\ \vdots \\ m_n \end{bmatrix}$$

which minimizes the objective function Eqn. (5) and satisfies the constraints

$$2T_{pl}\ Q^{-1} \cdot P_j^2 \cdot b_j^{-1} . N_{pj}^{-1} \prod_{k=1}^{j-1} m_k \leq 1 \tag{16}$$

$$2T_{pl}\ U^{-1} \cdot P_j^2 \cdot b_j^{-1} .\ N_{pj}^{-2} \prod_{k=1}^{j-1} m_k \leq 1 \tag{17}$$

$$\beta^{-1} \cdot b_j \cdot P_j \cdot N_{pj}^{-1} \leqslant 1 \tag{18}$$

$$N_{min} \cdot N_{pj}^{-1} \leqslant 1 \tag{19}$$

$$(m + \Delta m)^{-1} \prod_{j=1}^{n} m_j \leqslant 1 \tag{20}$$

$$(m - \Delta m) \prod_{j=1}^{n} m_j^{-1} \leqslant 1 \tag{20a}$$

The quantity $(No + N_k) - (N_v + 1)$ is termed as a degree of difficulty in the geometric programming. Since $No = (2n)$, $N_k = (4n + 2)$ and $N_v = (4n)$, this problem has $(2n+1)$ degree of difficulty, i.e., the degree of difficulty equals to twice the number of meshes (n) plus one.

-Dual Program:

The dual problem can be stated as follows [7]:

$$\text{Find} \quad W = \begin{Bmatrix} W_{o1} \\ \vdots \\ W_{o(2n)} \\ \hline W_{11} \\ W_{21} \\ \vdots \\ W_{(4n+2)1} \end{Bmatrix}$$

so as to maximize

$$V(W) = \prod_{j=1}^{2n} \left(\frac{C_{oj}}{W_{oj}}\right)^{W_{oj}} \prod_{k=1}^{(4n+2)} (C_{kl})^{W_{kl}} \qquad (21)$$

subject to:

the normality constraints

$$\sum_{j=1}^{2n} W_{oj} = 1 \qquad (22)$$

the orthogonality constraints

$$\sum_{j=1}^{2n} a_{oij} \cdot W_{oj} + \sum_{k=1}^{(4n+2)} a_{ikl} \cdot W_{kl} = 0 \qquad (23)$$

and the nonnegativity constraints

$$W_{oj} \geqslant 0 , \qquad (24)$$

$$W_{kl} \geqslant 0$$

$i = (1, 2, \dots \; 4n)$

Maximizing Eqn. (21) is equivalent to minimizing

$$\ln V(W) = -\left[\sum_{j=1}^{2n} W_{oj} \cdot \ln C_{oj} + \sum_{k=1}^{(4n+2)} W_{kl} \cdot \ln C_{kl}\right] + \left[\sum_{j=1}^{2n} W_{oj} \cdot \ln W_{oj}\right] \qquad (25)$$

subject to (22), (23) and (24).
The problem, as rewritten by Eqns. (25), (22), (23) and (24) can be solved by making a piece wise linear approximation [8] for the last term in Eqn. (25), i.e. for the function

$$f = \sum_{j=1}^{2n} W_{oj} \cdot \ln W_{oj}.$$

Hence, the entire linear dual formulation can be solved efficiently using the standard simplex

Once the optimum value of the dual function and the optimum values fo the dual variables (W_{oj} and W_{kl}) are obtained, the next step is to determine the optimum values of the design variables b_j, P_j, N_{pj} and m_j . This can be achieved by solving simultaneously the equations:

$$W_{oj} = \frac{\text{jth term in the objective function}}{X_o^*} \qquad (26)$$

$$\begin{aligned} j &= 1, 2, \ldots\ldots\ldots\ldots 2n \\ l &= \text{kth constraint.} \qquad (27) \\ K &= 1, 2, \ldots\ldots\ldots\ldots (4n+2) \end{aligned}$$

NUMERICAL EXAMPLE

A numerical example is considered to illustrate the effectiveness of the optimization procedure developed.

The following numerical data is used in the problem:

$m=8$, $m=0.5$, $n=3$, $T_{pl}=180$ lb.in, $S_{at}=19000$ PSi,

$K_l = K_T = K_i = C_s = C_f = C_l = C_H = C_T = 1$, $K_R = C_m = 1.2$,

$K_o = C_o = 1.25$, $K_n = 1.6$, $K_v = C_v = 0.62$, $G=0.32$,

$I = 0.15$, $C_R = 1.15$, $C_p = 2300$, $S_{ac} = 85000$, $=1.2$,

$N_{min} = 14$, and $= 0.28$ lb/in^3.

The optimization problem can be stated as follows:

$$\text{Find} \qquad X = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ p_1 \\ p_2 \\ p_3 \\ N_{p1} \end{Bmatrix}$$

$$\begin{pmatrix} N_{p2} \\ N_{p3} \\ m_1 \\ m_2 \\ m_3 \end{pmatrix}$$

as to minimize

$$F(x) = 0.027\ b_1.P_1^{-4}.N_{p1}^{4}+b_1.P_1^{-4}.N_{p1}^{4}m_1^{2}+b_2.P_2^{-4}.N_{p2}^{4}.$$
$$m_1^{-2}+b_2.P_2^{-4}.N_{p2}^{4}.m_1^{-2}\ m_2^{2}+b_3.P_3^{-4}.N_{p3}^{4}.m_1^{-2}.m_2^{-2}+$$
$$64.b_3.P_3^{-4}.N_{p3}^{4}.\ m_1^{-4}.m_2^{-4}. \qquad \text{(E1)}$$

Subject to

$$\left.\begin{array}{lcl}
0.2268\ P_1^{2}\ .\ b_1^{-2}.\ N_{p1}^{-1} & \leqslant & 1 \\
0.2268\ P_2^{2}\ .\ b_2^{-1}.\ N_{p2}^{-1}.\ m_1 & \leqslant & 1 \\
0.2268\ P_3^{2}\ .\ b_3^{-1}.\ N_{p3}^{-1}.\ m_1.m_2 & \leqslant & 1 \\
5.616\ P_1^{2}\ .\ b_1^{-1}.\ N_{p1}^{-2} & \leqslant & 1 \\
5.616\ P_2^{2}\ .\ b_2^{-1}.\ N_{p2}^{-2}\ .\ m_1 & \leqslant & 1 \\
5.616\ P_3^{2}\ .\ b_3^{-1}.\ N_{p3}^{2}\ .\ m_1.m_2 & \leqslant & 1 \\
0.833\ P_1\ .\ b_1\ .\ N_{p1}^{-1} & \leqslant & 1 \\
0.833\ P_2\ .\ b_2\ .N_{p2}^{-1} & \leqslant & 1 \\
0.833\ P_3\ .\ b_3\ .\ N_{p3}^{-1} & \leqslant & 1 \\
14\ N_{p1}^{-1} & \leqslant & 1 \\
14\ N_{p2}^{-1} & \leqslant & 1 \\
14\ N_{p3}^{-1} & \leqslant & 1 \\
7.5\ m_1^{-1}\ .\ m_2^{-1}\ .\ m_3^{-1} & \leqslant & 1 \\
0.1176\ m_1\ .\ m_2\ .\ m_3 & \leqslant & 1
\end{array}\right\}\ \text{(E2)}$$

The dual problem can be stated as follows:

$$\text{Find} \quad W = \begin{Bmatrix} W_{01} \\ W_{02} \\ W_{03} \\ W_{04} \\ W_{05} \\ W_{06} \\ W_{11} \\ W_{21} \\ W_{31} \\ W_{41} \\ W_{51} \\ W_{61} \\ W_{71} \\ W_{81} \\ W_{91} \\ W_{101} \\ W_{111} \\ W_{121} \\ W_{131} \\ W_{141} \end{Bmatrix}$$

So as to maximize

$$V(W) = \left(\frac{0.027}{W_{o1}}\right)^{W_{o1}} \cdot \left(\frac{0.027}{W_{o2}}\right)^{W_{o2}} \cdot \left(\frac{0.027}{W_{o3}}\right)^{W_{o3}} \cdot \left(\frac{0.027}{W_{o4}}\right)^{W_{o4}}$$

$$\left(\frac{0.027}{W_{o5}}\right)^{W_{o5}} \cdot \left(\frac{1.728}{W_{o6}}\right)^{W_{o6}} . 0.2268^{(W_{11}+W_{21}+W_{31})}$$

$$5.616^{(W_{41}+W_{51}+W_{61})} . \ 0.833^{(W_{71}+W_{81}+W_{91})} .$$

$$14^{(W_{101} + W_{111} + W_{121})} . \ 7.5^{(W_{131})} . 0.1176^{(W_{141})} \qquad \text{(E3)}$$

Subject to

$$
\left.
\begin{aligned}
&W_{01} + W_{02} + W_{03} + W_{04} + W_{05} + W_{06} &= 1\\
&W_{01} + W_{02} - W_{11} - W_{41} + W_{71} &= 0\\
&W_{03} + W_{04} - W_{21} - W_{51} + W_{81} &= 0\\
&W_{05} + W_{06} - W_{31} - W_{61} + W_{91} &= 0\\
&-4W_{01} -4 W_{02} + 2 W_{11} + 2 W_{41} + W_{71} &= 0\\
&-4W_{03} -4 W_{04} + 2 W_{21} + 2 W_{51} + W_{81} &= 0\\
&-4W_{05} -4 W_{06} + 2 W_{31} + 2 W_{61} + W_{91} &= 0\\
&4W_{01} + 4 W_{02} - W_{11} -2 W_{41} - W_{71} - W_{101} &= 0\\
&4W_{03} + 4 W_{04} - W_{21} -2 W_{51} - W_{81} - W_{111} &= 0\\
&2W_{02} - 2 W_{03} - 2 W_{04} - 2 W_{05} - 4 W_{06} + W_{21} +\\
&\quad W_{31} + W_{51} + W_{61} - W_{131} + W_{141} &= 0\\
&2W_{04} - 2W_{05} -4W_{06} + W_{31} + W_{61} - W_{131} + W_{141} &= 0\\
&-W_{131} + W_{141} &= 0
\end{aligned}
\right\} \text{(E4)}
$$

$$W_{oj} \geqslant 0 \qquad J = 1, 2, \ldots, 6$$

$$W_{k1} \geqslant 0 \qquad K = 1, 2, \ldots, 14$$

Maximizing (E3) is equivalent to minimizing

$$
\begin{aligned}
\ln V(W) = \Big[& 3.61(W_{01}+W_{02}+W_{03}+W_{04}+W_{05}) -\\
& 0.546\, W_{06} + 1.48(W_{11}+W_{21}+W_{31}) -\\
& 1.725(W_{41}+W_{51}+W_{61}) + 0.182(W_{71}+W_{31}+\\
& W_{91}) - 2.639(W_{101}+W_{111}+W_{121}) - 2.015(W_{131}) +\\
& 2.14(W_{141}) + W_{01} \cdot \ln W_{01} + W_{02} \ln W_{02} +\\
& W_{03} \ln W_{03} + W_{04} \ln W_{04} + W_{05} \ln W_{05} +\\
& W_{06} \ln W_{06} \Big]
\end{aligned}
\qquad \text{(E5)}
$$

The problem can be solved by linear programming by making a piecewise linear approximation to the each terms, $(W_{oj}.\ln W_{oj})$, in Eqn. (E5). This is accomplished by replacing each curve,

$(B_j = W_{oj}.\ln W_{oj})$, by a broken line, B=B (A). Linear programming can now be used to minimize (E5) subject to (E4), if (E5) is replaced by the approximation

$$\approx 3.61\ W_{01} + 3.61\ W_{02} + 3.61\ W_{03} + 3.61\ W_{04} + 3.61\ W_{05} - 0.546\ W_{06} + 1.48\ W_{11} + 1.48\ W_{21} + 1.48\ W_{31} - 1.725\ W_{41} - 1.725 W_{51} - 1.725\ W_{61} +$$

$$0.182\ W_{71} + 0.182\ W_{81} + 0.182\ W_{91} - 2639\ W_{101} - 2.639\ W_{111} - 2.639\ W_{121} - 2.015\ W_{131} + 2.14\ W_{141} +$$

$$(A_{11}.\ln A_{11} + A_{12}.\ \ln A_{12} + A_{13}.\ \ln A_{13}) +$$

$$(A_{21}.\ \ln A_{21} + A_{22}.\ \ln A_{22} + A_{23}.\ln A_{23}) +$$

$$(A_{31}.\ \ln A_{31} + A_{32}\ \ln A_{32} + A_{33}.\ \ln A_{33}) +$$

$$(A_{41}.\ \ln A_{41} + A_{42}\ .\ \ln A_{42} + A_{43}\ .\ln A_{43}) +$$

$$(A_{51}.\ \ln A_{51} + A_{52}.\ln A_{52} + A_{53}.\ \ln A_{53}) +$$

$$(A_{61}.\ \ln A_{61} + A_{62}.\ln A_{62} + A_{63}.\ \ln A_{63}). \qquad \text{(E6)}$$

Which is also subject to

$$\left.\begin{array}{ll} -W_{01} + A_{11}\ W_{011} + A_{12}\ W_{012} + A_{13} W_{013} & = 0 \\ -W_{02} + A_{21}\ W_{021} + A_{22}\ W_{022} + A_{23} W_{023} & = 0 \\ -W_{03} + A_{31}\ W_{031} + A_{32}\ W_{032} + A_{33} W_{033} & = 0 \\ -W_{04} + A_{41}\ W_{041} + A_{42}\ W_{042} + A_{43} W_{043} & = 0 \\ -W_{05} + A_{51}\ W_{051} + A_{52}\ W_{052} + A_{53} W_{053} & = 0 \\ -W_{06} + A_{61}\ W_{061} - A_{62} W_{062} + A_{63}\ W_{063} & = 0 \\ W_{01} + W_{012} + W_{013} = 1 & \\ W_{21} + W_{022} + W_{023} = 1 & \\ W_{031} + W_{032} + W_{033} = 1 & \\ W_{041} + W_{042} + W_{043} = 1 & \\ W_{051} + W_{052} + W_{053} = 1 & \end{array}\right\} \text{(E7)}$$

$$W_{061} + W_{062} + W_{063} = 1$$

Equation (E6) is minimized, subject to Eqns.(E4) and (E7), by a sequence of LP problems using linear approximations to each term of the form (A. ln A), i.e., first compute $B = A_1 . \ln A_1$ at $A_1 = 0, 0.5$, and 1 and use a grid refinement technique to set up each new LP problem. After each LP approximation has been solved by using the simplex algorithm, the grid size is halved using those new values which re adjacent to the current value of A_1. he procedure is depicted in Fig. (2). Assume that the previous grid points for a given variable were a, c, and e and the corresponding solution denoted by x fell in the intervial c to d. Then the new grid points for the new trial would be c, d and e. If the next solution fall in the interval from d to e, the new grid points would be d, (d+e)/2 and e; and so forth.

It is clear that the GP-LP formulation is larger than the original GP dual problem, but the efficiency and effectiveness of the simplex algorithm in determining the optimum solution more than makes up for the increased problem size, particulary with the availability and the capacity of modern computers.

The optimal solution of the GP-LP programming problem is given in Table 1. The previous example was solved using the available date in ref.[1], i.e., $m_1 = 1.65$ $m_2 = 1.8$ and $m_3 = 2.66$. The optimal solution listed in Table 2. It should be pointed out that the analytical study in ref.[1] only provides a method for determining the optimum reduction ratio for each mesh for restrictive case of idrntical pinions. Furthermore, the problem was formulated with no design and side constraints.

The results of which are listed in Tables 1 and 2 indicated that the suggested approach results in a reduction of inertia by a factor of 2.25 over the case of making the pinions identical and of equal thickness [1]. It should be pointed out that the variables N_{pj} and P_j must be integer and discrete values, respectively.

A comparison between the suggested method and the method in ref. [2] can not be made, because this method is valid only for restrictive case of two meshes.

CONCLUSIONS

The main objective of this paper was to determine the optimum diemensions of all the gears in a multi-stage reduction unit and the optimum reduction ratio of each gear mesh that will give minimum moment of inertia of all gears in the reduction unit.
optimum solution was determined using geometric programming which is considered as an effective programming technique for nonlinear constraints and higher order objective functions. This technique, when combined with linear programming in a sequential mode, can handle as many constraints as required. The effectiveness of the geometrical programming method is well demonstrated by the resulting optimum three-stage reduction unit solution, which involved a nonlinear objective function with twelve design variables and also accounted for 14 constriants-3 for designing spure gears for strength, 3 for contact stress, 3 for face width factor, 3 for minimum number of teeth and 2 for permissible error in the overall reductionratio. The advantage of the GP approach can also be demonstrated by the resulting inertia distribution which can be given by the values of the dual variables used in the GP solution method.

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NOMENCLATURE

a_{ikl} Exponent of the ith variable in the Kth constraint.
a_{oij} Exponent of the ith variable in the jth term of the objective function.
b_j Face width of the gear in the jth mesh. inch.
C Constant $=\rho\pi/32$ lb/in^3.
C_f Surface condition factor.
C_H Hardness ratio.
C_{Kl} Constant of the Kth constraint.
C_L Life factor.
C_m Load distribution factor.
C_o Overload factor.
C_{oj} Constant of the jth term in the objective function.
C_p Coefficient depending upon the elastic properties of the material.
C_R Factor of safty.
C_s Size factor.

C_T	Temperature factor.
C_v	Dynamic factor
d_{gj}	Pitch diameter of the jth gear inch.
d_{pj}	Pitch diameter of the jth pinion inch.
F_t	Transmitted load on the jth pinion lb.
G	Geometry factor.
I	Geometry factor.
J_{gj}	Moment of inertia of the jth gear lb.in^2.
J_{pj}	Moment of inertia of the jth pinion lb.in^2.
J_t	Total equivalent moment of inertia of all gears in the reduction unit lb.in^2.
K_L	Life factor.
K_n	Load distribution factor.
K_o	Overload correction factor.
K_R	Reliability factor.
K_s	Size correction factor.
K_t	Temperature factor.
K_v	Dynamic factor.
m	Overall reduction ratio.
m_j	Reduction ratio of the jth mesh.
N_{gj}	Number of teeth in the jth gear.
N_K	Number of terms in the constraints.
N_o	Number of terms in the objective function.
N_{pj}	Number of teeth in the jth pinion.
N_v	Number of design variables.
n	Number of meshes.
S_{ad}	Maximum allowable stress of material.
S_{at}	Allowable stress of material. psi.
T_{p1}	Applied torque on the driving pinion. lb. inch.
$V(W)$	Dual function.
W_{kl}	Dual variable corresponding to the kth constraint.
W_{oj}	Dual variable corresponding to the jth term in. the objective function.
X	Design vector.
X^*_o	Optimum value of the dual function, i.e., the minimum value of the objective function.

β FAce width factor.

Δm Permissible error.

ρ Material density lb/in^3.

σ_c Calculated contact stress number.

σ_t Calculated stress at the most of the tooth.psi.

Table (1)

Design Variables	Stage					
	1		2		3	
	Pinion	Gear	Pinion	Gear	Pinion	Gear
Pitch diameter	2	2.57	2	4.714	2.83	7.15
Diametral pitch	7		7		6	
Number of teeth	14	18	14	33	17	43
Face width	1.45		1.82		2.2	
Reduction ratio	1.285		2.357		2.529	

- Resultant overall reduction ratio (m) = 7.659
- Equivalent moment of inertia of the gears (J_t)=7.867 lb.in^2.
- Ratio of inertia of gear train to inertia of driving pinion J_t/J_{p1} = 12.55.

Table (2)

Design variables	Stage					
	1		2		3	
	Pinion	Gear	Pinion	Gear	Pinion	Gear
Pitch diameter (inch)	2.5	4.13	2.5	4.5	2.5	6.66
Diametral pitch	6		6		6	
Number of teeth	15	25	15	27	15	40
Face width (inch)	2.5		2.5		2.5	
Reduction ratio	1.65		1.8		2.66	

- Resultant overall reduction ratio (m) = 7.9.
- Equivalent moment of inertia of the gears (J_t)=17.69 $lb.in^2$.
- Ratio of inertia of gears train to inertia of driving pinion (J_t/J_{p1}) = 6.21.

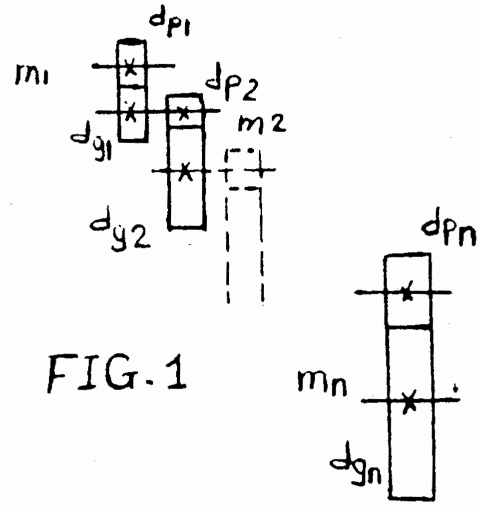

FIG. 1

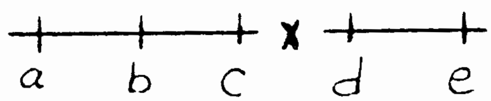

FIG. 2