Menofia University Faculty of Engineering Shebien El-kom Basic Engineering Sci. Department. First semester Examination, 2017-2018 Date of Exam : 6 / 6 / 2018 Grade 600	Minoufia University	Subject : Mathematical Physics(1) . Code: BES 631 Year : Master Time Allowed : 3 hrs Total Marks: 100 Marks

Answer the following questions

Question 1 (30 MARKS)

(A)

Determine the complex exponential Fourier series representation for each of the following signals:

$(a)\ x(t) = \cos\omega_0 t$

$(b)\ x(t) = \sin\omega_0 t$

$(c)\ x(t) = \cos\left(2t + \frac{\pi}{4}\right)$

$(d)\ x(t) = \cos 4t + \sin 6t$

$(e)\ x(t) = \sin^2 t$

(10 Marks)

(B)

Consider a rectified sine wave signal $x(t)$ *defined by* $x(t) = |A\sin\pi t|$

(a) *Sketch* $x(t)$ *and find its fundamental period.*

(b) *Find the complex exponential Fourier series of* $x(t)$.

(c) *Find the trigonometric Fourier series of* $x(t)$.

(10 Marks)

(C)

Consider a continuous − time LTI system described by $\frac{dy(t)}{dt} + 2y = x(t)$

Using the Fourier transform, find the output $y(t)$ *to each of the following input signals:*

$(a)\ x(t) = e^{-t}u(t)$

(10 Marks)

$(b)\ x(t) = u(t)$

Question 2 (40 MARKS)

(A) *Suppose that an "infinite string" has an initial displacement*

$$u(x,0) = f(x) = \begin{cases} x+1, & -1 \le x \le 0 \\ 1-2x, & 0 \le x \le \frac{1}{2} \\ 0, & Otherwise \end{cases}$$

and zero initial velocity $u_t(x,0) = 0$. *Write down the solution of the wave equation*

$$u_{tt} = u_{xx}$$

with ICs $u(x,0) = f(x)$ *and* $u_t(x,0) = 0$ *using D'Alembert's formula. Illustrate the nature of the solution by sketching the ux − profiles* $y = u(x,t)$ *of the string displacement for* $t = 0, \frac{1}{2}, 1, \frac{3}{2}$.

$x = 2t, y = t^2 + 3;$

(20 Marks)

(B) *Drive the d'Alembert solution of the wave equation:* $u_{tt} = a^2 u_{xx}$

given the conditions:

$u(0,t) = u(L,t) = 0, \quad for\ all\ t$

$u(x,0) = f(x), \quad 0 < x < L$

$u_t(x,0) = g(x), \quad 0 < x < L.$

(10 Marks)

(C) *Use the method of separation of variables to solve the following wave equation problem where the string is rigid, but not fixed in place, at both ends (i.e., it is inflexible at the endpoints such that the slope of displacement curve is always zero at both ends, but the two ends of the string are allowed to freely slide in the vertical direction).*

$u_{tt} = a^2 u_{xx}, \quad 0 < x < l, \quad t > 0$

$u_x(0,t) = u_x(L,t) = 0$

$u(x,0) = f(x)$

$u_t(x,0) = g(x).$

(10 Marks)

Question 3 (30 MARKS)

(A)

Use the method of separation of variables to solve the following heat equation with no sources, fixed temperature boundary conditions (that are also homogeneous) and an initial condition.

$u_t = k\, u_{xx}, \quad 0 < x < l, \quad t > 0$

$u(0,t) = u(L,t) = 0$

$u(x,0) = f(x).$

(15 Marks)

(B)

Using the Laplace transform, solve the second order linear differetial equation

$y''(t) + 5y'(t) + 6y(t) = x(t)$

with the initial conditions $y(0) = 2$, $y'(0) = 1$, *and* $x(t) = e^{-t}\, u(t)$.

(15 Marks)

This exam measures the following ILOs

Question Number	Q1-a	Q2-a	Q3-b	Q2-e	Q2-b	Q3-b	Q2-d		Q1-b	Q3-a	Q1-d
Skills			b-ii			b-i					
	Knowledge &understanding skills				Intellectual Skills				Professional Skills		

With my best wishes

Associate Prof. Dr. Islam M. Eldesoky