https://erjm.journals.ekb.eg
ISSN: 1110-1180

DOI: ---------

ERJ
Engineering Research Journal
Faculty of Engineering
Menoufia University

A Multi-Objective Optimization of Profitability and Capacity Utilization in a Feed Manufacturing Firm Using Goal Programming Techniques

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ABSTRACT

This study aimed to create a mathematical model for a feed manufacturing firm that maximizes profit, minimizes cost, and maximizes capacity utilization simultaneously. Data on materials, products, and targets were collected and analysed to form the necessary parameters for the model. Two models were developed: Weighted Lexicographic (WLGP) and Weighted Meta Goal Programming (WMGP). The models included various constraints such as goals, capacity, material proportion, material balance, and demand ratios. Lingo 20.0 optimization software was used to solve the models. The feed manufacturing firm dealt with 15 materials and produced 5 products. The target was to achieve a profit of ₦9,000,000 without exceeding a cost of ₦62,000,000, while utilizing 98% of the production capacity. In Case 1 (profit maximization), the firm achieved the profit target but slightly fell short of the capacity utilization goal by 0.14%. In Case 2 (cost minimization), the firm achieved the cost goal but had a profit shortfall and a slight underachievement in capacity utilization. In Case 3 (capacity utilization maximization), the firm attained the desired capacity utilization goal but had an excess profit and an overachievement in cost. Monthly feed type allocations varied based on the prioritized objective in each case. The WMGP method was also used, and all goals were achieved within acceptable limits. Material allocation aligned with the outcomes of Case 3 in the WLGP method. In conclusion, WLGP and WMGP approaches proved valuable for optimizing production in the poultry feed industry.

Keywords: *Capacity utilization; Feed manufacturing; Production planning; Weighted Lexicographic goal programming; Weighted Meta goal programming.*

1. Introduction

Manufacturing, in simple description, is the conversion of raw materials into finished products that are of greater use and importance than the raw materials converted [1]. In the 1970s, manufacturing contributed 10% of Nigeria's gross domestic product (GDP). However, this has dropped to about 6% overtime as a result of Nigeria's reliance on crude oil [2]. Due to its capacity to promote economic growth, increase productivity, create employment, job opportunities, and encourage trade, manufacturing is often considered as an important ingredient and highly essential to a country's economy.

Animals must consume a lot of protein at affordable prices, and feed is essential for this to happen [3]. This indicates that the feed manufacturing industry is the centre point of Livestock production. The cost of raw materials is one of the primary elements that affect the content of an animal feed. [4]. Animal feed produced by feed manufacturing firms are used in areas such as poultry, livestock and aquaculture. Some essential factors are needed for the success of a feed manufacturing firm and they include raw material purchase, cost of production, and quality control.

The rate of capacity utilization change has an impact on productivity growth [5,6]. A rise in capacity utilization of a company can lead to increase in demand for goods and services produced by the company. This in turn leads to higher production levels and potential growth in productivity. On the contrary, capacity utilization declining may be as a result of drop in demand and production levels, hereby slowing down productivity levels.

It is imperative for companies to adopt management science production guidelines in order to maintain competitiveness [7]. The implementation of effective production planning, while considering practical resource limitations, plays a crucial role in the success of a production system. To address the complexities of multiple objectives, various models of multi-criteria optimization have been developed to determine the optimal level of criterion fulfilment. These models utilize different techniques such as goal programming, compromise programming, integer programming, linear programming, weighted

sum scalarization, and compromise constraints bi-objectives. This study specifically concentrates on the combination of goal programming approaches.
Underutilization of the facility results in lower profits, a failure to meet demand, and excessive raw material inventory costs. Finding an optimized compromise solution that minimizes raw material costs while boosting profitability and maximizing facility utilization is a challenging task for manufacturing companies.
Previous studies have employed goal programming techniques in various domains such as production planning in factories and manufacturing plants [8, 9, 10], financial management and budgetary planning [11, 12], process optimization on farms [13], preferences setting in multi-product manufacturing systems [14], aggregating individual preferencesinto global preferences [15], production optimization in the bakery sector [7], and textile manufacturing industry [16] amongst others. Additionally, research conducted in the Nigerian feed manufacturing sector has focused on feed ration optimization in order to reduce cost and improve quality simultaneously [17, 18, 19], challenges faced by feed manufacturing firms [20, 21], optimizing production units and profit [22], and optimizing raw material procurement given limited liquid capital [23].
However, these studies have not adequately addressed the optimization of production in the feed manufacturing sector in terms of profit maximization, cost minimization, and capacity maximization simultaneously, while considering the production stages and available facilities. Furthermore, while studies have combined weighted and lexicographic goal programming [24], as well as weighted and meta goal programming [9], these combinations have not been applied to a single case study or specifically in the context of feed manufacturing.

2. Research significance

Consequently, there is a need to assess the effectiveness of these goal programming techniques in multi-criteria decision making within the feed manufacturing industry.To bridge this gap, this study aimed to develop a model that utilizes a combination of weighted and lexicographic goal programming, as well as weighted and meta goal programming, to optimize a feed manufacturing firm's production planning decisions. By maximizing profit, minimizing costs, and optimizing capacity utilization, the authors believe that this model will enhance the performance and competitiveness of feed manufacturing operations, contributing to the overall advancement of the industry.

3. Research Objectives

The objectives of this study include:

1. Investigating the activities of a typical feed manufacturing firm and collection of necessary data.
2. Using combined weighted, lexicographic, and meta goal programming as a tool to develop mathematical models to solve the model objectives (maximize profit, minimize cost and maximize capacity utilization) based on target goals.
3. Solving the mathematical models developed using LINGO 20.0 optimization software.

4. Methodology

4.1 Data collected

The methods used to collect necessary data were majorly through interview, observation, and past records of a feed manufacturing firm in Sagamu, Ogun State, South-Western Nigeria. Two staffs and a management staff were interviewed and chosen purposefully based on their years of experience at the firm. They were also interviewed independently to better validate answers. The poultry feed production section was observed for a week to further validate the results of the interview. The coverage area of the data collected include product list, bill of materials, quantity ratio of materials, product demand ratio, production process, cost price of materials, selling price of products, maximum capacity utilization per month and target goal.
Others were calculated and derived from the data collected as follows:

1. Cost price of products (C_i): The cost price of each product per kg was calculated by multiplying the quantity ratio of materials by the cost price of materials as shown in Eq. (1).

$$C_i = \sum_{i}^{n}\sum_{j}^{m}\sum_{s=1}^{d} R_{ijs}C_j \qquad (1)$$

2. Profit margin on products (P_i): The profit margin was derived by subtracting the cost price per kg of product from the selling price per kg for each product. This is expressed in Eq. (2).

$$P_i = S_i - C_i \ \forall\ i = 1, 2 \ldots n \qquad (2)$$

Where i is product number, n is number of products, j is raw material/additives number, m is number of raw materials/additives, s is stage of production number, d is number of stages of production, R_{ijs} is quantity ratio of j[th] raw material/additive under s[th] stage of production of i[th] product, C_j is Cost of j[th] raw material, S_i is selling price of i[th]product, C_i is cost price of i[th] product, and P_i is profit margin on i[th] product.

4.2 Model assumptions

1. The various production stages are made up of work centres that house a number of machines that carry out related tasks. "k" stands for the machine number and represented by 1 ... h. The labour centres are organized according to
2. production technology. "s" stands for the stage number and represented by 1, 2, … d.
3. The unit production cost (C_{isk}) varies from product to product and from machine to machine within a stage due to variations in the model and age of the equipment.
4. Stage s follows stage s-1 immediately. Inventory that is still being produced is prohibited, and production losses are minimal.
5. The access of raw materials is unrestricted.
6. The data values demand ratio, selling price, cost price, etc. are assumed to be deterministic values.

4.3 Model variables

1. Decision variables: The decision variables of the formulated models are quantity of product (Q_i), quantity of raw materials (X_{ijk}), quantity of additives (A_{ijk}), and quantity of intermediate products (Y_{ik}).
2. Deviation variables: The deviation variables of the formulated model are U_t and F_t to measure how much the goals were underachieved, E_t and $B_t$to measure how much the goals were overachieved, and D represents the maximum percentage weighted deviation.
3. Binary variables: They are represented in the model formulated with G_t.

4.4 Model objectives

The objectives of the formulated GP model are as follows. These objectives become soft/goal constraints in the GP model.

1. Maximization of Profit made from the feed products based on firm's profit goal (Z_1).

$$\sum_{i=1}^{n} P_i Q_i + U_1 - E_1 = Z_1 \qquad (3)$$

2. Minimization of cost of raw materials used in production process based on firm's cost goal (Z_2).

$$\sum_{i=1}^{n}\sum_{j=1}^{m} C_{i11}X_{ij1} + \sum_{i=1}^{n}\sum_{k=1}^{h}\sum_{j=1}^{m} C_{i2k}A_{ijk} + U_2 - E_2 = Z_2 \qquad (4)$$

3. Maximization of capacity utilization of the production facilities based on firm's capacity utilization goal (Z_3).

Where k is facility/machine number, h is number of facilities/machines, P_i is profit margin on ith product, Q_i is quantity of ithproduct, X_{ijk} is quantity of jth raw material used in kth facility/machine of ith product, A_{ijk} is quantity of jth additive used in kth facility/machine of ith product, C_{isk} is cost of raw materials used in kth facility/machine under sth stage of production of ith product, U_{isk} is maximum

$$\frac{100}{15}\left(\sum_{i=1}^{n}\frac{\sum_{j=1}^{m} X_{ij1}}{U_{i11}} + \sum_{i=1}^{n}\sum_{k=1}^{h}\frac{\sum_{j=1}^{m} A_{ijk}}{U_{i2k}}\right) + U_3 - E_3 = Z_3 \qquad (5)$$

capacity utilisation of kth facility/machine under sth stage of production of ith product, U_t is underachieved deviation variable of tth model objective, and E_t is overachieved deviation variable of tth model objective.

4.5 Model constraints

The constraints used in the model developments are as follows:

1. Capacity constraint of the grinding and mixing machines for each of the product (poultry feed): The level of production cannot exceed the capacity.

$$\sum_{j=1}^{m} X_{ij1} \le U_{i11} \qquad (6a)$$

$$Y_{i1} + \sum_{j=1}^{m} A_{ij1} \le U_{i21} \qquad (6b)$$

$$Y_{i2} + \sum_{j=1}^{m} A_{ij2} \le U_{i22} \qquad (6c)$$

$$\forall\, i = 1, 2 \ldots n$$

2. Material balance constraint: The quantity of materials entering a junction must match the quantity of materials leaving the junction because production losses are negligible.

At junction a:

$$\sum_{j=1}^{m} X_{ij1} = \sum_{k=1}^{h} Y_{ik} \qquad (7a)$$

$$\sum_{j=1}^{m} X_{ij1} - \sum_{k=1}^{h} Y_{ik} = 0 \qquad (7b)$$

$$\forall\, i = 1, 2 \ldots n$$

At junction b:

$$\sum_{k=1}^{h} Y_{ik} + \sum_{j=1}^{m}\sum_{k=1}^{h} A_{ijk} = Q_i \qquad (7c)$$

$$\sum_{k=1}^{h} Y_{ik} + \sum_{j=1}^{m}\sum_{k=1}^{h} A_{ijk} - Q_i = 0 \qquad (7d)$$

$$\forall\, i = 1, 2 \ldots n$$

Where Q_i is quantity of i[th]product, Y_{ik} is quantity of intermediate product fed into k[th] facility/machine of i[th] product, X_{ijk} is quantity of j[th] raw material used in k[th] facility/machine of i[th] product, A_{ijk} is quantity of j[th] additive used in k[th] facility/machine of i[th] product, and U_{isk} is maximum capacity utilisation of k[th] facility/machine under s[th] stage of production of i[th] product.

3. Demand Ratio Constraint: All other products were in ratio to the first product (chick mash) as mathematically expressed in Eq. (8a) and Eq. (8b).

$$\frac{Q_i}{Q_1} = \frac{D_i}{D_1} \tag{8a}$$

$$D_i Q_1 - D_1 Q_i = 0 \tag{8b}$$

$$\forall\, i = 2, 3 \dots n$$

Where D_i is demand ratioof i[th] product.

4. Material proportion constraint for each product (poultry feed): The quantity of each material per kg in stage 1 were in ratio to that of chick mash (X_{i11}) as shown in Eq. 9a and Eq. (9b). Also, the quantity of each material (additives) per kg in stage 2 are in ratio to the intermediate product coming into each facility from the first stage of production (Y_{ik}) as shown in Eq. (10a) and Eq. (10b).

For stage 1 of production:

$$\frac{X_{ij1}}{X_{i11}} = \frac{R_{ij1}}{R_{i11}} \tag{9a}$$

$$X_{ij1} - \frac{R_{ij1}}{R_{i11}}(X_{i11}) = 0 \tag{9b}$$

$$\forall\, i = 1, 2 \dots n \; and \; j = 1, 2, \dots m$$

For stage 2 of production:

$$\frac{A_{ijk}}{Y_{ik}} = R_{ij2} \tag{10a}$$

$$A_{ijk} - R_{ij2}\,(Y_{ik}) = 0 \tag{10b}$$

$$\forall\, i = 1, 2 \dots n, j = 2, 3, \dots m \; and \; k = 1 \dots h$$

Where R_{ijs} is quantity ratio of j[th] raw material/additive under s[th] stage of production of i[th] product.

The constraints above are hard constraints that is they must be met exactly as specified. The only inequality constraint is capacity constraint while the others are equality constraints. The non-negativity constraint was not specifically stated because all variables in the LINGO 20.0 program are automatically assumed to be non-negative.

4.6 Weighted Lexicographic goal programming

This is the combination of weighted GP and Lexicographic or pre-emptive GP. The objective of the model is expressed mathematically in Eq. (11).

$$MIN = \sum_{t=1}^{l} r_t \left(\frac{w_t U_t}{Z_t} + \frac{v_t E_t}{Z_t} \right) \tag{11}$$

Three cases or scenarios were considered in applying this model

Where t is model objective number, l is number of model objectives, U_t is underachieved deviation variable of t[th] model objective, E_t is overachieved deviation variable of t[th] model objective, v_t is weight associated with overachievement of objective "t", $w_t$is weight allocated to underachievement of objective "t",Z_t is target goal of t[th]objective, and$r_t$ispriority ranking of objective "t"

Case 1: Profit Maximization was the main goal or has the highest priority level while cost minimization and capacity utilization maximization has the same but lower priority level. This means for Case 1;

$$r_1 = 10,\ w_1 = 1,\ v_1 = 0 \tag{12a}$$

$$r_2 = 1,\ w_2 = 0,\ v_2 = 1 \tag{12b}$$

$$r_3 = 1,\ w_3 = 1,\ v_3 = 0 \tag{12c}$$

Case 2: Cost minimization was the main goal or has the highest priority while the profit and capacity utilization maximization have the same but lower priority level. Therefore, for Case 2;

$$r_1 = 1,\ w_1 = 1,\ v_1 = 0 \tag{13a}$$

$$r_2 = 10,\ w_2 = 0,\ v_2 = 1 \tag{13b}$$

$$r_3 = 1,\ w_3 = 1,\ v_3 = 0 \tag{13c}$$

Case 3: Capacity Utilization was the main goal or has the highest priority while profit maximization and cost minimization have the same but lower priority level.

$$r_1 = 1,\ w_1 = 1,\ v_1 = 0 \tag{14a}$$

$$r_2 = 1,\ w_2 = 0,\ v_2 = 1 \tag{14b}$$

$$r_3 = 10,\ w_3 = 1,\ v_3 = 0 \tag{14c}$$

All the objectives were allocated weight of 1 as shown above so w_t or v_t is the same in all cases.

4.7 Weighted Meta goal programming

This is the combination of weighted GP and meta GP. The objective of the model is expressed mathematically in Eq. (15).

$$MIN = B_1 + B_2 - B_3 \tag{15}$$

Where B_t is overachieved deviation variable of type "t" under meta-goal.

The weighted Meta G.P is divided into three types.

Type 1: The percentage sum of unwanted deviation variables cannot surpass a certain bound M_1, thus, imposing the constraint in Eq. (16).

$$\sum_{t=1}^{l}\left(\frac{w_t U_t}{Z_t} + \frac{v_t E_t}{Z_t}\right) + F_1 - B_1 = M_1 \tag{16}$$

Where F_t is underachieved deviation of type "t" under meta-goal.

Type 2: The maximum percentage deviation (D) variables cannot surpass a certain bound M_2, thus, imposing constraints in Eq. (17a) and Eq. (17b).

$$\frac{w_t U_t}{Z_t} + \frac{v_t E_t}{Z_t} - D \le 0 \tag{17a}$$

$$D + F_2 - B_2 = M_2 \tag{17b}$$

$$\forall\ t = 1,2 \ldots l$$

Type 3: The percentage of achieved goals cannot surpass a certain bound M_3, thus, imposing constraints in in Eq. (18a) and Eq. (18b).

$$U_t + E_t - Z_t G_t \le 0 \tag{18a}$$

$$\frac{\sum_{t=1}^{l} G_t}{3} + F_3 - B_3 = M_3 \tag{18b}$$

Where G_t is binary variable of model objective "t"

The values of w_t and $v_t$are as follows for each objective:

For t= 1 (profit maximization), w_1=1, v_1= 0 (19a)

For t= 2 (cost minimization), w_2= 0, v_2= 1 (19b)

For t= 3 (capacity utilization maximization), w_3= 1, v_3= 0 (19c)

The formulated model was solved using Lingo 20.0 software.

5. Results and Discussion

5.1 Data collected

A total of 15 materials (6 out of which are additives) were used in production. The first 9 materials used in larger quantities are maize/corn, soya bean meal, full fat soya, soya oil, groundnut Cake (GNC), wheat offal, corn bran, bone meal and limestone. The remaining 6 materials are additives, like toxin binder, lysine, methionine, enzymes, salt, and premixes, needed in very small quantities. The 5 products produced at the firm include chick mash, grower mash, layer mash, broiler starter, and broiler finisher. The target goal of the feed manufacturing firm per month is to actualize a minimum profit of ₦9,000,000 while spending not more than ₦62,000,000 on materials needed for production and utilizing at least 98% of their capacity. Other data gotten or calculated include:

1. Production process

The production process starts with the acquisition of raw materials which are weighted into appropriate quantities for each batch of production and poured into the Grinding or Crunching machine where the major raw materials (macro nutrients) are grinded together. It then moves from there to the mixing chamber where the grinded raw materials are mixed thoroughly. It is at this stage that the additives (micro nutrients) are added. The mashed feeds are then dispensed from the mixer into the packaging bags and weighted to ensure it is up to the required weight. This entire production process is summarized into two main production processes for the purpose of this study. They are:

A. Grinding/Crunching Operation (Stage 1)
B. Mixing & Filling Operation (Stage 2)

2. Quantity ratio of materials

Table (1) illustrates the quantity of each material that makes up the five products. The quantity of raw materials presented in the table are per 1 kilogram of feed. Some values are zero because some materials are not required for the production of such a product.

Table 1-Quantity of Materials Required for Each Product

S/N	Raw Materials	Names of Products and their Material Ratio Per Kg of Feed				
		Chick Mash	Grower Mash	Layer Mash	Broiler Starter	Broiler Finisher
1	Maize/Corn	0.52000	0.4300	0.5000	0.50000	0.5300
2	Soya bean meal	0.29000	0.0800	0.2000	0.33000	0.3000
3	Full fat soya	0	0	0.0300	0.03000	0
4	Soya oil	0	0	0	0.00800	0.0120
5	Groundnut Cake (GNC)	0	0.1000	0	0	0
6	Wheat offal	0.14500	0.2780	0.1520	0.08000	0.1110

S/N	Raw Materials	Names of Products and their Material Ratio Per Kg of Feed				
		Chick Mash	**Grower Mash**	**Layer Mash**	**Broiler Starter**	**Broiler Finisher**
7	Corn bran	0	0.0700	0	0	0
8	Bone meal	0.02000	0.0185	0.0230	0.02700	0.0170
9	Limestone	0.01400	0.0150	0.0850	0.01400	0.0170
Additives						
1	Toxin binder	0.00120	0.0010	0.0010	0.00150	0.0012
2	Lysine	0.00160	0.0010	0.0010	0.00250	0.0025
3	Methionine	0.00200	0.0015	0.0030	0.00150	0.0030
4	Enzymes	0.00025	0	0	0.00025	0
5	Premix	0.00300	0.0025	0.0025	0.00250	0.0030
6	Salt	0.00300	0.0025	0.0025	0.00300	0.0030

3. Cost price of materials

Table (2) shows the cost price of the materials used in the production process in ₦ per kg of material.

Table 2-Cost price of materials

S/N	Raw Material	Cost Price/Kg (₦)
1	Maize/Corn	230
2	Soya bean meal	340
3	Full fat soya	440
4	Soya oil	1090
5	Groundnut Cake (GNC)	270
6	Wheat offal	180
7	Corn bran	120
8	Bone meal	110
9	Limestone	30
Additives		
1	Toxin binder	1000
2	Lysine	2500
3	Methionine	3500
4	Enzymes	350
5	Layers premix	2020
6	Grower premix	1650
7	Chicks and starter premix	2500
8	Broiler starter and finisher premix	2050
9	Salt	130

4. Product Demand Ratio

Table (3) shows the demand ratio of feeds produced by the firm relative to the demand of chick mash. The broiler starter and broiler finisher have the highest demand ratio because broilers are mainly used for consumption purposes. They are followed by the Layer mash, grower mash then chick mash with the lowest demand ratio.

Table 3-Product Demand Ratio

S/N	Names of Products	Demand Ratio
1	Chick Mash	1
2	Grower Mash	2
3	Layer Mash	3
4	Broiler Starter	4
5	Broiler Finish	4

5. Cost price of products

Table (4) shows the cost price of each product per kg as well as the cost associated with each stage of production. The total cost for each product was calculated using Eq. (1). Broiler starter is the most expensive to produce with a total cost of ₦268.83 while grower mash is the least expensive to produce with a total cost of ₦223.225.

Table 4-Cost Price of Products per Kg, Cost at Each Stage of Production, and cost of materials based on quantity ratio per Kg

S/N	Raw Materials	Names of Products and their Cost Per Kg of Feed				
		Chick Mash (₦)	**Grower Mash (₦)**	**Layer Mash (₦)**	**Broiler Starter (₦)**	**Broiler Finisher (₦)**
1	Maize/Corn	119.6000	98.900	115.000	115.0000	121.90
2	Soya bean meal	98.6000	23.200	58.000	95.7000	87.00
3	Full fat soya	0	0	13.200	13.2000	0
4	Soya oil	0	0	0	8.7200	13.08

S/N	Raw Materials	Names of Products and their Cost Per Kg of Feed				
		Chick Mash (₦)	Grower Mash (₦)	Layer Mash (₦)	Broiler Starter (₦)	Broiler Finisher (₦)
5	Groundnut Cake	0	27.000	0	0	0
6	Wheat offal	26.1000	50.040	27.360	14.4000	19.98
7	Corn bran	0	8.400	0	0	0
8	Bone meal	2.2000	2.035	2.530	2.9700	1.87
9	Limestone	0.4200	0.450	2.550	0.4200	0.51
	Cost of Raw Materials in Stage 1	246.9200	210.025	218.640	250.4100	244.34
Additives						
1	Toxin binder	1.2000	1.000	1.000	1.5000	1.20
2	Lysine	4.0000	2.500	2.500	6.2500	6.25
3	Methionine	7.0000	5.250	10.50	5.2500	10.50
4	Enzymes	0.0875	0	0	0.0875	0
5	Premix	7.5000	4.125	5.050	5.1250	6.15
6	Salt	0.39000	0.325	0.325	0.3900	0.39
	Cost of Raw Materials in Stage 2	20.1775	13.200	19.375	18.6025	24.49
	Total	267.0975	223.225	238.015	269.0125	268.83

6. Maximum capacity utilization per month

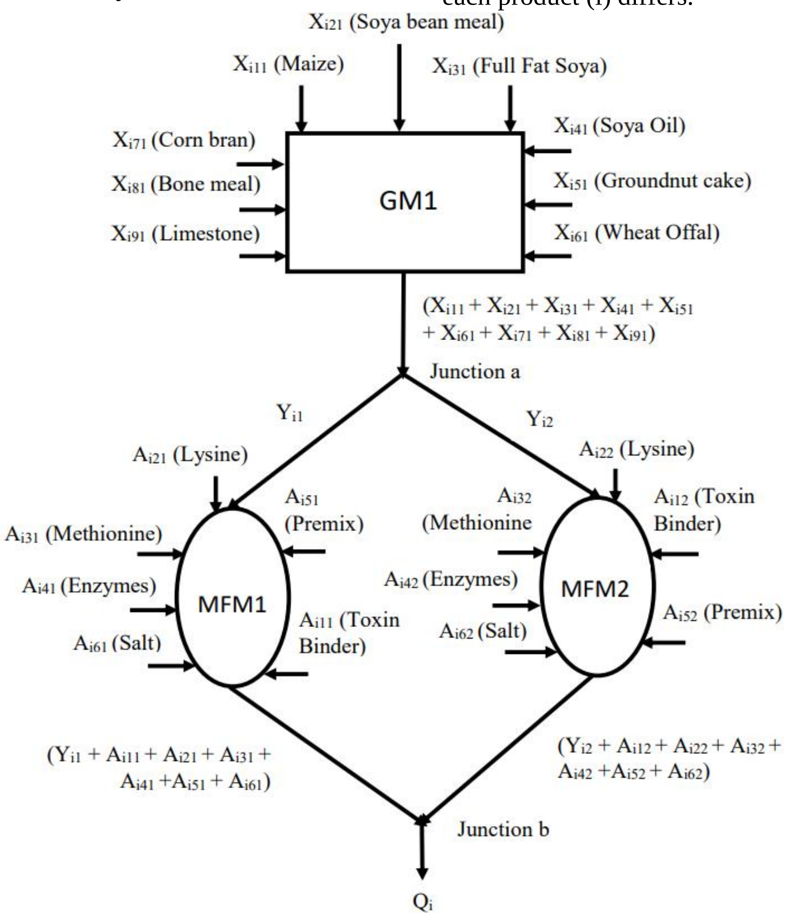
Table (5) shows the capacity of each of the facilities per month based on the stage of production and product. It also shows the cost per kg of material processed for each facility under each stage of production and type of product. The broiler starter and broiler finisher have the highest values of capacity utilization per month for all facilities while chick mash has the lowest value.

Table 5-Major production facilities with corresponding capacities/month and cost/kg of materials processed per product

Stage of Production	Facility Name	Chick Mash		Grower Mash		Layers Mash		Broiler Starter		Broiler Finisher	
		Capacity/month (kg)	Cost/kg of materials processed (₦)	Capacity/month (kg)	Cost/kg of materials processed (₦)	Capacity/month (kg)	Cost/kg of materials processed (₦)	Capacity/month (kg)	Cost/kg of materials processed (₦)	Capacity/month (kg)	Cost/kg of materials processed (₦)
Grinding/ Crushing	Grinding machine 1 (GM1)	17900	246.920	35700	210.025	53600	218.640	71400	250.410	71400	244.34
Mixing & Filling	Mixing and Filling machine 1 (MFM1)	8950	20.1775	17850	13.200	26800	19.375	35700	18.6025	35700	24.49
	Mixing and Filling Machine 2	8950	20.1775	17850	13.200	26800	19.375	35700	18.6025	35700	24.49

7. Process flow diagram

Fig. (1) shows the production process and materials used at each stage of production for a product (i). It also shows the junction a, and junction b considered in the material balance constraint. It is important to note that not all materials are used in the manufacturing of each product so the materials entering each facility at each stage of production for each product (i) differs.



GM1- Grinding machine 1; MFM1 & MFM2- Mixing and Filling machine 1 and 2 respectively

Fig. 1-Process Flow Diagram for Production of i^{th} Product

8. Selling price, cost price and profit margin

Table (6) shows the profit on each product produced as computed using Eq. (2) using the selling price/kg and cost price/kg. Broiler finisher has the highest profit margin while layer mash has the lowest profit margin.

Table 6-Profit Margin on Products

S/N	Products	Selling Price/kg (₦)	Cost Price/kg (₦)	Profit (₦)
1	Chick Mash	302	267.0975	34.9025
2	Grower Mash	240	223.2250	16.7750
3	Layer Mash	250	238.0150	11.9850
4	Broiler Starter	320	269.01250	50.9875
5	Broiler Finisher	320	268.8300	51.1700

The constraints of the model from Eq. (20) to Eq. (131) are shown in Appendix **1**.

5.2 Weighted-Lexicographic goal programming

Case 1: Profit Priority

$$MIN = \left(\frac{10}{9000000}\right)U_1 + \left(\frac{1}{62000000}\right)E_2 + \left(\frac{1}{98}\right)U_3 \quad (132)$$

Subject to constraints in Eq. (20) to Eq. (131).

Case 2: Cost Priority

$$MIN = \left(\frac{10}{62000000}\right)E_2 + \left(\frac{1}{9000000}\right)U_1 + \left(\frac{1}{98}\right)U_3 \quad (133)$$

Subject to constraints in Eq. (20) to Eq. (131).

Case 3: Capacity Utilization Priority

$$MIN = \left(\frac{10}{98}\right)U_3 + \left(\frac{1}{9000000}\right)U_1 + \left(\frac{1}{62000000}\right)E_2 \quad (134)$$

Subject to constraints in Eq. (20) to Eq. (131).

5.3 Weighted-Meta goal programming

The following equations were derived;

$$MIN = B_1 + B_2 + B_3 \text{ (Objective function)} \quad (135)$$

Type 1: The percentage maximum deviation from all goals should be less than or equal to 0.015.

$$\left(\frac{1}{9000000}\right)U_1 + \left(\frac{1}{62000000}\right)E_2 + \left(\frac{1}{98}\right)U_3 + F_1 - B_1 = 0.015 \quad (136)$$

Type 2: The maximum percentage deviation from any goal should be less than or equal to 0.005.

$$U_1 - 9000000D \leq 0 \quad (137)$$

$$E_2 - 62000000D \leq 0 \quad (138)$$

$$U_3 - 98D \leq 0 \quad (139)$$

$$D + F_2 - B_2 = 0.005 \quad (140)$$

Type 3: Number of goals unsatisfied should be less than or equal to 1

$$U_1 - 9000000G_1 \leq 0 \quad (141)$$

$$E_2 - 62000000G_2 \leq 0 \quad (142)$$

$$U_3 - 98G_3 \leq 0 \quad (143)$$

$$\frac{G_1 + G_2 + G_3}{3} + F_3 - B_3 = \frac{1}{3} \quad (144)$$

Subject to constraints in Eq.(20) to Eq.(131).

5.4 Solving the Problem Using Lingo 20.0

Using the WMGP technique, three different types were considered. The results obtained as shown in **Table (7)** shows that all the overachievement variables, B_1, B_2, and B_3, were equal to zero so all meta-goals were achieved within the specified limits. The underachievement variable for type 1 (F_1) which was 0.0111, is not greater than the specified limit of 0.015. This implies that the percentage maximum deviation from all goals was within the acceptable range. Also, the underachievement variable for type 2 (F_2), with a value of 0.0011, was within the acceptable limit of 0.005, indicating that the maximum percentage deviation from any goal was also within the acceptable range.

Furthermore, the maximum percentage weighted deviation (D), being 0.0039 and within the acceptable limit of 0.005, indicated that that none of the solutions deviated beyond what is acceptable to the firm. The values of binary variables G_1, G_2, and G_3 determined if the goals were satisfied or not. G_1 and G_3 were 0, indicating that the profit and capacity utilization goals were adequately met or exceeded, while G_2 was 1, meaning the cost goal was exceeded (an unwanted outcome). However, the number of goals not achieved was still within limit as only one goal was unmet which made F_3 zero.

Table 7-Solution to WMGP variables

S/N	Variable		Values
1	B_1	Overachievement of percentage maximum deviation	0
2	B_2	Overachievement of maximum percentage deviation	0
3	B_3	Overachievement of goals unsatisfied	0
4	F_1	Underachievement of percentage maximum deviation	0.0111
5	F_2	Underachievement of maximum percentage deviation	0.0011
6	F_3	Underachievement of goals unsatisfied	0
7	D	Maximum percentage weighted deviation	0.0039
8	G_1	Binary variable for goal 1 satisfaction	0
9	G_2	Binary variable for goal 2 satisfaction	1.0000
10	G_3	Binary variable for goal 3 satisfaction	0

It can be observed from **Table (8)** that the result of the WMGP method in terms of monthly allocation ($Q_1...Q_5$) to the products corresponds with case 3 of WLGP method. It was only in case 3 of the WLGP method that two of the objective goals were attained in a desirable way which is capacity being used optimally at 98% and excess profit of ₦13,000.94 from the desired goal of ₦9,000,000. Case 1 and Case 2 on the other hand only gave one desirable outcome.

For case 1, the company was able to achieve the exact desirable profit, as it was the priority, but couldn't meet up with the capacity utilization goal, which was slightly less than the desired 98% by 0.14% (U_3). The cost incurred on raw materials was also in excess (E_2) of ₦150,358. For case 2, the company achieved the cost goal perfectly, as it was the main priority, but couldn't meet up with the profit goal as it fell short by ₦21,773.35 (U_1). Utilization of capacity was also underachieved (U_3) by 0.38%. Therefore, based on the restrictions given on the deviation variables by the WMGP method, case 3 gave the best compromise solution for the firm.

Additionally, the monthly allocations of the five types of product were determined, with quantities (Q_1...Q_5) slightly higher than gotten in cases 1 and 2. This is expected as capacity utilization was underachieved by 0.14% and 0.38% in Cases 1 and 2, respectively. The result for all products (Q_1...Q_5) also showed that the quantities derived in case 2 are lower than those derived in case 1. It can be inferred from results that the company's cost target of ₦62,000,000 is insufficient to achieve the profit goal of ₦9,000,000. Therefore, a trade-off between profit, cost, and capacity utilization goals is necessary.

Table 8-Quantity of product allocations and goal deviation variables solutions provided by Lingo 20.0 for WLGP and WMGP techniques

S/N		Variable	WLGP			WMGP
			Case 1	Case 2	Case 3	
1	U_1	Underachieved profit goal (₦)	0	21773.35	0	0
2	U_2	Underachieved cost goal (₦)	0	0	0	0
3	U_3	Underachieved capacity utilization goal (%)	0.14	0.38	0	0
4	E_1	Overachieved profit goal (₦)	0	0	13000.94	13000.94
5	E_2	Excess cost goal (₦)	150358.00	0	240137.20	240137.20
6	E_3	Overachieved capacity utilization goal (%)	0	0	0	0
7	Q_1	Quantity of Chick mash (kg)	17542.58	17500.14	17567.92	17567.92
8	Q_2	Quantity of Layer mash (kg)	35085.15	35000.27	35135.84	35135.84
9	Q_3	Quantity of Grower mash (kg)	52627.73	52500.41	52703.76	52703.76
10	Q_4	Quantity of Broiler starter (kg)	70170.31	70000.55	70271.67	70271.67
11	Q_5	Quantity of Broiler finisher (kg)	70170.31	70000.55	70271.67	70271.67

The breakdown of the monthly allocation of raw materials that make up each product based on the models and cases used appropriately are represented in tables as shown in **Table (9)** to **Table (12)**. It was divided based on the type of product, stages of production, number of facilities and intermediate product going from one stage of production to the next. **Table (9)**, **Table (10)** and **Table (11)** are monthly allocation to facility using WLGP model with profit maximization as priority (case 1), cost minimization as priority (case 2), and capacity utilization maximization as priority (case 3) respectively. However, **Table (12)** shows the result of the WMGP model in terms of monthly allocation of materials to facility. The result in **Table (11)** is similar to that of **Table (12)** which is more confirmation to the similarity between the WMGP result and the capacity utilization maximization priority result from the WLGP method.

Table 9-Monthly allocation to facility per product using cost minimization as the main priority in WLGP model

Stage	Raw Material	Monthly Allocation to Facility per Product [Kg]				
		Chick Mash	**Grower Mash**	**Layers Mash**	**Broiler Starter**	**Broiler Finisher**
Grinding/ Crunching		GM1	GM1	GM1	GM1	GM1
	Maize/Corn	9122.42	15087.80	26316.50	35080.73	37208.85
	Soya bean meal	5087.58	2806.33	10526.60	23153.28	21060.21
	Full fat soya	0	0	1578.99	2104.84	0

Stage	Raw Material	Monthly Allocation to Facility per Product [Kg]									
		Chick Mash		**Grower Mash**		**Layers Mash**		**Broiler Starter**		**Broiler Finisher**	
Grinding/ Crunching		GM1		GM1		GM1		GM1		GM1	
	Soya oil	0		0		0		561.29		840.92	
	Groundnut Cake	0		3509.42		0		0		0	
	Wheat offal	2544.24		9754.26		8000.22		5612.92		7791.53	
	Corn bran	0		2456.29		0		0		0	
	Bone meal	351.21		648.78		1210.56		1894.36		1194.40	
	Limestone	245.39		526.56		4473.81		982.26		1194.40	
Mixing/ Filling	**Additives**	MFM1	MFM2	MFM1	MFM2	MFM1	MFM2	MFM1	MFM2	MFM1	MFM2
	Intermediate product	8852.18	8498.67	17699.55	17089.89	26534.65	25572.01	35302.84	34086.83	35252.30	34038.03
	Toxin binder	10.62	10.20	17.70	17.09	26.53	25.57	52.95	51.13	42.30	40.85
	Lysine	14.16	13.60	17.70	17.09	26.53	25.57	88.26	85.22	88.13	85.0951
	Methionine	17.70	17.00	26.55	25.63	79.60	76.72	52.95	51.13	105.76	102.11
	Enzymes	2.21	2.12	0	0	0	0	8.83	8.52	0	0
	Premix	26.56	25.50	44.25	42.72	66.34	63.93	88.26	85..22	105.76	102.11
	Salt	26.56	25.50	44.25	42.72	66.34	63.93	105.91	102.26	105.76	102.11

Table 10-Monthly allocation to facility per product using cost minimization as the main priority in WLGP model

Stage	Raw Material	Monthly Allocation to Facility per Product [Kg]									
		Chick Mash		**Grower Mash**		**Layers Mash**		**Broiler Starter**		**Broiler Finisher**	
Grinding / Crunching		GM1		GM1		GM1		GM1		GM1	
	Maize/Corn	9100.35		15051.30		26252.83		34995.86		37118.83	
	Soya bean meal	5075.27		2799.54		10501.13		23097.27		21009.26	
	Full fat soya	0		0		1575.17		2099.75		0	
	Soya oil	0		0		0		559.94		838.89	
	Groundnut Cake	0		3500.93		0		0		0	
	Wheat offal	2538.09		9730.66		7980.86		5599.34		7772.68	
	Corn bran	0		2450.35		0		0		0	
	Bone meal	350.36		647.21		1207.63		1889.78		1191.52	
	Limestone	244.80		525.29		4462.98		979.88		1191.52	
Mixing/ Filling	**Additives**	MFM1	MFM2	MFM1	MFM2	MFM1	MFM 2	MFM1	MFM2	MFM1	MFM2
	Intermediate Product	8852	8457	17699	17005	26534.6	25446	35303	33919	35252	33870
	Toxin binder	10.62	10.15	17.70	17.01	26.53	25.45	52.95	50.89	42.30	40.64
	Lysine	14.16	13.53	17.70	17.01	26.53	25.45	88.26	84.80	88.13	84.68
	Methionine	17.70	16.91	26.55	25.51	79.60	76.34	52.95	50.89	105.76	101.61
	Enzymes	2.21	2.11	0	0	0	0	8.83	8.48	0	0
	Premix	26.56	25.37	44.25	42.51	66.34	63.61	88.26	84.80	105.76	101.61
	Salt	26.56	25.37	44.25	42.51	66.34	63.61	105.91	101.76	105.76	101.61

Table 11-Monthly allocation to facility per product using capacity utilization maximization as the main priority in WLGP model

Stage	Raw Material	Monthly Allocation to Facility per Product [Kg]									
		Chick Mash		**Grower Mash**		**Layers Mash**		**Broiler Starter**		**Broiler Finisher**	
Grinding/ Crunching		GM1		GM1		GM1		GM1		GM1	
	Maize/Corn	9135.60		15109.59		26354.51		35131.40		37262.60	
	Soya bean meal	5094.93		2810.38		10541.81		23186.72		21090.63	
	Full fat soya	0		0		1581.27		2107.88		0	
	Soya oil	0		0		0		562.10		842.13	
	Groundnut Cake	0		3514.49		0		0		0	
	Wheat offal	2547.92		9768.24		8011.77		5621.02		7802.79	
	Corn bran	0		2459.84		0		0		0	
	Bone meal	351.72		649.71		1212.31		1897.10		1196.13	
	Limestone	245.75		527.32		4480.27		983.68		1196.13	
Mixing/ Filling	**Additives**	MFM1	MFM2	MFM1	MFM2	MFM1	MFM2	MFM1	MFM2	MFM1	MFM2
	Intermediate product	8852.18	8523.73	17699.55	17140.15	26534.65	25647.28	35302.84	34187.07	35252.30	34138.12
	Toxin binder	10.62	10.23	17.67	17.14	26.53	25.65	52.95	51.28	42.30	40.97
	Lysine	14.16	13.64	17.67	17.14	26.53	25.65	88.26	85.47	88.13	85.35
	Methionine	17.70	17.05	26.55	25.71	79.60	76.94	52.95	51.28	105.76	102.41
	Enzymes	2.21	2.13	0	0	0	0	8.83	8.55	0	0
	Premix	26.56	25.57	44.25	42.85	66.34	64.12	88.26	85.47	105.76	102.41
	Salt	26.56	25.57	44.25	42.85	66.34	64.12	105.91	102.56	105.76	102.41

Table 12-Monthly allocation to facility per product using WMGP model

Stage	Raw Material	Monthly Allocation to Facility per Product [Kg]									
		Chick Mash		**Grower Mash**		**Layers Mash**		**Broiler Starter**		**Broiler Finisher**	
Grinding/ Crunching		GM1		GM1		GM1		GM1		GM1	
	Maize/Corn	9135.60		15109.59		26354.51		35131.40		37262.60	
	Soya bean meal	5094.93		2810.38		10541.81		23186.72		21090.63	
	Full fat soya	0		0		1581.27		2107.88		0	
	Soya oil	0		0		0		562.10		842.13	
	Groundnut Cake	0		3514.49		0		0		0	
	Wheat offal	2547.92		9768.35		8011.77		5621.02		7802.79	
	Corn bran	0		2459.84		0		0		0	
	Bone meal	351.72		649.71		1212.31		1897.10		1196.13	
	Limestone	245.75		527.32		4480.27		983.68		1196.13	
Mixing/ Filling	**Additives**	MFM 1	MFM 2	MFM 1	MFM 2	MFM1	MFM 2	MFM 1	MFM2	MFM1	MFM2
	Intermediate product	8852	8523	17699	17140	26534	25647	35302	34187	35252	34138
	Toxin binder	10.62	10.23	17.70	17.14	26.53	25.65	52.95	51.28	42.30	40.97
	Lysine	14.16	13.64	17.70	17.14	26.53	25.65	88.26	85.47	88.13	85.35
	Methionine	17.70	17.05	26.55	25.71	79.60	76.94	52.95	51.28	105.76	102.41
	Enzymes	2.21	2.13	0	0	0	0	8.83	8.55	0	0
	Premix	26.56	25.57	44.25	42.85	66.34	64.12	88.26	85.47	105.76	102.41
	Salt	26.56	25.57	44.25	42.85	66.34	64.12	105.9	102.56	105.76	102.41

The above results indicate that the WMGP and WLGP method can be successfully applied to optimize feed manufacturing operations. However, each of this goal programming techniques, based on this application and result obtained, have individual strength and limitations. The WLGP method allows prioritizing individual objectives while comparing their trade-offs. The WMGP method, on the other hand, gives more power to the decision maker allowing more stringent control on the deviational variables. The WMGP method gives one solution within the compromising region, which in this case of application to the feed manufacturing firm was similar to the result obtained for capacity utilization maximization priority (case 3) of the WLGP method.

From these results, it can be inferred that WMGP method is a more direct way to get the most optimized solution limiting the deviational variables. However, if the decision maker needs a more comprehensive view of trade-offs and results possible considering different priority levels, then the WLGP method is more appropriate. The choice of which technique to use is dependent on the priorities of the firm and level of trade-offs they are willing to accept. The results obtained from this study can also be applied to the production planning of feeds in feed industry.

6. Conclusions

A manufacturing firm has been studied and a multi-objective model developed and solved using Lingo 20.0 optimization software. Necessary data about a typical manufacturing firm were obtained and studied. A model was developed that maximises profit, minimizes cost, and maximizes capacity utilization hereby helping producers in production planning and identifying trade-offs for better decision making. This study offers a comprehensive decision-making framework for feed manufacturing firms. The findings provide valuable insights and optimization strategies that can enhance the performance and competitiveness of feed manufacturing operations. It is recommended that feed manufacturing firms adopt the combined weighted, lexicographic, and meta goal programming approach to optimize operations. Customizing objective weights and incorporating robust decision-making strategies will enhance profitability, reduce costs, and maximize capacity utilization. Continuous monitoring, collaboration, and future research will further improve optimization efforts in the industry.

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8. Appendix 1

Model constraints

1. Goal Constraint: It was formulated based on method discussed for model objectives from Eq. (3) to Eq. (5).

$$34.9025Q_1 + 16.775Q_2 + 11.985Q_3 + 50.9875Q_4 + 51.17Q_5 + U_1 - E_1 = 9000000 \text{ (Profit goal constraint)} \quad (20)$$

$$246.92(X_{111} + X_{121} + X_{161} + X_{181} + X_{191}) + 20.1775(Y_{11} + A_{111} + A_{121} + A_{131} + A_{141} + A_{151} + A_{161} + Y_{12} + A_{112} + A_{122} + A_{132} + A_{142} + A_{152} + A_{162}) + 210.025(X_{211} + X_{221} + X_{251} + X_{271} + X_{281} + X_{291}) + 13.2(Y_{21} + A_{211} + A_{221} + A_{231} + A_{251} + A_{261} + Y_{22} + A_{212} + A_{222} + A_{232} + A_{252} + A_{262} + 218.64(X_{311} + X_{321} + X_{331} + X_{361} + X_{381} + X_{391}) + 19.375(Y_{31} + A_{311} + A_{321} + A_{331} + A_{351} + A_{361} + Y_{32} + A_{312} + A_{322} + A_{332} + A_{352} + A_{362}) + 250.41(X_{411} + X_{421} + X_{431} + X_{441} + X_{461} + X_{481} + X_{491}) + 18.6025(Y_{41} + A_{411} + A_{421} + A_{431} + A_{441} + A_{451} + A_{461} + Y_{42} + A_{412} + A_{422} + A_{432} + A_{442} + A_{452} + A_{462}) + 244.34(X_{511} + X_{521} + X_{541} + X_{561} + X_{581} + X_{591}) + 24.49(Y_{51} + A_{511} + A_{521} + A_{531} + A_{551} + A_{561} + Y_{52} + A_{512} + A_{522} + A_{532} + A_{552} + A_{562}) + U_2 - E_2 = 62000000 \quad (21)$$

(Cost goal constraint)

$$\frac{100}{15}\Bigg(\left(\frac{X_{111}+X_{121}+X_{161}+X_{181}+X_{191}}{17900}\right) + \left(\frac{X_{211}+X_{221}+X_{251}+x_{261}+X_{271}+X_{281}+X_{291}}{35700}\right) + \left(\frac{X_{311}+X_{321}+X_{331}+X_{361}+X_{381}+X_{391}}{53600}\right) + \left(\frac{X_{411}+X_{421}+X_{431}+X_{441}+X_{461}+X_{481}+X_{491}}{71400}\right) + \left(\frac{X_{511}+X_{521}+X_{541}+X_{561}+X_{581}+X_{591}}{71400}\right) + \left(\frac{Y_{11}+A_{111}+A_{121}+A_{131}+A_{141}+A_{151}+A_{161}}{8950}\right) + \left(\frac{Y_{21}+A_{211}+A_{221}+A_{231}+A_{251}+A_{261}}{17850}\right) + \left(\frac{Y_{31}+A_{311}+A_{321}+A_{331}+A_{351}+A_{361}}{26800}\right) + \left(\frac{Y_{41}+A_{411}+A_{421}+A_{431}+A_{441}+A_{451}+A_{461}}{35700}\right) + \left(\frac{Y_{51}+A_{511}+A_{521}+A_{531}+A_{551}+A_{561}}{35700}\right) + \left(\frac{Y_{12}+A_{112}+A_{122}+A_{132}+A_{142}+A_{152}+A_{162}}{8950}\right) + \quad (22)$$

$\left(\frac{Y_{22}+A_{212}+A_{222}+A_{232}+A_{252}+A_{262}}{17850}\right)+\left(\frac{Y_{32}+A_{312}+A_{322}+A_{332}+A_{352}+A_{362}}{26800}\right)+\left(\frac{Y_{42}+A_{412}+A_{422}+A_{432}+A_{442}+A_{452}+A_{462}}{35700}\right)+\left(\frac{Y_{52}+A_{512}+A_{522}+A_{532}+A_{552}+A_{562}}{35700}\right)\Big)+U_3-E_3=98$ (Capacity Utilization goal constraint)

2. Capacity constraint: The result derived from applying Eq. (6a) to Eq. (6c) are as follows.

$$X_{111}+X_{121}+X_{161}+X_{181}+X_{191} \leq 17900 \quad (23)$$

$$X_{211}+X_{221}+X_{251}+X_{261}+X_{271}+X_{281}+X_{291} \leq 35700 \quad (24)$$

$$X_{311}+X_{321}+X_{331}+X_{361}+X_{381}+X_{391} \leq 53600 \quad (25)$$

$$X_{411}+X_{421}+X_{X_{431}}+X_{441}+X_{461}+X_{481}+X_{491} \leq 71400 \quad (26)$$

$$X_{511}+X_{521}+X_{541}+X_{561}+X_{581}+X_{591} \leq 71400 \quad (27)$$

$$Y_{11}+A_{111}+A_{121}+A_{131}+A_{141}+A_{151}+A_{161} \leq 8950 \quad (28)$$

$$Y_{21}+A_{211}+A_{221}+A_{231}+A_{251}+A_{261} \leq 17850 \quad (29)$$

$$Y_{31}+A_{311}+A_{321}+A_{331}+A_{351}+A_{361} \leq 26800 \quad (30)$$

$$Y_{41}+A_{411}+A_{421}+A_{431}+A_{441}+A_{451}+A_{461} \leq 35700 \quad (31)$$

$$Y_{51}+A_{511}+A_{521}+A_{531}+A_{551}+A_{561} \leq 35700 \quad (32)$$

$$Y_{12}+A_{112}+A_{122}+A_{131}+A_{141}+A_{151}+A_{161} \leq 8950 \quad (33)$$

$$Y_{22}+A_{212}+A_{222}+A_{232}+A_{252}+A_{262} \leq 17850 \quad (34)$$

$$Y_{32}+A_{312}+A_{322}+A_{332}+A_{352}+A_{362} \leq 26800 \quad (35)$$

$$Y_{42}+A_{412}+A_{422}+A_{432}+A_{442}+A_{452}+A_{462} \leq 35700 \quad (36)$$

$$Y_{52}+A_{512}+A_{522}+A_{532}+A_{552}+A_{562} \leq 35700 \quad (37)$$

3. Material balance constraint:The result derived from applying Eq. (7a) to Eq. (7d) are as follows.

$X_{111}+X_{121}+X_{161}+X_{181}+X_{191}-Y_{11}-Y_{12}=0$ (Junction a) (38)

$Y_{11}+A_{111}+A_{121}+A_{131}+A_{141}+A_{151}+_{161}+Y_{12}+A_{112}+A_{122}+A_{132}+A_{142}+A_{152}+A_{162}-Q_1=0$ (Junction b) (39)

$X_{211}+X_{221}+X_{251}+X_{261}+X_{271}+X_{281}+X_{291}-Y_{21}-Y_{22}=0$ (Junction a) (40)

$Y_{21}+A_{211}+A_{221}+A_{231}+A_{251}+A_{261}+Y_{22}+A_{212}+A_{222}+A_{232}+A_{252}+A_{262}-Q_2=0$ (Junction b) (41)

$X_{311}+X_{321}+X_{331}+X_{361}+X_{381}+X_{391}-Y_{31}-Y_{32}=0$ (Junction a) (42)

$Y_{31}+A_{311}+A_{321}\sim+A_{331}+A_{351}+A_{361}+Y_{32}+A_{312}+A_{322}+A_{332}+A_{352}+A_{362}-Q_3=0$ (Junction b) (43)

$X_{411}+X_{421}+X_{431}+X_{441}+X_{461}+X_{481}+X_{491}-Y_{41}-Y_{42}=0$ (Junction a) (44)

$Y_{41}+A_{411}+A_{421}+A_{431}+A_{441}+A_{451}+A_{461}+Y_{42}+A_{412}+A_{422}+A_{432}+A_{442}+A_{452}+A_{462}-Q_4=0$ (Junction b) (45)

$X_{511}+X_{521}+X_{541}+X_{561}+X_{581}+X_{591}-Y_{51}-Y_{52}=0$ (Junction a) (46)

$Y_{51}+A_{511}+A_{521}+A_{531}+A_{551}+A_{561}+Y_{52}+A_{512}+A_{522}+A_{532}+A_{552}+A_{562}-Q_5=0$ (Junction b) (47)

4. Demand constraint:The result derived from applying Eq. (8a) to Eq. (8b) are as follows.

$$2Q_1-Q_2=0 \quad (48)$$

$$3Q_1-Q_3=0 \quad (49)$$

$$4Q_1-Q_4=0 \quad (50)$$

$$4Q_1-Q_5=0 \quad (51)$$

5. Material proportion constraint:The result derived from applying Eq. (9a) to Eq. (10b) are as follows.

Chick Mash:

$X_{121}-0.5577X_{111}=0$ (Soya bean meal) (52)

$X_{161}-0.2789X_{111}=0$ (Wheat Offal) (53)

$X_{181}-0.0385X_{111}=0$ (Bone meal) (54)

$X_{191} - 0.0269X_{111} = 0$ (Limestone) (55)

$A_{111} - 0.0012Y_{11} = 0$ (Toxin Binder MFM1) (56)

$A_{112} - 0.0012Y_{12} = 0$ (Toxin Binder MFM2) (57)

$A_{121} - 0.0016Y_{11} = 0$ (Lysine MFM1) (58)

$A_{122} - 0.0016Y_{12} = 0$ (Lysine MFM2) (59)

$A_{131} - 0.002Y_{11} = 0$ (Methionine MFM1) (60)

$A_{132} - 0.002Y_{12} = 0$ (Methionine MFM2) (61)

$A_{141} - 0.00025Y_{11} = 0$ (Enzymes MFM1) (62)

$A_{142} - 0.00025Y_{12} = 0$ (Enzymes MFM2) (63)

$A_{151} - 0.003Y_{11} = 0$ (Premix MFM1) (64)

$A_{152} - 0.003Y_{12} = 0$ (Premix MFM2) (65)

$A_{161} - 0.003Y_{11} = 0$ (Salt MFM1) (66)

$A_{162} - 0.003Y_{12} = 0$ (Salt MFM2) (67)

Grower Mash:

$X_{221} - 0.186X_{211} = 0$ (Soya bean meal) (68)

$X_{251} - 0.2326X_{211} = 0$ (GNC) (69)

$X_{261} - 0.6465X_{211} = 0$ (Wheat Offal) (70)

$X_{271} - 0.1628X_{211} = 0$ (Corn Bran) (71)

$X_{281} - 0.043X_{211} = 0$ (Bone meal) (72)

$X_{291} - 0.0349X_{211} = 0$ (Limestone) (73)

$A_{211} - 0.001Y_{21} = 0$ (Toxin binder MFM1) (74)

$A_{212} - 0.001Y_{22} = 0$ (Toxin Binder MFM2) (75)

$A_{221} - 0.001Y_{21} = 0$ (Lysine MFM1) (76)

$A_{222} - 0.001Y_{22} = 0$ (Lysine MFM2) (77)

$A_{231} - 0.0015Y_{21} = 0$ (Methionine MFM1) (78)

$A_{232} - 0.0015Y_{22} = 0$ (Methionine MFM2) (79)

$A_{251} - 0.0025Y_{21} = 0$ (Premix MFM1) (80)

$A_{252} - 0.0025Y_{22} = 0$ (Premix MFM2) (81)

$A_{261} - 0.0025Y_{21} = 0$ (Salt MFM1) (82)

$A_{262} - 0.0025Y_{22} = 0$ (Salt MFM2) (83)

Layer Mash;

$X_{321} - 0.4X_{311} = 0$ (Soya bean meal) (84)

$X_{331} - 0.06X_{311} = 0$ (Full fat soya) (85)

$X_{361} - 0.304X_{311} = 0$ (Wheat offal) (86)

$X_{381} - 0.046X_{311} = 0$ (Bone meal) (87)

$X_{391} - 0.17X_{311} = 0$ (Limestone) (88)

$A_{311} - 0.001Y_{31} = 0$ (Toxin binder MFM1) (89)

$A_{312} - 0.001Y_{32} = 0$ (Toxin binder MFM2) (90)

$A_{321} - 0.001Y_{31} = 0$ (Lysine MFM1) (91)

$A_{322} - 0.001Y_{32} = 0$ (Lysine MFM2) (92)

$A_{331} - 0.003Y_{31} = 0$ (Methionine MFM1) (93)

$A_{332} - 0.003Y_{32} = 0$ (Methionine MFM2) (94)

$A_{351} - 0.0025Y_{31} = 0$ (Premix MFM1) (95)

$A_{352} - 0.0025Y_{32} = 0$ (Premix MFM2) (96)

$A_{361} - 0.0025Y_{31} = 0$ (Salt MFM1) (97)

$A_{362} - 0.0025Y_{32} = 0$ (Salt MFM2) (98)

Broiler starter;

$X_{421} - 0.66X_{411} = 0$ (Soya bean meal) (99)

$X_{431} - 0.06X_{411} = 0$ (Full fat soya) (100)

$X_{441} - 0.016X_{411} = 0$ (Soya oil) (101)

$X_{461} - 0.16X_{411} = 0$ (Wheat offal) (102)

$X_{481} - 0.054X_{411} = 0$ (Bone meal) (103)

$X_{491} - 0.028X_{411} = 0$ (Limestone) (104)

$A_{411} - 0.0015Y_{41} = 0$ (Toxin binder MFM1) (105)

$A_{412} - 0.0015Y_{42} = 0$ (Toxin Binder MFM2) (106)

$A_{421} - 0.0025Y_{41} = 0$ (Lysine MFM1) (107)

$A_{422} - 0.0025Y_{42} = 0$ (Lysine MFM2) (108)

$A_{431} - 0.0015Y_{41} = 0$ (Methionine MFM1) (109)

$A_{432} - 0.0015Y_{42} = 0$ (Methionine MFM2) (110)

$A_{441} - 0.00025Y_{41} = 0$ (Enzymes MFM1) (111)

$A_{442} - 0.00025Y_{42} = 0$ (Enzymes MFM2) (112)

$A_{451} - 0.0025Y_{41} = 0$ (Premix MFM1) (113)

$A_{452} - 0.0025Y_{42} = 0$ (Premix MFM2) (114)

$A_{461} - 0.003Y_{41} = 0$ (Salt MFM1) (115)

$A_{462} - 0.003Y_{42} = 0$ (Salt MFM2) (116)

Broiler finisher;

$X_{521} - 0.566X_{511} = 0$ (Soya bean meal) (117)

$X_{541} - 0.0226X_{511} = 0$ (Soya oil) (118)

$X_{561} - 0.2094X_{511} = 0$ (Wheat offal) (119)

$X_{581} - 0.0321X_{511} = 0$ (Bone meal) (120)

$X_{591} - 0.0321X_{511} = 0$ (Limestone) (121)

$A_{511} - 0.0012Y_{51} = 0$ (Toxin binder MFM1) (122)

$A_{512} - 0.0012Y_{52} = 0$ (Toxin Binder MFM2) (123)

$A_{521} - 0.0025Y_{51} = 0$ (Lysine MFM1) (124)

$A_{522} - 0.0025Y_{52} = 0$ (Lysine MFM2) (125)

$A_{531} - 0.003Y_{51} = 0$ (Methionine MFM1) (126)

$A_{532} - 0.003Y_{52} = 0$ (Methionine MFM2) (127)

$A_{551} - 0.003Y_{51} = 0$ (Premix MFM1) (128)

$A_{552} - 0.003Y_{52} = 0$ (Premix MFM2) (129)

$A_{561} - 0.003Y_{51} = 0$ (Salt MFM1) (130)

$A_{562} - 0.003Y_{52} = 0$ (Salt MFM2) (131)