N-Line Numerical Model to Predict The Morphological Changes around Groins System

نموذج عددى منعدد الخطوط للتنهؤ بالتغيرات المورفولوجية حول نظام الرؤوس المجرية

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ملفص البحث: التبسو بتناكلات قاع البحر في منطقة الناطي فاته تم اعداد نصوذج عددي شكلت الأبعداد وأدرجت العديد من العواصل الهيدرو بيناميكية السوش على الأسواح والتبارات والرسوييات. وقد استخدم نصوذج ن- الفيطي التثيل حركة الرسوييات في منطقة القدامي، بما ينبح دراستها واعطاء صورة حقيقية لتنكلات قاع البحر وحمسوما مع الساطي، بما ينبح دراستها واعطاء صورة حقيقية لتنكلات قاع البحر وحمسوما مع استخدام نظام العصاية بالرزوس العبارية وكذا التبو بالتغييرات العروفولوجية الناجمة عن استخدام فيذا الناسع من الحمساية . تسم استخدام نظرية Mild-Slope Equation في المناسبة من المناسبة عليه المناسبة والمناهدي المناسبة من المناسبة من المناسبة المناسبة والمناهدي المناسبة مناسبة المديث المديث المناسبة والمناهدي المناسبة مناسبة المناسبة والمناسبة والم

Abstract

A three-dimensional numerical model for predicting the beach evolution has been developed. A complete numerical model (SLMG2) was designed to represent most of the hydrodynamic factors affecting the problem. The N-Line model was used to simulate the sediment transport at the vicinity of groins system and to predict the shoreline morphological changes around it. The wave computations include wave refraction, diffraction and breaking. The validity of the model has been confirmed by applying it to the calculation of deformation of bottom topography around groins system. The comparison between model results and field data measurements showed a good agreement and indicated the flexibility of the model in handling such cases.

I. Introduction

Natural beaches are generally more or less in dynamical equilibrium; their deformation due to change in wave climates is rather seasonal, featured by alternate erosion and accretion. Once however, a structure is built near the shore, sediment budget on the neighboring beach will be unbalanced and an irreversible deformation will often take place. Prediction of beach deformation is needed in order to prevent erosion and siltation or to take rational counter measures against them. Physical models used to be exclusively adopted

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for the prediction, but they have been regarded to have some deficiencies such as high cost and scaling problems. Numerical models are now gradually replacing physical models.

Numerical models of beach deformation will be classified, according to the object and method of modeling, into two groups: shoreline model (One Line Model) and three dimensions model (N-Line Model). The shoreline models, often referred to as one-line models, have been well developed and applied to many parctical problems[15]. They may however not be used to predict local change of bottom topography, for which three-dimensional models will be required. Few attempts to develop such general topographical change models have been reported so far by such as Wang et al.[15], Hanson et al.[10], Watanabe[21], Mizuguchi and Mori [16], in all of which quantitative verification of the model is not given and the computation methods of waves, nearshore currents or sediment transport rates can not be regarded as adequate.

This paper presents a three-dimensional predictive model of beach evolution, which has been developed by using a new computation scheme of both the wave field and modified formulae for local sediment transport rates. The applicability of the model will be discussed through comparisons with field data measurments on the beach deformation around groin system.

II. Structure of The N-Line Beach Deformation Model

The present model is to predict the three-dimensional beach deformation from the spatial distribution of sediment transport rates, which are computed at local points, the wave conditions calculated in advance. The total model should therefore consist of two submodels for: 1) waves, and 2) sediment transport and beach deformation.

In the previous models mentioned above, the nearshore wave field has been computed by the wave energy equation (Wang et al.[15], Hanson et al.[10], Watanabe[21]) or by the wave ray method (Mizuguchi and Mori [16]). These methods may not be applicable to the computation of wave field under general conditions of complicated bottom topography and structure geometry. The author have recently developed a numerical model of the wave field under combined refraction, diffraction and breaking, depending on the solution of the Mild-Slope Equation (Tanaka 1990), which will be adopted in the present predictive model of beach deformation as the submodel for the nearshore wave computations. For more details of this wave model, refere to Sarhanl[18].

The submodel for sediment transport and beach deformation will be described in detail in the following chapter. In the present model, the wave-current interaction and shore flooding are not taken into account. Since the computations of waves and currents are time-consuming, it will not be practical to iterate them with those of beach deformation at a short time interval, and therefore the quasi-steady wave field will be assumed.

III. Modeling of Sediment Transport and Beach Deformation

There are several methods of modeling bathymetric changes due to the presence of a littoral barrier. An attempt can be made to model either the complete hydrodynamics and the resulting sediment transport or to use a combination of analytical and emperical sediment transport equations. In this model the second method was chosen due to the relative simplicity and previous effectivness of this latter approach.

At least two methods of employing sediment transport equations exist: a fixed longshore and cross-shore grid system where the depth is allowed to vary, or a fixed longshore and

depth system where the cross-shore distance is allowed to change. The latter system was chosen for the model. This method represents bathymetric changes due to a littoral barrier in terms of contour displacements.

The model is an N-Line representation of the littoral zone in which the longshore direction, x, is divided into equal segments, each Δx in length. The bathymetry is represented by N-contour lines, each of a specified depth, the location of which changes in offshore location according to equation of continuity. There are two components of sediment transport at each of the contour lines, a longshore component, Q_x , and an offshore component, Q_y . Figure (1) is a definition sketch showing the beach profile representation in a series of steps and the planform profile representation and notation used.

Governing Equations of Sediment Transport :-

Three basic equations are used to simulate the sediment transport and bathymetry changes according to the wave field. The equation of sediment continuity is:

$$\partial y/\partial t + 1/m \left(\partial Q_x/\partial x + \partial Q_y/\partial y\right) = 0$$
(1) in which m is the beach slope.

It requires as input, knowledge of the longshore and cross-shore components of sediment transport. The total transport alongshore has been studied by several investigators and many equations exist. However, the distribution of sediment transport across the surf zone is not well known as Thornton 1972^[20], Kraus et al. 1981^[14], Fulford 1982^[17], Ishida 1983^[12] and Kamphuis 1991^[13].

Fulford [17], based on laboratory data from Savage, developed a distribution of longshore sediment transport across the surf zone for the case of straight and parallel contour lines. The final form of Fulford's equation after Perlin 1983 is

f = constant to allow sediment transport above mean water line (MWL) (swash transport or transport in region of wave setup) to be represented. This equation predicts the relative transport at point y. To obtain the fraction of transport between two y coordinates, the integral of equ. (2) from y_1 to y_2 must be used

$$Q_{xD} = \exp \left\{ -((y_1 + f)/(1.25 y_b))^3 \right\} - \exp \left[-((y_2 + f)/(1.25 y_b))^3 \right] \dots \{3\}$$

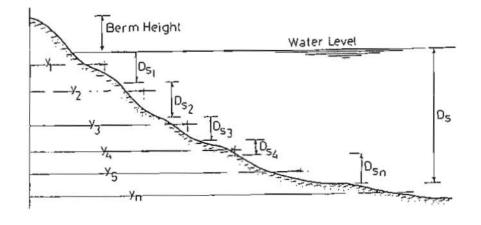
in which Q_{XD} = is the dimensionless quantity of the longshore sediment transport, y_1 , y_2 is the two limits as shown in figure (1).

To compute the value of the sediment transport between y₁ and y₂ in m³/sec., it must be multiplied by the total transport across a plan normal to the shoreline. The total longshore sediment transport rate is determined with the modified CERC formula;

where Q is the total longshore sediment transport rate given in m3/sec.,

Hb is the breaking wave height,

b subscript denoting the breaking condition,



(a)

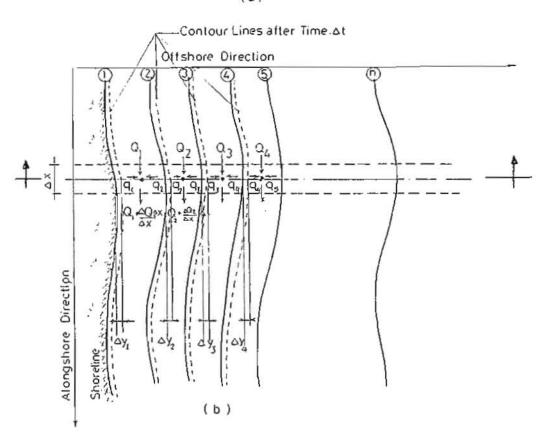


Fig.(1):Definition sketch :(a)Beach profile representation (b)Beach planform representation

C is a coefficient related to the soil properties.

Hanson et al. (1989)(10), derived the value of C as follows:

$$C = (a_1 \cdot \sin 2\alpha_{bs} - a_2 \cdot \cos \alpha_{bs} \cdot \partial H_b/\partial x)$$
(5)
where:

 α_{0s} is the angle of breaking waves to the local shoreline. $a_1 \& a_2$ are two nondimensional coefficients and are given by,

0.58 to 0.77 by Karus et al. (1982) and k2 is typically 0.50 to 1.0 times that of k1.

S is the relative density, P is the soil porosity, and $\tan \beta$ is the average bottom slope from the shoreline to the depth of active longshore sand transport which is calculated as a function of grain size. $\partial H_b/\partial x$ is the longshore gradient in breaking wave height.

This contribution to the longshore transport rate was introduced into shoreline change modeling by Ozasa (1980). The inclusion of the second term of eq. (5) provides an improved modeling results (Karus and Horikai 1983, Mimura et al. 1983)[10].

The second input required by the continuity equation is the onshore-offshore sediment transport. The cross shore sediment transport rate was investigated by many researchers and they tried to find a reliable formula joining the different variables affecting the problem. In this research, Bakker's formula (1968)[5] is used. This equation is written as:

 Q_y is the onshore-offshore sediment transport rate, Δx is the space step between two profiles normal to the shoreline as shown in figure (1). C_y is the activity factor and y' is the positive equilibrium profile distance between y_2 and y_1 , determined from the equilibrium profile used in the numerical model.

h(y) = A.y^{2/3}
where A is the scale parameter. The scale parameter has been calculated by Moore (1982)^[11] and it depends on the beach grain size;

$$A = 0.41 (d_{50})^{0.94}, d_{50} < 0.40$$

$$A = 0.23 (d_{50})^{0.32}, 0.40 \le d_{50} < 10.0$$

$$A = 0.23 (d_{50})^{0.28}, 10.0 \le d_{50} < 40.0$$
 (10)

$$A = 0.46 (d_{50})^{0.11}, 40.0 \le d_{50}$$

where do is the median nearshore beach grain size.

The value of Cy inside the surf zone is given by Bakker⁽⁵⁾ and equal to $3x10^{-6}$ m/sec. To generalize this equation outside the surf zone, the wave energy dissipation per unit volume is used as a measure of movement of the bottom sediment. Inside the surf zone, the breaking wave is the main cause of the wave energy dissipation. But outside the surf zone, the

dominant mode of wave energy dissipation is due to bottom friction. The wave energy dissipation by breaking waves, which is studied by Koniar et al. in 1972, and is given by D1 $D_1 = 5/24 \cdot \rho \cdot g^{3/2} \cdot k^2 \cdot A^{3/2} \dots (11)$ The wave dissipation by bottom friction is investigated by Nielsen (1983)[11], Sleath (1985), Perlin (1983), and is given by D2 $D_2 = 1/(6.\pi) \cdot \rho/h \cdot C_f \cdot [H^3 \cdot \sigma^3 / \sinh^3 kh]$ [12] where Cf is the bottom friction coefficient and a is the surface tension. The activity coefficient C, outside surf zone is given by $C_y = 1/\omega \cdot D_2/D_1 \cdot C_y$ in which w is a parameter relating to the efficiency with which breaking wave energy moves the sediment bottom $(0 < \omega < 1)$. The activity coefficient reduces rapidly with the increase of water depth. An implicit scheme that simultaneously solves the three governing equations, was developed[2]. The total longshore transport equation is obtained $Q_{x,i,i} = (\exp\{[(h_{i,i-1}^{3/2} + H_{bi} A^{3/2})/1.25.b_{bi}]^3\}$ - exp $\{[(h_{i,i}^{3/2} + H_{bi}.A^{3/2})/1.25.h_{bi}]\}) * C . H_{i,i}^{5/2} . sin (2\alpha_b)(14)$ at, is the breaking wave angle and is given by $\alpha_b = \theta - 3\pi/2 \cdot \alpha_c \qquad (15)$ where θ is the averaged wave angle at location of $Q_{\mathbf{r}}(i,j)$ and $\alpha_{\mathbf{r}}$ is the local contour orientation angle. Equation (14) could be written in the following form; n+1 and n+1/2 denoting the time step number. The value of the contour orientation wave angle could be calculated from the following equation $\Delta y^{n+1} = y_{i,j}^{n+1} - y_{i-1,j}^{n+1} \cdot \Delta y^n = y_{i,j}^n - y_{i-1,j}^n \cdot \Delta y = y_{i,j}^n \cdot y_{i-1,j}^n$ substitute from equation (17) into equation (16), and assume that the change in the denominator in equation (17) is small for a reasonable time step. The following equation is obtained;

 $Q_{xi,i}^{n+1} - \Psi_{i,i} \cdot \cos(2\theta) \cdot \cos(\alpha_c) \cdot 1/(\Delta x^2 + \Delta y^2)^{1/2} \cdot y_{i,i}^{n+1}$

To simplify equation (18), put that

$$\xi 1_{i,j} = \Psi_{i,j} \cdot \cos(2\theta) \cdot \cos(\alpha_c) \cdot 1/(\Delta x^2 + \Delta y^2)^{1/2}$$

 $\xi 2_{i,j} = \cdot \Psi_{i,j} \cdot (2 \sin \theta \cdot \cos \theta) \cdot (\cos^2 \alpha_c \cdot 1) + \xi 1_{i,j} \cdot (y_{i,j}^n \cdot y_{i-1,j}^n)$

Equation (18) could be written as;

$$Q_{x_{i,j}}^{n+1} - (\xi 1_{i,j}) \cdot y_{i,j}^{n+1} + (\xi 1_{i,j}) \cdot y_{i-1,j}^{n+1} = (\xi 2_{i,j})^n \dots (19)$$

From equation (19), the total longshore sediment transport could be calculated and this equation could be considered as the final form of it in the model.

Also, the final form of the cross-shore sediment transport in the model is as follows;

$$Q_{y_{i,j}} = \xi 3_{i,j} \cdot \left[\frac{1}{2} \cdot \left(y_{i,j-1}^{n+1} + y_{i,j-1}^{n} - y_{i,j}^{n+1} - y_{i,j}^{n} \right) + y'_{i,j} \right] \dots (20)$$

Substitute from equations (19) and (20) into continuity equation, the following equation is obtained.

$$y_{i,j}^{n+1} - y_{i,j}^{n} / \Delta t = 1/(2.\Delta x.\Delta h). [Q_{xi,j}^{n+1} + Q_{xi,j}^{n} - Q_{xi+1,j}^{n+1} - Q_{xi+1,j}^{n} + Q_{yi,j}^{n+1} + Q_{xi,j}^{n} - Q_{xi+1,j}^{n+1} - Q_{xi+1,j}^{n} + Q_{xi,j}^{n} + Q_{xi,j}^{n$$

$$+ Q_{yi,j}^{n} - Q_{yi,j+1}^{n+1} - Q_{yi,j+1}^{n}$$
(21)

Equation (21) is a weighted, centered scheme in which $y_{i,j}^{n+1}$ is computed using a weighting of itself and its four adjacent grid. By writing simultaneous equations, one for each $y_{i,j}$, a banded matrix can be constructed. The banded matrix is solved to yield the new values of y. (i.e. the new contour locations)

Model Construction

The model consists of one main program and ten subprograms. Figure (2) shows the flow chart of the program (SLMG2).

Boundary Conditions

- Longshore conditions: The longshore boundary conditions are treated by modeling a sufficient stretch of shoreline so that effects of a structure's presence are minimal. The y values along these boundaries can therefore be fixed at their initial locations.
- Onshore offshore conditions: In this case both the onshore and offshore are separated into two cases. In the onshore boundary condition, the berm and beach face are assumed to move in conjunction with the shoreline position. In the offshore boundary condition is treated by keeping the contour line beyond the last simulated contour fixed, until the angle of repose is exceeded.
- The structure boundary condition: There are no-flow boundary condition required at each of the structure be modeled^[3]. If the contour lines advance to the tip of the groin, another partial-flow conditions are needed. The sand bypassing is taken into consideration^[8,9].

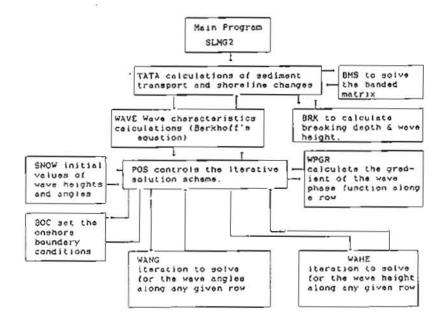


Fig.(2).Flow chart of the model (SLMG2).

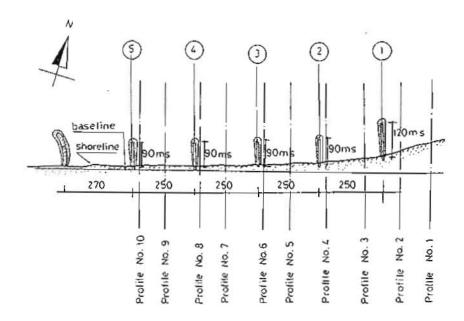


Fig. (3): Alignment of El-Arish shore protection system.

Model Verification:

The SLMG2 model predicts the morphological changes around groins system after any particular simulation period. This will help in designing the groins system along eroded beaches. To verify the model, the monitoring of El-Arish coastal area was used. The required input data are:

· Wave Characteristics :

The wave heights, periods, directions and durations at deep water are required as input data. The recording of wave characteristics at Port-Said is used for the simulation during the year. The wave data during the period from April 1984 to July 1985 are tabulated, arranged and the sequence of each storm is taken into consideration.

- Sediment and Beach Properties :

Soil investigation from the geotechnical map of El-Arish coastal area shows a very deep firm strata of culourous sand with shells and organic malters^[6]. The soil is mostly salty sand with specific weight in air equals to $1.55~\rm Vm^3$. The mean diameter (d₅₀) of soil ranges from 0.08 to 0.18 mm and the soil porosity is about 0.35. The activity coefficient inside the surf zone is $3\times10^{-6}~\rm m/sec$. The depth of closure, which is the critical depth of sea bottom changes, is calculated with Hallermeter's equation and equals to 4.0 ms. The berm height ranges from 1.0 to 4.0 ms in some places along the shoreline of the simulated coastal area. The beach slope equals to 1:40 in average. The value of scale parameter (A) ranges from 0.07 to 0.12.

The value of Δx is equal to 25 ms. The number of simulated profiles are 57 sections. The simulated contour lines are equal to 8. The dimensions and positions of each grain are feeded to the model, figure (3).

The simulated period is chosen to match with the available recording contour lines map. This period is about 15 monthes, started from April 1984 to 17 July 1985. The monitoring of shoreline during the period from April 1984 to September 1984 was used to calibrate the model and during the period from September 1984 to 17 July 1985 to verify the model. The time step is checked and is chosen to be 6.0 hours. This time step gives the most accurate results.

The boundary conditions are required along the extremities of the modeled area. The longshore boundaries can either be input as fixed or the sediment transport can be specified [11]. In this model, longshore boundaries are put to be fixed. The onshore boundary is treated by assuming that the berm and beach face are fixed relative to the shoreline contour. The offshore contour is fixed unless the angle of repose is exceeded. Additional conditions are required along the structure. There is no sediment transport along the structure position in the longshore direction ($Q_x = 0.0$)

Figures (4,5) show the results of the simulation compared with the field observations. These figures show good cabability of the model to simulate the sea bed changes at the vicinity of groins system. With reference to these figures, it is concluded that the model results show a good agreement with the measured field data. The sequence of storms have a significant effect on the final results of the sea bed configurations.

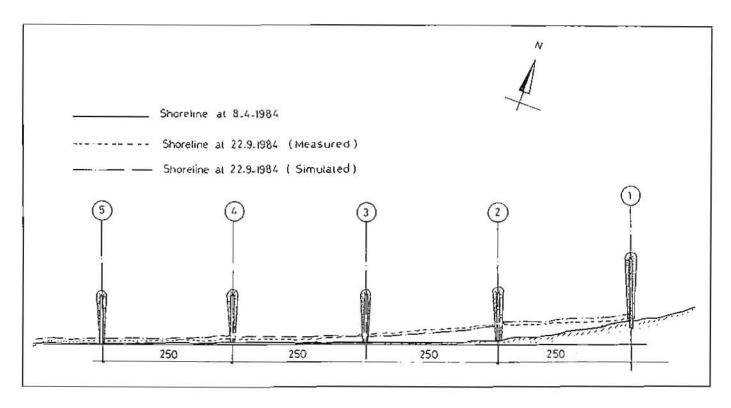


Fig.(4):Comparison between simulated and measured shoreline along El-Arish coast (at 22/9/1984)

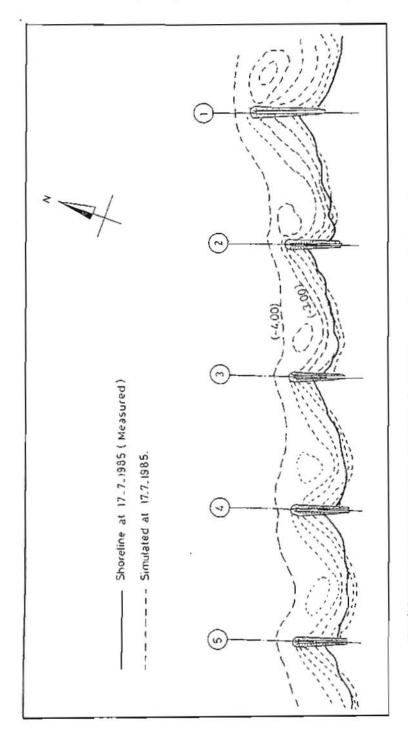


Fig.(5):Comparison between simulated and measured morphological changes along El-Arish coast. (at 17/7/1985)

To check the advance and the retarding of the sea bed bathymetry with relative to the base line, ten bed profiles were compared with the field data. Figures (6) to (15) indicate the comparisons between simulted and measured beach profiles. These figures show a good agreement between theoretical and field data with acceptable accuracy.

The results of the comparison approve the reliability of the model calculation model scheme. Owing to the comparison results and under the calculation assumpations, the model could be applied to many cases for the simulation of sediment transport around groins system.

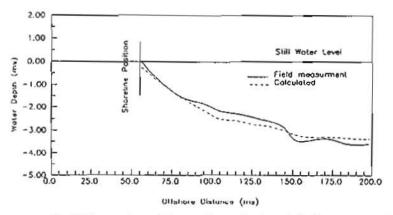


Fig.(6):Comparison between theoretical and field measurment of sea bed profile No. (1)

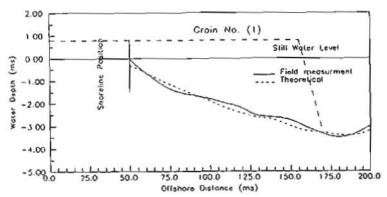


Fig.(7):Comparison between theoretical and field measurment of sea bed profile No. (2)

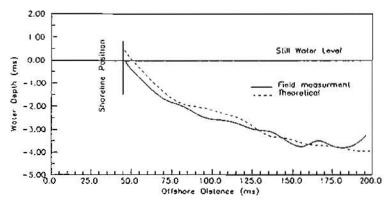


Fig.(8):Comparison between theoretical and field measurment of sea bed profile No. (3)

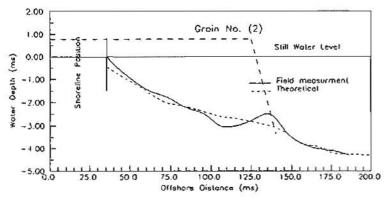


Fig.(9):Comparison between theoretical and field measurment of sea bed profile No. (4)

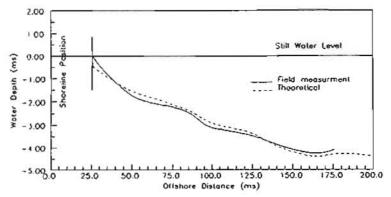


Fig.(10):Comparison between theoretical and field measurment of sea bed profile No. (5)

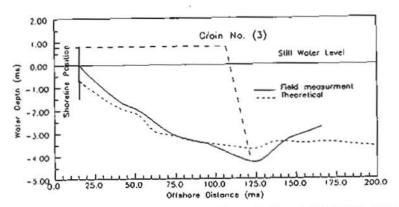


Fig.(11):Comparison between theoretical and field measurment of sea bed profile No. (6)

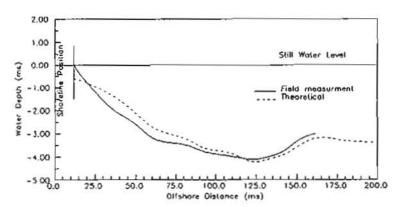


Fig.(12):Comparison between theoretical and field measurment of sea bed profile No. (7)

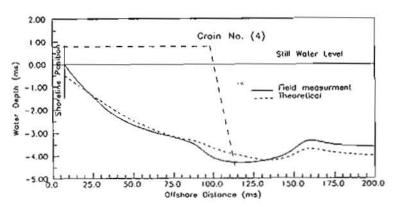


Fig.(13):Comparison between theoretical and field measurment of sea bed profile No. (8)

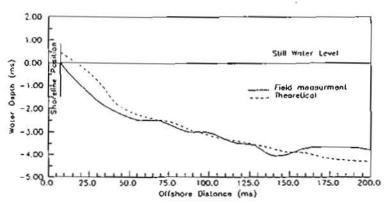


Fig.(14):Comperison between theoretical and field measurment of sea bed profile No. (9)

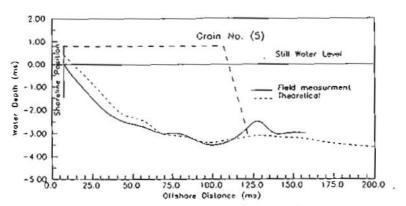


Fig.(15):Comperison between theoretical and field measurment of see bed profile No. (10)

Conclusions:

An N-Line shoreline morphology model (SLMG2) was designed to simulate the sea bed changes along any coastal area protected with groins system. Thus model can be used to design the protection system, to predict the morphological changes in future around the groins system and to choose the optumum alignments with munimum side effects on the neighboring areas.

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Notations

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The following symbols are used in this research.
       is the scale parameter;
a1, a2 : are dimensionless coefficients :
       is a coefficient related to the characteristics of the soil properties;
C
Cf
       : is the coefficient of bottom friction;
       is the activity factor inside the surf zone;
       is die wave energy dissipation by breaking waves;
         is the wave energy dissipation by bottom friction;
D_2
       is the median grain size;
d50
ť
       : is the constant of integration.;
       : the gravational acceleration ;
h
       : is the still water depth;
H
         the wave height;
рР
         is the breaking water depth;
       is the breaking wave height;
Hb
k1
       : 18 a dimensionless coefficient;
P
       : is the soil porosity;
        is the volume rate of longshore sediment transport;
Q
Q_x
       is the longshore component of sediment transport;
Qy
       : is the onshore-offshore sediment transport;
       is the rate of sediment entering and leaving the profile from the
         land and seaward boundaries;
T
       ; is the wave period;
       : honzontal coordinates ;
X,Y
         is the average separation distance between the two lines when the
         beach profile is in equilibrium:
       . is the distauce to the point of breaking;
50
y1,y2 is two limits as shown in figure (1);
αδ
       : is the angle of breaking waves to the local shoreline;
B
       : is the average bottom slope from the shoreline to the depth of active longshore
         sand transport;
       . is the incremental time step;
Δx
       : is the incremental space step in the x-direction;
dy
       : is the incremental space step in the y-direction;
\theta_{\text{c}}
       . is the angle between the contour lines and the x-axis;
       : water density;
       : is the soil density;
Ps
       is the surface tension; and
σ
      the first partial derivative.
```