Menofia University
Faculty of Engineering Shebien El-kom
Basic Engineering Sci. Department.
First semester Examination, 2017-2018
Date of Exam : 3 /1/2018

Subject: Partial Diff. Eqs.
Code: BES 507
Year : Master
Time Allowed : 3 hrs
Total Marks: 100 Marks

***Answer the following questions*** ***Two Pages***

**Question 1** ( 30 MARKS)

*(A) Classify according to type and determine the characteristics of the following P.D.Es.:*

(i) $2u_{xx} - 4u_{xy} - 6u_{yy} + u_x = 0$ (ii) $4u_{xx} + 12u_{xy} + 9u_{yy} - 2u_x + u = 0$

(iii) $u_{xx} - x^2 y u_{yy} = 0$ ( $y>0$) (iv) $e^{2x} u_{xx} + 2e^{x+y} u_{xy} + e^{2y} u_{yy} = 0$ (5 Marks)

*(B) Find all eigenvalues and eigenfunctions for the problem:*

$$-w''(x) = \lambda w(x) \quad 0 < x < l$$
$$w'(0) = w'(l) = 0$$

(5 Marks)

*(C) Solve the following PDE.*

$$u_t(x,t) = k u_{xx}(x,t) + F(x,t) \quad 0 < x < l,\ t > 0$$
$$u(x,0) = f(x) \quad 0 < x < l$$
$$u(0,t) = u(l,t) = 0 \quad t > 0$$

(10 Marks)

*(D) Solve the following Dirichlet problem for Laplace 's equation:*

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r) + \frac{1}{r^2}u_{\theta\theta} = 0 \quad r < 1, \quad -\pi < \theta < \pi$$
$$u(1,\theta) = f(\theta) \quad -\pi < \theta < \pi$$
$$u(r,-\pi) = u(r,\pi),\ u_\theta(r,-\pi) = u_\theta(r,\pi) \quad r < 1$$

(10 Marks)

**Question 2** ( 40 MARKS)

(A) For the initial-boundary value problem

$$u_t = a^2 u_{xx} \quad 0 < x < 1,\ t > 0$$
$$u(x,0) = f(x) \quad 0 < x < 1$$
$$u(0,t) = p(t), u(1,t) = q(t) \quad t > 0$$

Show how to imbed the implicit method and the Crank- Nicolson method in a single algorithm. (10 Marks)

(B) Use the numerical method of characteristics to estimate, at y = 0.01, 0.1, 0.2, 0.3, the solution to

$$uu_x + u_y = -2u^3$$
$$u(x,0) = x$$

on the characteristic through Q = (1, 0). Then compare the numerical results with the exact solution (10 Marks)

**(C)** Show how to apply finite differences to

$$u_{xx} + u_{yy} = f(x,y) \quad in\ \Omega$$
$$u = g(x,y) \quad on\ S$$

in the case that $\Omega$ has a curved boundary. (10 Marks)

(D) Making use of Problem in Part C, approximate the solution to

$$u_{xx} + u_{yy} = 0 \quad x^2 + y^2 < 1, y > 0$$
$$u(x,y) = 100 \quad x^2 + y^2 < 1, y > 0$$
$$u(x,y) = 0 \quad y = 0, -1 < x < 1$$

hoose a square grid with h = 0.5. (10 Marks)

**Question 3** ( 30 MARKS)

**(A)** *Construct the Rayleigh- Ritz approximation to the solution of*

$$-u''(x) + u(x) = 1 - x \quad 0 < x < 1$$
$$u'(0) = u'(1) = 0$$

*using the trial functions*

(a ) $\emptyset_1(x) = 1, \emptyset_2(x) = x, \emptyset_3(x) = x^2$;

(b) $\psi_1(x) = x^2(1-x)^2, \psi_2(x) = x^3(1-x)^2, \psi_3(x) = x^2(1-x)^3$.

(c ) Compare both approximate solutions with the exact solution,

$$u^*(x) = \frac{\cosh x - \cosh(1 - x)}{\sinh 1} + 1 - x$$

(15 Marks)

**(B)** Construct the Galerkin approximation to the weak solution of the problem

$$-(u_{xx} + u_{yy}) + 2u_x - u_y = 1 \quad in\ \Omega = \{x > 0, y > 0, x + 2y < 2\}$$
$$u = 0 \quad on\ S_1 = \{x + 2y = 2\}$$
$$u_x(0,y) = y \quad on\ 0 < y < 1$$
$$u_y(x,0) = 0 \quad on\ 0 < x < 2$$

Use the single trial function $\phi(x,y) = (2 - x - 2y)(1 + x + y)$ and the single weight function $\psi(x,y) = 2 - x - 2y$. (15 Marks)

| This exam measures the following ILOs | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Question Number | Q1-a | Q2-a | | | Q2-b | Q3-b | | | Q1-b | Q3-a | |
| Skills | | b-i | | | b-i, b-iii | | | | | | |
| | Knowledge &understanding skills | | | | Intellectual Skills | | | | Professional Skills | | |

*With my best wishes*

*Associate Prof. Dr. Islam M. Eldesoky*