"Static Analysis of Three Span Cable Stayed bridges having three levels of cable attachments with pylons"

التحليل الاستاتيكى للكبارى ذات الكابلات ذو الثلاثة بحور المربوطة بثلاثة مستويات من الكابلات بالصوارى

By

Mohamed Naguib Abo El-Saad
Professor,Struct.,Eng.,Dept.
Faculty of Engineering
Mansoura University-Egypt

Shaaban Ibrahim Selim
Dr.,Struct.,Eng.,Dept.
Faculty of Engineering
Mansoura Unversity-Egypt

Mohamed Mohamed Mortagy
M. Sc. In Structural Enggineering
Faculty of Engineering
Mansoura University-Egypt

الخلاصة:

إن هدف هذا البحث الرئيسى هو الوصول الى التمثيل الرياضي الأمثل لاجراء التحليل الاستاتيكي للكبارى ذات ثلاث مستويات من الكابلات المربوطة فى الجزء العلوى من الأبراج والجزء السفلى مع كمرات أرضية الكوبرى حيث تم عمل دراسة لكوبرى بثلاثة بحور بأطوال ٢٦٥ متر لكل من البحرين الخارجين و ٥٣٠ متر للبحر الداخلى وبطول اجمالى ١٠٦٠ متر. وقد تم الاخذ فى الأعتبار أربعة حالات تحميل مختلفة وقد تركزت الدراسة على تأثيرالشد الابتدائي في الكابلات وذلك بإتباع طرق التقارب للوصول إلى أفضل شد إبتدائى فى الكابلات الذى يحقق أقل ترخيم بكمرات أرضية الكوبرى [١]. مستخدما طريقة الطاقة المبنية علي تصغير طاقة الوضع باستخدام الانحدارات المتبادلة.

Abstract

The main object of this research is concerned about the best choice of the mathematical model to carry out the static analysis for cable stayed bridges having three levels of cables attached with pylons of floor beams in radiating shapes. A studying case on cable stayed bridge has two equal exterior spans of 265m, each, and interior span of 530 m is carried out with four cases of loading which include the most symmetric traffic loads are considered. The work is concerned with the effect of initial tension in inclined cables on the outcome responses of some cable bridges. A technique for the choice of the best initial tension in cables depending on an iterative scheme to give the minimum static responses [1]. This technique is termed " circle of solution". In the static analysis, the energy method, based on the minimization of the total potential energy of structural elements, via conjugate gradient method.

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1 - Introduction

Cable stayed bridge are the bridge systems in which the decks are supported by cables. The load from the deck are transferred to the towers then to the ground, or directly to the base rocks if no tower adopted instead of ground anchors. Generally, the cable stayed bridge consist of the following parts: stiffening girder, cable systems ,towers, and anchors bodies. The cable-stayed bridge has been developing rapidly since World War II, and becomes one of the most competitive types of bridges for main spans ranging from 300 to 1200 meters. The Normandy bridge (865 meter, 1995, France) and the Tatara bridge (890 meter, 1999, Japan) showed the potential to compete with the suspension bridge in the lower end of its rational span range. For some soft soil bridge sites, on which building of the anchor will dramatically increase the overall cost, a long-span cable-stayed bridge would be the first candidate. It is feasible to build a cable-stayed bridge with a main span as long as 1200 meters. The results of some feasibility studies on building a cable-stayed bridge with a main span over 1000 meters motivated some huge bridge projects in Southeast Asia [2].

The most common cable stayed bridges may be classified as harp, radiating, star and fan shapes depending on the arrangements of cables and their connections with pylons and decks. The own weight of all structural elements with all various considered cases of equivalent traffic loads are considered. The analysis is carried out considering cable and space frame elements for cables and pylons and floor beams, respectively. The Energy method is a unifying approach to the analysis of both linear and non-linear structures. It is an indirect method of analysis and valid for both small and large structures. The energy method is applied to the analysis of general pin-ended truss and cable structures. A summary together with a step-by-step iterative procedure is presented. Main sources of knowledge about this method are given in[3], [4], [5], [6] and [7]. The obtained numerical results for all cases are discussed and compared. Finally, the major conclusions are presented.

2. Step- by –step static response analysis by minimization of the total potential energy using the conjugate gradient technique.

The point at which W (total potential energy) is a minimum defines the equilibrium position of the loaded structure. Mathematically, the equilibrium condition in the i direction at joint j may be expressed as:

$$\frac{\partial W}{\partial x_{ji}} = [g_{ji}] = 0 \quad , i=1, 2 \text{ and } 3 \quad (1).$$

The location of the position where W is a minimum is achieved by moving down the energy surface along a descent vector v a distance S_v until W is a minimum in that direction, i.e , to a point where :

$$\frac{\partial W}{\partial S} = 0 \quad \text{--------------------------} \quad (2).$$

Where:

x_{ji}= the displacement of joint j corresponding to a particular degree of freedom in direction, i , and

g_{ji} = the corresponding gradient of the energy surface.

From this point a new descent vector is calculated and the above process is repeated.The method is mathematically expressing the displacement vector at the (k+1) th iteration as:

$[x]_{k+1} = [x]_k + S_k v_k$ -----------------(3)

Where:

v_k = the descent vector at the K^{ith} iteration from x_k in the displacement space, and

S_k = the step length determining the distance along v_k to the point where W is a minimum.

The method is shown diagrammatically in Fig. (1).

Summary of circle of solution[1].

The main steps in optimum outcome responses shape technique through the iterative processes required to achieve structural equilibrium by minimization of the total potential energy may be summarized as follows:

I-Consider in the first cycle an initial tension in all cable members as a 10% of the minimum ultimate value , T_o, then:

II-First, before the start of the iteration scheme Calculate the tension coefficients for the pretension forces in the cables by:

$$t_{jn} = \left[\left(T_0 + \frac{EA}{L_0}e\right) / L_0\right]_{jn} \text{----------------}(4)$$

Where: t_{jn} = the tension coefficient of the force in member jn ;

e =elongation of cables due to applied load;

T_0 = initial force in a pin-jointed member or cable link due to pretension;

E = modulus of elasticity;

A = area of the cable element, and

L_0 = the unstrained initial length of the cable link.

Assume the elements in the initial displacement vector to be zero.

Calculate the lengths of all the elements in the pretension structure using the following equation:

$$L_0^2 = \sum_{i=1}^{3} \left(X_{ni} - X_{ji}\right)^2 \quad \text{------------------(5)}$$

Where: X = element in displacement vector due to applied load only, for using method of conjugate gradients is used, calculate the elements in the scaling matrix.:

$$H = diag\left\{k_{11}^{-1/2}, k_{22}^{-1/2}, \ldots\ldots\ldots\ldots k_{nn}^{-1/2}\right\} \quad \text{----(6)}$$

Where: n = total number of degrees of freedom of all joints, and

k is the main diagonal element in frame stiffness matrix for flexural elements.

III- The steps in the iterative procedure are then summarized as:

Step (1) Calculate the elements in the gradient vector of the TPE, using:

$$[g_i]_n = \sum_{n=1}^{f_n} \sum_{r=1}^{12} (k_{nr} x_r)_n + \sum_{n=1}^{P_n} \left(T_0 + \frac{EA}{L_0} e\right)_n \left[\frac{\partial e_n}{\partial x_i}\right]_n - [F_i]_n \quad \text{--------(7)}$$

Step (2) Calculate the Euclidean norm of the gradient vector, $R_k = \left[g_k^T g_k\right]^{1/2}$, and check if the problem has converged. If $R_k \le R_{\min}$ stop the calculations and print the results. If not, proceed to step (3).

Step (3) Calculate the elements in the descent vector, v using:

$$[v]_{k+1} = -[H][g]_{k+1} + \beta_K [v]_k \quad \text{--------(8)}$$

Where $[v]_0 = -[g]_0$ ----------------(9)

and ,

$$\beta_k = \frac{[g]_{k+1}^T [H]^T [H][g]_{k+1}}{[g]_k^T [H]^T [H][g]_k} \quad \text{------------(10)}$$

Step (4) Calculate the coefficients in the step-length polynomial from:

$$C_4 = \sum_{n=1}^{P} \left(EAa_3^2 / 2L_0^3\right)_n$$

$$C_3 = \sum_{n=1}^{P} \left(EAa_2 a_3 / L_0^3\right)_n$$

$$C_2 = \sum_{n=1}^{P} \left[t_0 a_3 + EA\left(a_2^2 + 2a_1 a_3\right) / 2L_0^3\right]_n + \sum_{n=1}^{f} \sum_{s=1}^{12} \sum_{r=1}^{12} \left(\frac{1}{2} v_s k_{sr} v_r\right)_n$$

$$C_1 = \sum_{n=1}^{P} \left[t_0 a_2 + EAa_1 a_2 / L_0^3\right]_n + \sum_{n=1}^{f} \sum_{s=1}^{12} \sum_{r=1}^{12} \left(x_s k_{sr} v_s\right)_n - \sum_{n=1}^{N} F_n v_n \quad \text{------------------(11)}$$

Where:

$$a_1 = \sum_{i=1}^{3} \left[\left(X_{ni} - X_{ji}\right) + \frac{1}{2}\left(x_{ni} - x_{ji}\right)\right]\left(x_{ni} - x_{ji}\right)$$

$$a_2 = \sum_{i=1}^{3} \left[\left(X_{ni} - X_{ji}\right) + \left(x_{ni} - x_{ji}\right)\right]\left(v_{ni} - v_{ji}\right)$$

$$a_3 = \sum_{i=1}^{3} \frac{1}{2}\left(v_{ni} - v_{ji}\right)^2 \quad \text{--------------------(12)}$$

Where:

f = number of flexural members, P = number of cable links,

F = element in applied load vector, and

K_{sr} = Element of stiffness matrix in global coordinates of a flexural element.

Step (5) Calculate the step-length S using Newton's approximate formula as:

$$S_{k+1} = S_k - \frac{4C_4 S^3 + 3C_3 S^2 + 2C_2 S + C_1}{12C_4 S^2 + 6C_3 S + 2C_2} \quad \text{----------------(13)}$$

Where: k is an iteration suffix and $S_{k=0}$=0 is taken as zero

Step (6) Update the tension coefficients using the following equation.

$$(t_{ab})_{k+1} = (t_{ab})_k + \frac{EA}{(L_0^3)_{ab}} (a_1 + a_2 s + a_3 s^2)_{ab} \quad \text{--(14)}$$

Step (7) Update the displacement vector using equation (3).

Step (8) Repeat the above iteration by returning to step (1).

IV– Take the final tension in cables for each group of cables as an initial tension and start again with II.

V– In each cycle of solution, the ratio of the vertical displacements in deck floor or lateral sway in pylons in cable-stayed bridges and shafts in guyed towers at control points , to those at main structural points, μ , will be checked. i.e. $\mu < \epsilon$, where ϵ is the convergence tolerance. The cycles of solutions will be repeated until the convergence tolerance is achieved otherwise continue with step II.

4. Geometry and properties of bridge

A three span cable stayed bridge has two equal exterior spans of 265m, each, and interior span of 530 m. The deck girder has a total span of 1060 m . The bridge is symmetric and is composed of three major elements: (a) the deck girder, (b) the pylons and (c) eight cables on each side of pylon shown in Figs.(2,3) . The cables types are spiral strand bridge cable. [11] . The properties of cables ,pylons and deck are given in Table [1].

5. Analysis Considerations

The model is carried out for the four cases of loading as shown in Fig.(4). This model has 1302 degrees of freedom .The analysis is carried out for all four cases of loading. Many examples are solved to explain the factors affecting the analysis of cable-stayed bridges. These factors are pylon height relative to central span, and cable pretension.

The pylon height relative to the central span of the bridge (H/L) varies between 0.2 and 0.5, with interval of 0.1.The initial tension in cables in all cases is taken (5%,10 %,15 % and 20%)of of the minimum ultimate value , Tu. Using circle solution technique .Four circle were carried out.

All prepared computer programs used in this research with its verification is given by [13].

Figures (5) to (8) show the values of the displacement along the floor beam due to changing in the initial tension, and (H/L) ratio for case 1.

Figures (9) to (12) show the values of the bending moment along the floor beam due to changing in the initial tension and (H/L) ratio for case 1.

Figures (13) to (16) show the of the normal force along the floor beam due to changing in the initial tension and (H/L) ratio for case 1.

Table [2] gives some obtained results for other cases.

Fainally , the model with (H/L) = 0.3 is carried out, using circle of solution technique .

Figure (17) show some of the obtained results for this solution..

The obtained results for circle of solution is give in Table (3).

Figures (18) to (20) show some of the obtained results for circle of solution.

6. *Conclusions*

It may be concluded that

1. Increasing (H/L) ratio led to decreasing the deflection along the floor beam.
2. Increasing the intial tension from 5% to 20% causes decrasing the deflection along floor beam.
3. Using optimum outcome response shape technique gave a significant reduction in deflection and moment in floor beam.This reduction varies between 75 to 85% for deflection,and 20 to 30% for moment.
4. The initial tension in cables plays an important role in the analysis of cable structures.

Finally: to get up the minimum deflection, it is required to increasing (H/L) ratio up to 0.3 then the effect of intial tension can be taken in second category.

Using optimum outcome response shape technique gave a significant reduction in deflection and moment in floor beam.

8-Symbols

T.P.E.= total potential energy,

H = the height of pylon above floor level,

H/L =pylon height to span ratio,

D.L.=own weight of structural elements including weight of asphalt (dead load).

L.L.= an equivalent uniform traffic loads including impact as live loads.

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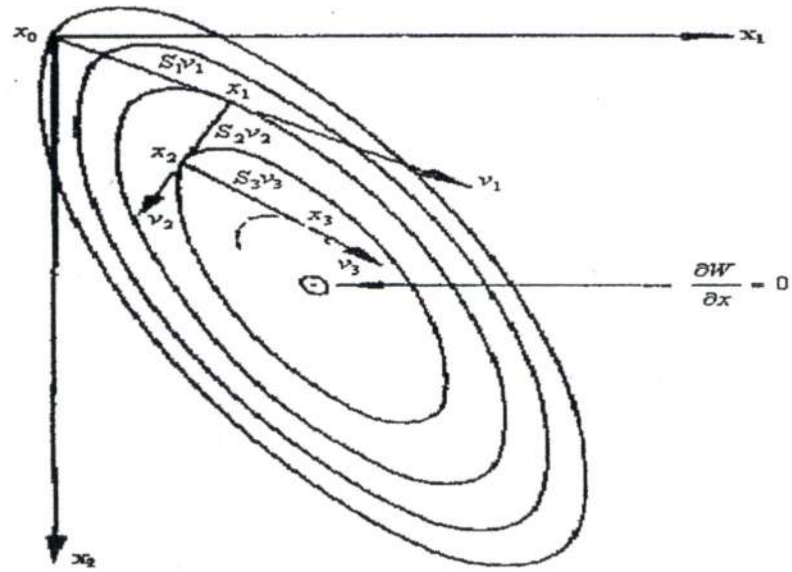


Fig .(1). Contour map indicating diagrammatically the basic method, based on the minimization of the total potential energy.

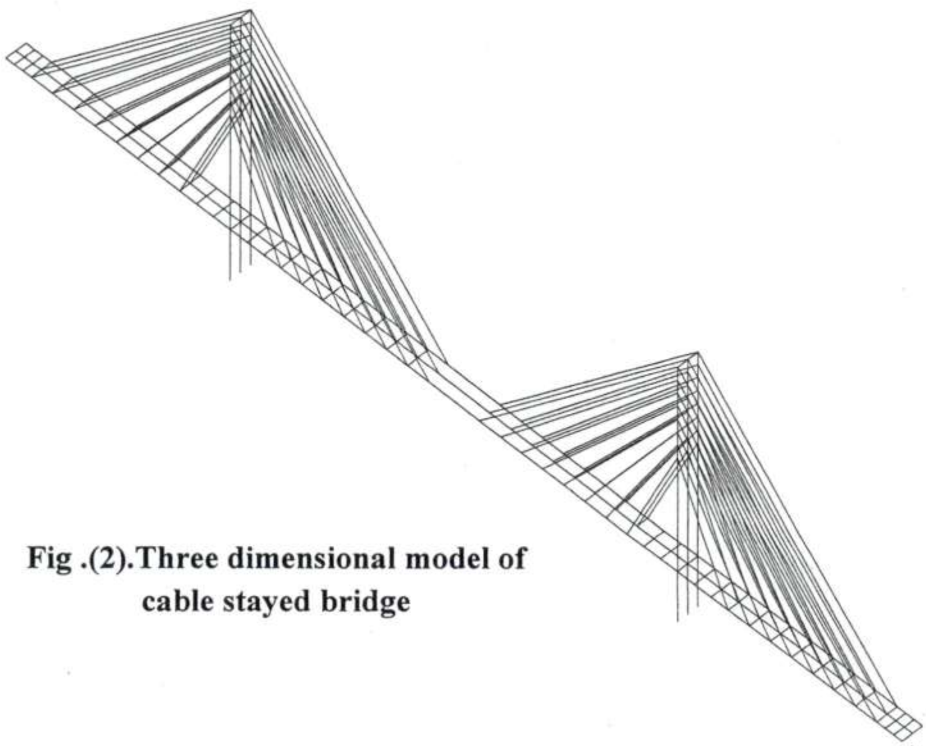
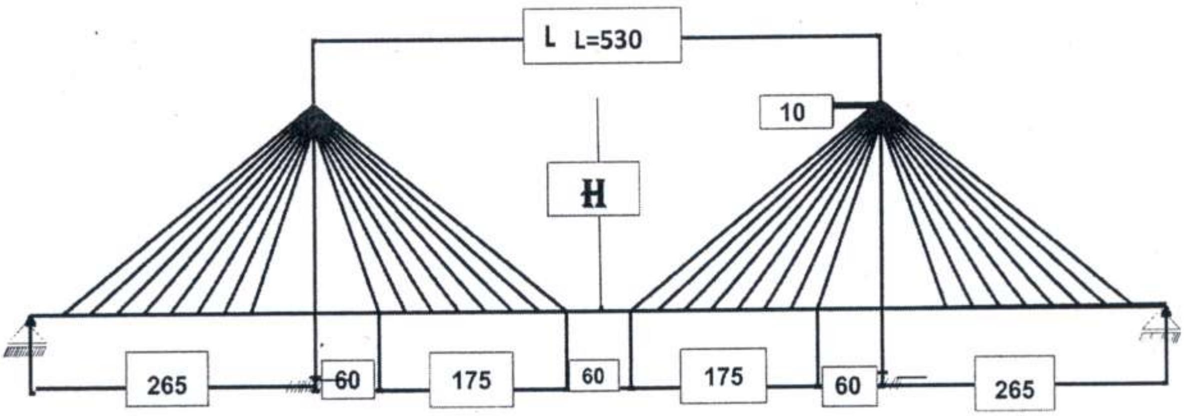
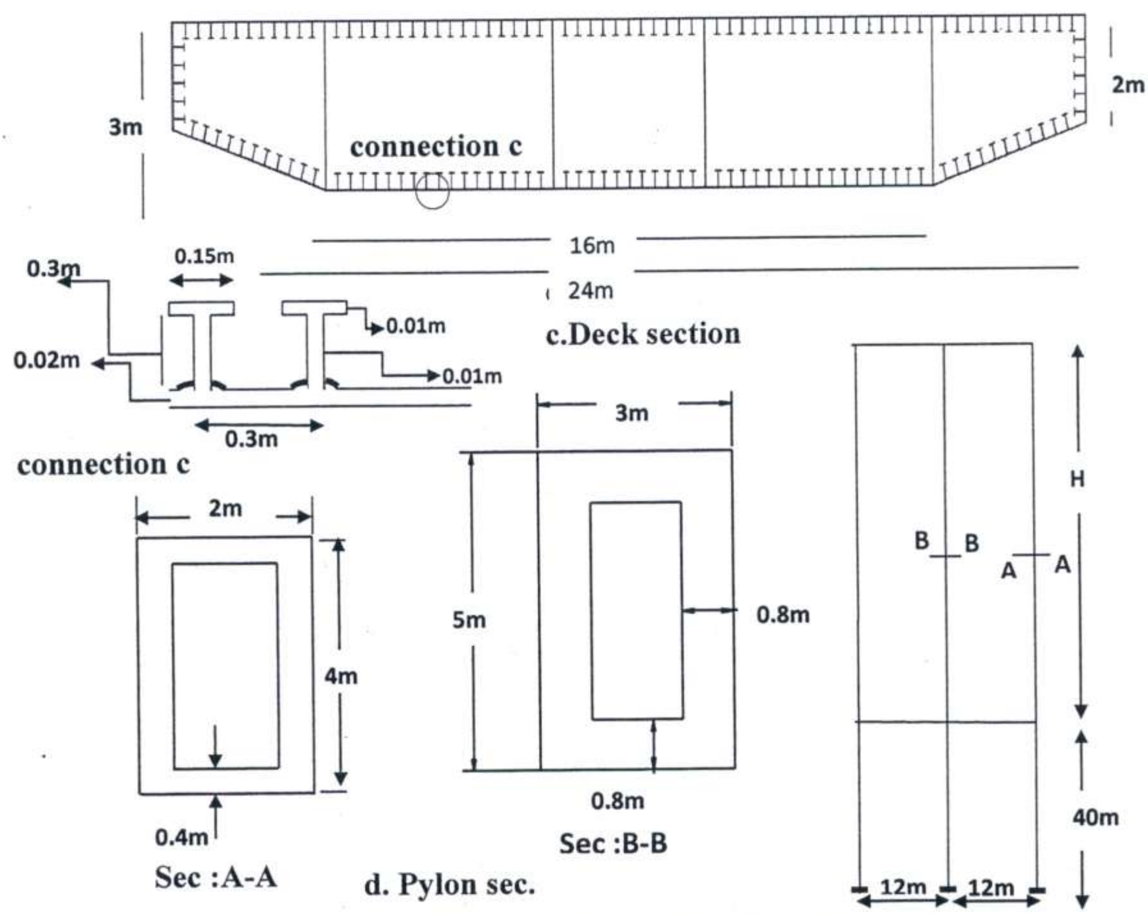


Fig .(2).Three dimensional model of cable stayed bridge



b.Bridge elevation



c.Deck section

d. Pylon sec.

Fig .(3) A general layout of cable stayed bridge.

Table (1) : propere s of pylon, deck o or and cables-stayed bridges.

Structural element		Description of structural elements		Properties of sections					loads	
				Young's modulus t/cm2	Area m^2	inertia I_x m^4	Inertia I_y m^4	Torsion Constant m^4	Dead load t/m	Live load t/m
Pylon	Vertical member	Hollow rectangular R.C. section	Sec A	300	7.04	9.5	2.655	5.4	17.6	0
			Sec B	300	11.64	23.31	10.473	21.334	29.10	0
	trasverse beam	a square cross reinforced concrete		with dimensions 1x1 m. The width of the bridge is 24 m.						
Deck	Longitudinal beam	Steel box girder in orthotropic plate shape		2100	2.5771	4.6585	134.222	69.44	Figure (3)	
	trasverse beam	built I- section		2100	0.12	0.01042	0.0544	0.0543	2.5	0
Cables		spiral strand bridge cable		1472	0.01101	Diameter =17.3 cm			0.0891	

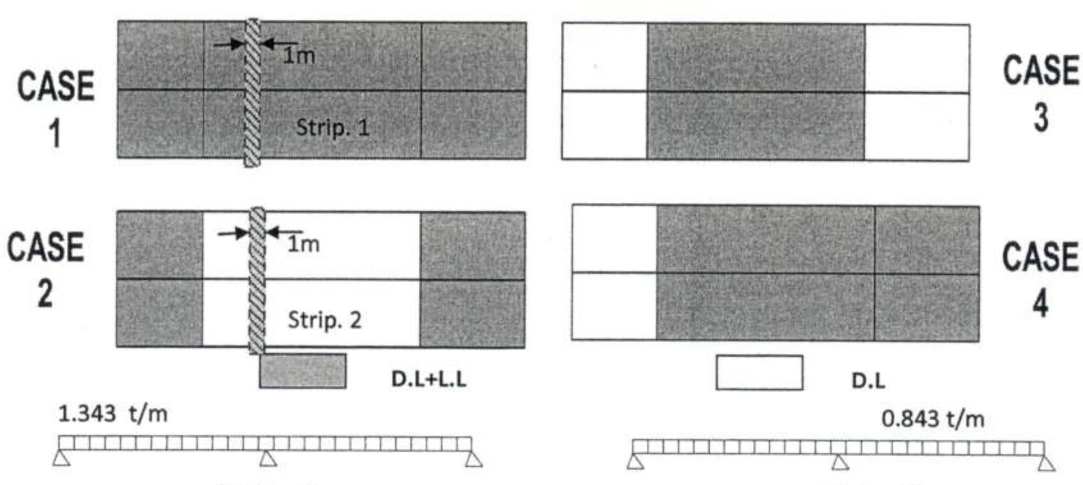


Fig.(4). Case of loading

case (1)

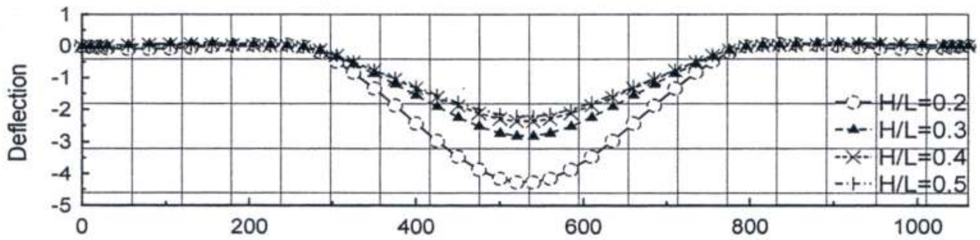


Fig.(5): Deflection for floor beam with various H/L at 5% initial tension

Case (1)

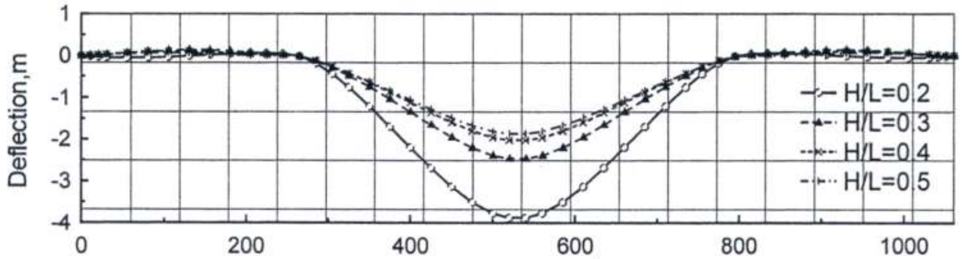


Fig.(6): Deflection for floor beam with various H/L at 10% initial tension

Case (1)

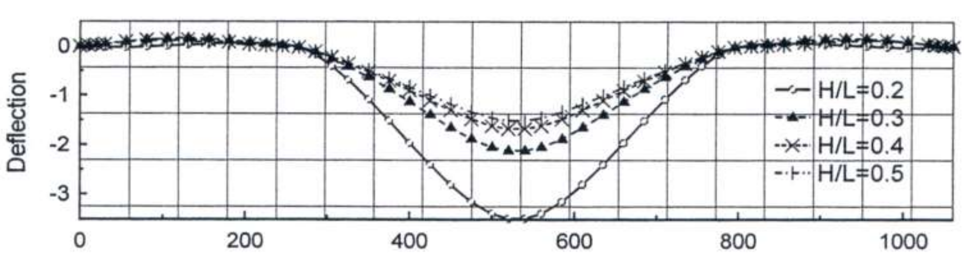


Fig.(7): Deflection for floor beam with various H/L at 15% initial tension

Case (1)

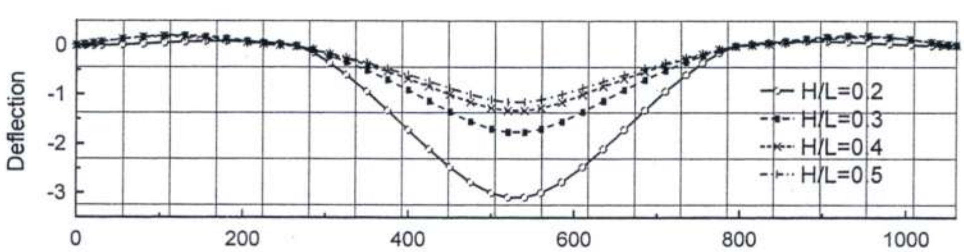


Fig.(8): Deflection for floor beam with various H/L at 20% initial tension

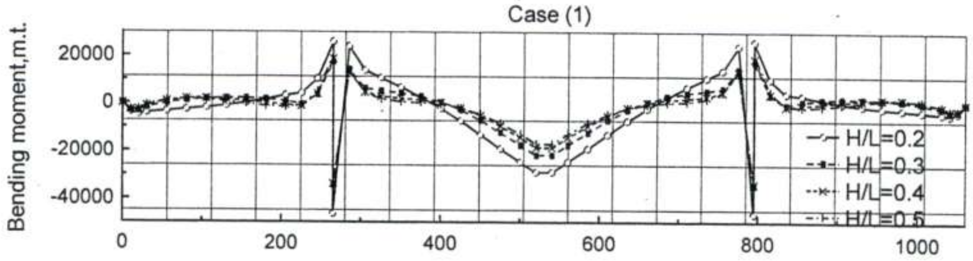


Fig.(9):Bending moment along floor beam for various H/L at initial tension = 5%

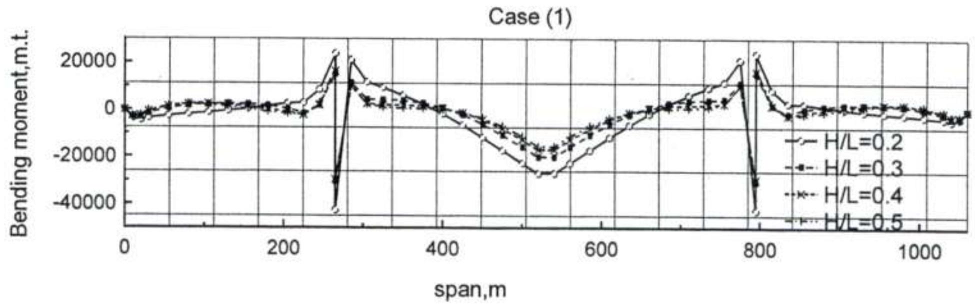


Fig.(10):Bending moment along floor beam for various H/L at initial tension = 10%

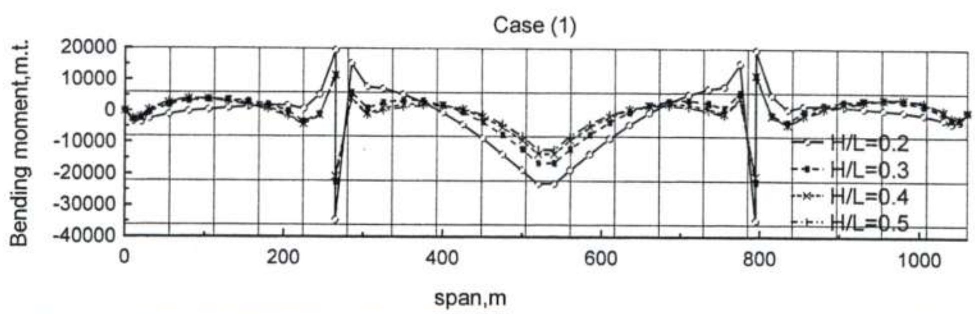


Fig.(11):Bending moment along floor beam for various H/L at initial tension = 15%

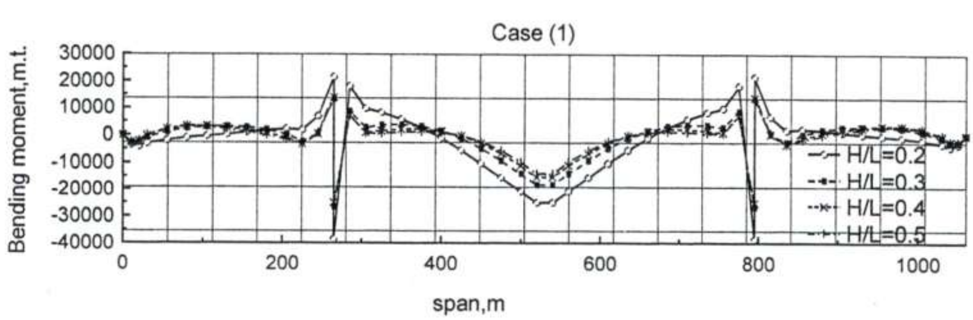


Fig.(12):Bending moment along floor beam for various H/L at initial tension = 15%

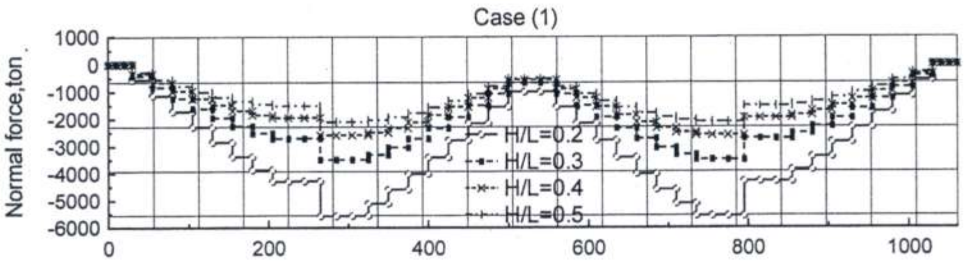


Fig.(13):Normal force along floor beams for various of H/L at 5% initial tension.

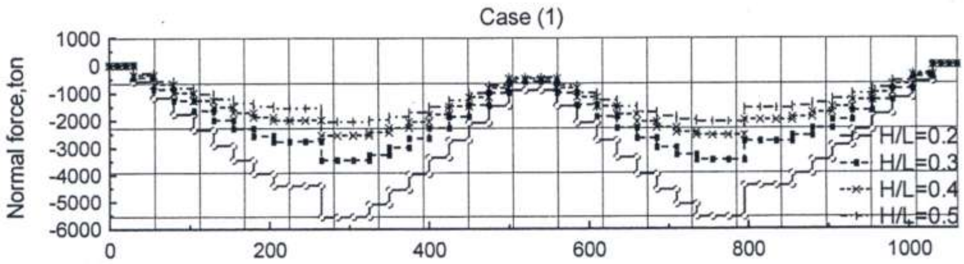


Fig.(14):Normal force along floor beams for various of H/L at 10% initial tension.

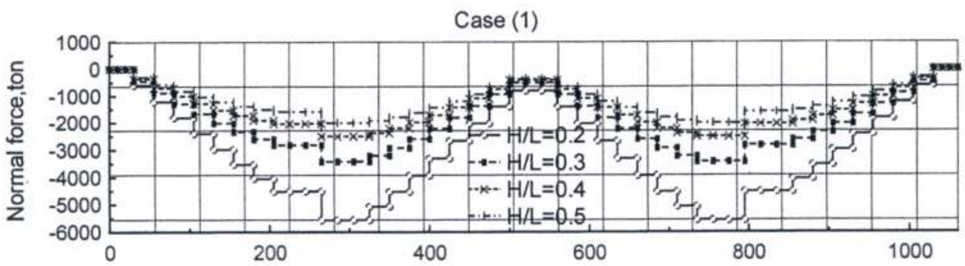


Fig.(15):Normal force along floor beams for various of H/L at 15% initial tension.

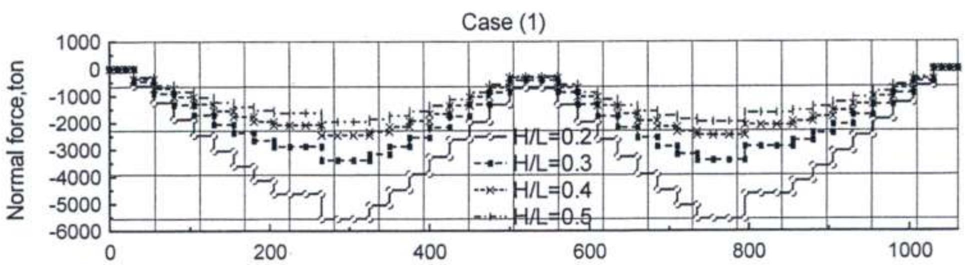


Fig.(16):Normal force along floor beams for various of H/L at 20% initial tension.

	To %	Case 2			Case 3			Case 4		
		Deflection ,m	Bending moment,t,m	Normal force,t	Deflection ,m	Bending moment,t,m	Normal force,t	Deflection ,m	Bending moment,t,m	Normal force,t
H/L=0.2	5	-1.712	37174	-3375	-5.1554	-51074	-6645	-2.3973	33222	-3393
	10	-1.3134	35019	-3487	-4.764	-40128	-6635	-1.8976	30618	-3520
	15	-0.9638	32859	-3599	-4.3721	-35209	-6604	-1.4664	28617	-3635
	20	-0.923	30693	-3708	-3.9799	-30269	-6619	-1.0274	26690	-3754
H/L=0.3	5	-0.8686	29534	-2092	-3.6747	-40144	-4489	-1.4952	26675	-2097
	10	-0.8339	27514	-2149	-3.3251	-36239	-4453	-1.0062	23132	-2175
	15	-0.7992	25493	-2207	-2.9751	-32328	-4400	-0.8033	20832	-2240
	20	-0.7643	23470	-2260	-2.6247	-28413	-2304	-0.7192	18824	-2304
H/L=0.4	5	-0.8543	30264	-1498	-3.1802	-39248	-4382	-1.2905	28584	-1495
	10	-0.8133	27802	-1542	-2.837	-34873	-3494	-0.7527	23708	-1558
	15	-0.7722	25336	-1586	-2.4934	-30487	-3435	-0.7676	20686	-1607
	20	-0.731	22864	-1625	-2.1495	-26093	-3406	-0.6743	18131	-1654
H/L=0.5	5	-0.871	31876	-1169	-3.0141	-40128	-3021	-1.300	31412	-1157
	10	-0.8212	28938	-1244	-2.6597	-35209	-2970	-0.6833	25204	-1212
	15	-0.7712	25984	-1058	-2.3052	-30269	-2904	-0.761	21429	-1255
	20	-0.7209	23018	-1276	-1.9505	-25317	-2868	-0.6577	18280	-1295

Table (3) : The obtained results for other cases...

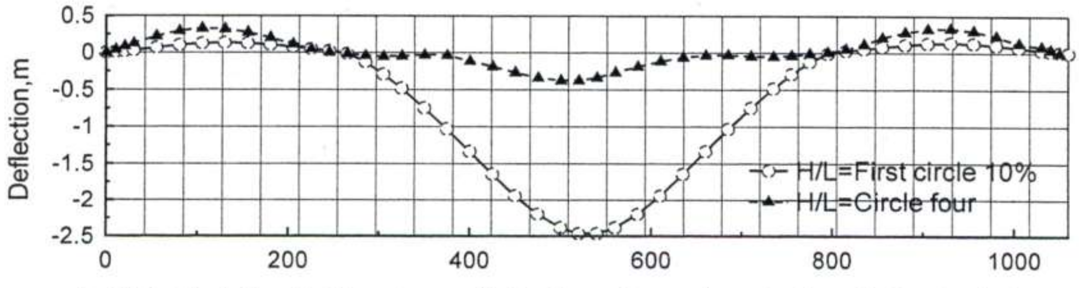


Fig.(17): Variation in floor beam deflection after using circle solution technique

Itreation	1	2	3	4
(Max.Deflection/530)*100	0.465 %	0.347 %	0.20628 %	0.0706 %
Max.Bending moment,mt	31860	22899	14820	7100
Max.Normal force. t	25017	2705.7	2932*	3249

Table4: the obtained results for circle of solution..

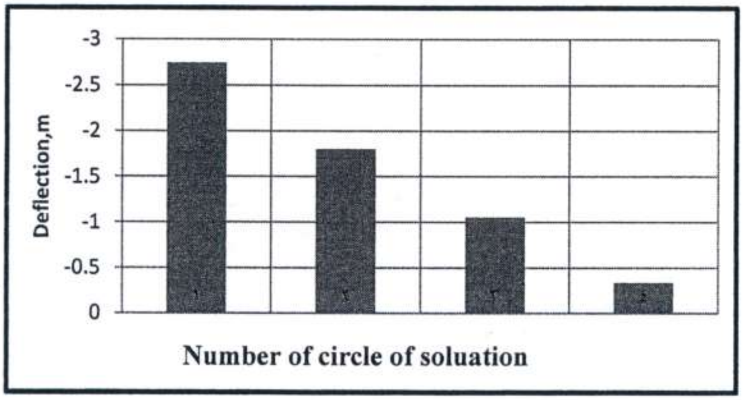


Fig.(18):Max.Deflection in floor beam for each circle of solution.

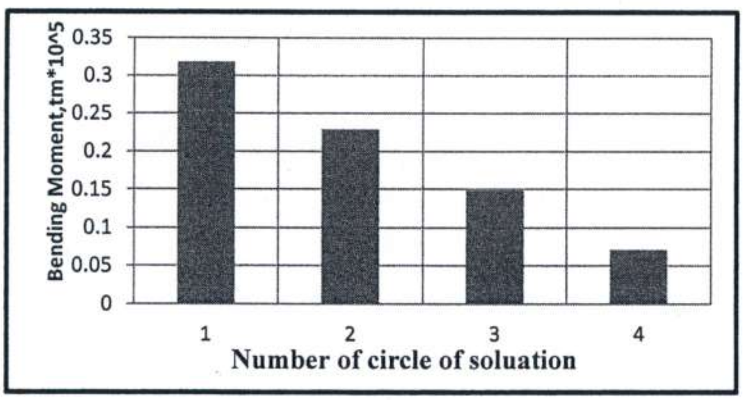


Fig.(19):Max .Moment in floor beam for each circle of solution.

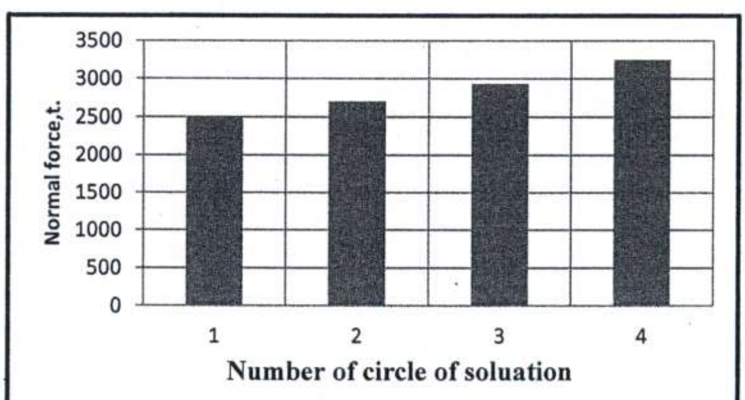


Fig.(20):Max .Normal Force in floor beam for each circle of solution.