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TRAJECTORY PLANNING FOR AUTONOMOUS UNMANNED AIR VEHICLES

تخطيط المسارات للطائرات الذاتية الحركة بدون طيار

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إن الطائرات بدون طيار UAVs ستصبح فى المستقبل أكثر خطورة وفعالية عنها الان ، حيث ستخدم كمنصات إطلاق توجه عن بعد. والاتجاه الحالى هو استخدامها فى مهام هجومية ، وهذا يشكل تحديا لأسباب عدة منها حاجاتها لتخطيط المسارات لها. وهذه المسارات لابد أن تكون على درجة من التخفى من الإجراءات المضادة وهى الرادارات المعادية والصواريخ المضادة للطائرات. كما انها يجب أن تقلل التكلفة إلى أقصى حدٍ ممكن.

وهذه التكلفة هى دالة فى كلا من طول المسار ومدى التهديد الذى يتعرض له ، وتصبح هذه المشكلة اكثر تعقيدا اذا كانت لسرب من عدة طائرات موجه لعدة أهداف. وهناك العديد من التقنيات المنشورة لتخطيط المسارات مثل : تحديد الخلايا ، وخارطة الطريق ، وشدة المجالات ، وتقنيات التعظيم . وكل منها تعاني من عيوب خطيرة مثل : الوقوع فى قمة صغرى محلية – طول زمن احتساب المسارات – طول وقت عملية البحث فيها عن المسار الأمثل تتناسب بطريقة تسلسلية وغير محددة مع مفردات المشكلة ، كما أنه ليس هناك ضمان لوجود حل أمثل أو قريب منه .

وهذا البحث يشرح طريقة جديدة لتوليد المسار لوحدة UAV مكلفة بهدف محدد. وهذا المسار يختار من بين المسارات الممكنة كلها بدءًا من موقع البدء للـ UAV وحتى الهدف. ويعتمد تخطيط هذا المسار على إعتبار دائرة أمنة حول كل موقع للصواريخ المضادة للطائرات SAMs وهى دائرة لايمكن الطيران فوقها، وقطر هذه الدائرة هو المدى الفعال لهذا الموقع إضافة إلى نسبة زيادة للأمان. وهذا المدى الفعال يختلف من موقع إلى أخر ، وتعتبر المماسات لهذه الدوائر بدءً من موقع UAV إلى الهدف أحد مفاصل مسار محتمل (إذا كان هذا المفصل مسموحا به بمعنى أنه لايمر فوق أى من مواقع عدم الطيران) . ومن كل نقاط التماس يتم إستنتاج مجموعة أخرى من المماسات للمواقع التى ما زالت تقع فى الطريق إلى الهدف. كما يتم دوران المسار حول هذه المواقع إذا لزم الأمر لتصبح الطائرة مواجهة ناحية الهدف ، ومن نهاية نقطة الدوران هذه يتم إعادة تخطيط مجموعة أخرى من المسارات للمواقع التى مازالت بين هذه النقطة والهدف ، وبذلك تتشكل مفاصل جديدة للمسارات متصلة بالمجموعة الأولى ، ويتكرر هذا التوليد للمفاصل حتى تصل جميع المسارات ذات المفاصل الممكنة إلى الهدف . وأثناء ذلك يحتسب طول وموقع كل مسار كما يحتسب التهديد الواقع نتيجة إمكانية إكتشافها بالرادارات المعادية باحتساب قيمة هذا التهديد عند ثلاث نقاط على كل مفصل من مفاصل المسار. وإجمالي الأطوال لمفاصل المسار يمثل طول المسار كما أن إجمالي التهديد على مفاصل مسار يمثل التهديد الواقع على المسار كله. ومن ثم يمكن إحتساب تكلفة كل مسار كدالة في طوله والتهديد الواقع عليه.ومن بين المسارات الممكنة جميعها يختار البرنامج أقل المسارات تكلفة اي أقلها طولا وتهديداً.

وقد تم تطبيق نفس البرنامج على مجموعة من UAVs مكلفة بمجموعة من الأهداف المحددة لها . حيث تزداد المشكلة تعقيداً،إلا أن البرنامج له العديد من المميزات والتي تجعله قادرا على أن يتم هذا التخطيط وإعادة التخطيط أثناء الطيران على العكس من كل الطرق السابقة .

ABSTRACT:

Unmanned Air Vehicles (UAVs) will be more lethal and strength air force than the remotely-piloted platforms in use today. The new tendency is to implement it in combat missions. Among the many open issues in their development is that of trajectory planning (TP). This is a challenging problem for several reasons. The algorithm must compute a stealthy path, and minimizes a cost function .The cost function is composed of : trajectory length and threat of adversarial sites which may be Radar Sites or Surface to Air Missiles (SAMs). This problem becomes more complicated if a UAV fleet is considered to attack multi targets simultaneously. A lot of techniques are reported for UAV TP such as : cell decomposition, road map, potential field and optimization techniques. Each one suffers from serious disadvantages such as : local minima, long computation time, the search for the optimal path can be NP – hard problem, and no guarantee to find an optimal (or near optimal) path.

This paper explains a new methodology to generate a trajectory for a single UAV assigned for a specific target. This trajectory is chosen among all feasible trajectories from the UAV starting position to

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the target position, each trajectory has to : avoid the SAMs effective range circles, minimize the radar signature and length. From the UAV starting position all feasible tangents (on safe circles wider than the SAMs range circles) are considered to be a leg of a possible path. From each tangent point other generations of possible path legs is produced as tangent points on the safe circles of the way till the target position. Each leg length and threat is computed and added, then the overall path length and threat are computed, The algorithm searches for the minimum cost trajectory.

This study is extended to multi UAVs attacking multi targets with rendezvous arrival time. The algorithm computation cost is sufficiently feasible to be executed in real time, besides the increase in arguments (computation cost due the increase in the number of targets, SAM's sites and UAVs) causes the computation cost to be increased in a deterministic manner, so this increase is bounded.

1. INTRODUCTION:

Unmanned vehicles–Air borne, under sea, and land based have become an integral part of the battlefield environment. They also have civilian applications such as disaster relief, environmental monitoring and planetary exploration. There has recently been considerable interest in making these unmanned vehicles autonomous, giving rise to the search area of UAVs. These are usually seen as rather simple vehicles, acting cooperatively in teams to accomplish difficult missions in dynamics purely known in hazardous environment.

The Capabilities of UAVs are evolving continuously, some versions were built to carry a payload 2000 Ib for altitude range 65000 ft with 42 hr endurance [1-3].UAVs have the potential to significantly improve operational effectiveness of the Air force. They have many useful military applications, including reconnaissance, search and destroy and search and rescue missions in hazardous environments such as battlefields or disaster areas. Recently there has been considerable interest in the possibility of using it in combat missions i.e. attacking a pre-specified targets[4].

1.1 The potential advantages of UAVs

There are a lot of advantages that give the implementation of UAVs a considerable interest such as maneuverability of a UAV could vastly exceed that of a manned aircraft, even with G suits to keep blood from pooling in the lower extremities, seated human beings lose consciousness if subjected to maneuvers harder than 4 to 10G. Humans are also quickly exhausted by continuous heavy maneuvering. By removing men from the vehicles, UAVs may be able to extend their performance envelope to approach performance limits achieved by modern missile systems (40 to 50 G) negative G is allowed too, [5].

UAVs are suitable for missions where the risk to pilots would be deemed unacceptably high. for example suppression of enemy air defense missions involve attacking well defended locations where risk to craft attacking in the earliest stages of the offensive are extremely high. In this situation, UAVs could be used in initial attacks to degrade or destroy enemy air defense systems, while manned aircraft could be used in subsequent bombing sorties.

There is significant weight savings because there is no pilot or cockpit in a UAV. Most of the weight savings is not from the pilot but from the support hardware that a pilot requires. Up to 40% of the weight and volume of a manned fighter is dedicated to the human interface and life support systems. This weight savings can be dedicated to increase the payload or to improve performance by maintaining a lighter weight platform. Superior coordination by taking advantage of modern sensing, computing and communication capabilities, UAVs, have the potential to offer superior coordination of activities among aircrafts. Currently coordination is accomplished by visual and voice communication among pilots. This limits in a fundamental way the level of coordination that can be attained among air crafts.

It is assumed that coordination strategies for UAVs will draw on the superior cognitive and decision making capabilities of humans, while also taking advantage of both the superior computing, sensing and communication capabilities that modern technology provides, as well as the maximum maneuverability available to UAVs. To take full advantages of UAVs, it is necessary to develop new operational paradigms that draw on the unique strengths and capabilities of UAVs[6],[7]. They have the potential to fulfill many of the same duties performed today by manned aircraft such as : intelligence, surveillance, reconnaissance, communications node, suppression of enemy air defenses, fixed target attack, air combat, and coordinated jamming capability. Using UAVs as unmanned compact air vehicles represents a significant cost savings. The effectiveness of a system is tied not only to its combat effectiveness i.e. ability to destroy targets, but also to such factors as theater integration effectiveness and costs of acquisition and operation [5]. UAVs will cost less than their manned aircraft counterparts. Most of the cost savings will come from the reduced need for multiple highly trained pilots per aircraft. Other savings, will result from the mass production of a common UAV platform capable of fulfilling multiple roles. For example, the role of a large payload bomber could be accomplished by many smaller payload UAVs operating in a cooperative fashion.

1.2 Trajectory planning approaches:

Generally, the UAV trajectory planning approaches can be classified as : cell decomposition, road map, and potential fields or virtual forces [8]. Most of the algorithms implementing these techniques suffer from critical disadvantages e.g. local minima, SAMs influence (threat) are not considered.

Besides, searching for the minimum cost path in these techniques is an NP-hard problems [9-12]. The computation cannot be completed in real time which is a necessity for the vehicle to be autonomous.

These approaches assume all threat sites equally lethal, and treat all the threats on the same level. Optimization techniques are used to solve the trajectory planning problem [12-15] but they also suffer from the computation cost which increases exponentially with the number of sites and targets. Some work is done using linear programming with a receding horizon and computes the optimal trajectory in that part of the path after reaching it. Another horizon is stated and a new optimized leg is computed and so on, till it reaches the target, but no guarantee for the overall optimization [16]. In this paper, a proposed optimal trajectory planning technique is explained. The technique has powerful advantages. It does not suffer from local minima, considers both radar threat and SAMs. No unnecessary flying over adversary area., takes the UAV dynamic constraints into consideration i.e. maximum turning angle of the UAV. More importantly the computation cost is considerably low that makes the trajectory to be planned in real time. This enables the UAV to be autonomous and to deal with pop up threats. If a pop up threat is detected, replanning for a new trajectory can be done. Another important advantage of the proposed technique is that the increase in computation cost with the number of targets, UAVs and NFZs is deterministic. It cannot turned out to be NP-hard. From the UAV starting position the algorithm answers the two questions. First, What are all feasible paths to the target?. Second, which path of these is optimal. The obstacle space is determined by the SAMS range circles, any point outside these obstacle space is in the free space. Searching points in the free space is determined by all possible tangent points from UAV position to each circle. The length and threat for each path leg is considered and added. The technique is introduced for multi UAVs multi targets with rendezvous time.

2 Trajectory Planning

Without loss of generality, the vehicle altitude is assumed to be constant, so the planning process is in a 2-D plane.

List of symbols :

N_r : number of radar sites.

N_s : number of SAMs sites, which are the no fly zones NFZs.

n : number of UAVs.

m : number of targets.

Φ_c : the center line angle.

Φ_r : angle of right tangent of NFZ circle.

Φ_l : angle of left tangent of NFZ circle.

Φ_d : angle of direct line (LOS) to the target.

L_d : length of (LOS) to the target.

L_t : length of the tangent.

L_c : length of the line to the circle center.

α : the angle between center line and the tangent.

ω_u: the center angle UAV side.

ω_t : the center angle target side.

f_{zx} , f_{zy} : coordinates of the NFZ center.

f_{zr} : radius of the (NFZ) circle.

t_x , t_y : target position coordinates.

x_r , y_r : tangent point coordinates R.H.S. } From UAV position or tangent points

x_L, y_L : tangent point coordinates L.H.S. } From UAV position or tangent points

$\bar{x}_r$, $\bar{y}_r$: tangent point coordinates R.H.S. } From target position

$\bar{x}_L$, $\bar{y}_L$: tangent point coordinates L.H.S. } From target position

h_t : tangent length from target to NFZ.

3 The Algorithm

3.1 Arranging the numbers of NO-Fly Zones (NFZs)

The tangent points from UAV starting position to all NFZs are called level 0 points Fig.1 and Fig.2

1. The line of site (LOS) From UAV to the target and the first tangent is

$$L_d = \sqrt{(t_x - u_x)^2 + (t_y - u_y)^2} \quad (1)$$

(If the altitude is going to be considered for variable flying altitude then Equ.1 will be

$$L_d = \sqrt{(t_x - u_x)^2 + (t_y - u_y) + h^2_{alt}}$$; (Where h_{alt} is the altitude)

$$\Phi_d = \arctan\left[(t_y - u_y)/(t_x - u_x)\right] \quad (2)$$

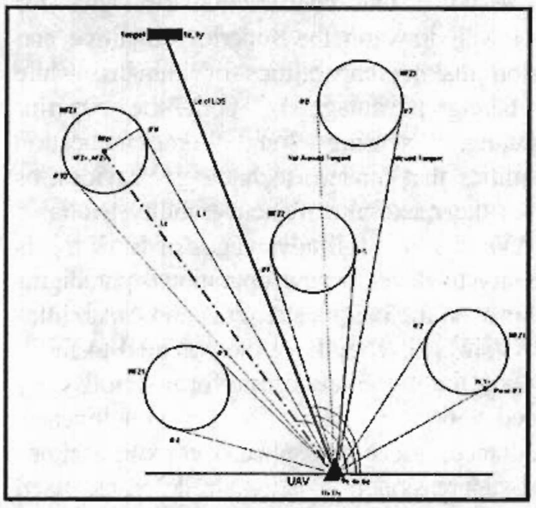

Fig 1 Tangents, Angles and LOS From UAV Position

2. From the vehicle starting position to the first NFZ (in the given NFZs order) The length of the center line

$$L_c = \sqrt{(fz_x - u_x)^2 + (fz_y - u_y)^2} \quad (3)$$

It's angle is

$$\Phi_c = \arctan\left[(fz_y - u_y \;/\; fz_x - u_x)\right] \quad (4)$$

The other center lines (L_c) and their angles (φ_c) to all NFZ are computed in a similar way.

3. Getting L_c for all sites, the sites numbers are arranged W.R.T. the UAV position, the NFZ number 1 will be the closest to the UAV i.e. which has the shortest L_c length.. This arrangement is considerable since every tangent point will be considered in order as a node for other points. Generation of tangents will be from these nodes to the NFZs which lay between it and the target.

4. The tangent length on a NFZ is

$$L_t = \sqrt{(L_c - fz_r)^2} \quad (5)$$

And the angle between the center line and the tangent is $\alpha = \arctan(fz_r / L_c)$ (6)

Getting L_t and α the angle of the R.H.S. tangent can be determined as:

$$\Phi_r = \Phi_c - \alpha \quad (7)$$

And the angle of the L.H.S. tangent

$$\Phi_l = \Phi_c + \alpha \quad (8)$$

5. The coordinates of the R.H.S. tangent Point is computed as shown in Fig. 2 :

$$\left.\begin{aligned} x_r &= u_x + L_r \cos \Phi_r \\ y_r &= u_y + L_r \sin \Phi_r \end{aligned}\right\} \quad (9)$$

Similarly, the coordinates of the L.H.S. tangent Point is computed as

$$\left.\begin{aligned} x_l &= u_x + L_l \cos \Phi_l \\ y_l &= u_y + L_l \sin \Phi_l \end{aligned}\right\} \quad (10)$$

6. The LOS shown in Fig. 1 (L_d) has an angle Φ_d computed by Equ. 2 is feasible and is considered a candidate path to the target If this condition is not true

$$\Phi_{ri} < \Phi_d < \Phi_{li} \quad (i = 1,\ldots\ldots,N_s) \qquad (11)$$

If this condition in Equ.(11) is fulfilled, this means that the LOS will pass over one or more of the NFZs. If this condition is not true then the LOS is considered one of the candidate path to the target.

7. For a tangent number j , its length is L_{lj} and its angle Φ_j . it is not feasible if :

$$\Phi_{ri} < \Phi_j < \Phi_{li} \text{ and} \qquad (12)$$

$$L_j > Li \quad (i= 1,\ldots\ldots,N_s) \qquad (13)$$

Equation 12 and 13 have 3 conditions :

a) Equ. 12 is true and Equ. 13 is not true then the tangent j is allowed (feasible) Fig.3A.
b) Equ. 12 is not true and Equ. 13 is true then the tangent j is allowed (feasible) Fig.3B.
c) Equ. 12 is true and Equ. 13 is true then the tangent j is not allowed (unfeasible). Fig3C

Clearly if both Eqns. are (12, 13) not true then the path is feasible.

In case (c) the tangent point is not considered as a node to generate new points by the algorithm from this U to T. Else if

the tangent is allowed , the tangent is considered as an edge of a possible path. A path will be feasible if all of its edges (legs) are allowed. Its tangent point is a node point and is used to generate more new legs.

8. Repeating steps 1 to 7 for NFZ's i, i = , 2,3,... N_s, these arguments are computed:

* x_{rj} , y_{rj}. The coordinates of the right tangents to NFZ i, its length L_{rj} and its angle Φ_{rj}.
* x_{lj} , y_{lj} The coordinates of the left tangent to NFZ i, its length L_{lj} and its angle Φ_{Lj}.
* The length of the LOS L_d , its angle Φ_d , and its allowance.
* The state of each tangent allowed or not allowed.

• *Computing the tangents on NFZ from the target position Fig. 4. :*

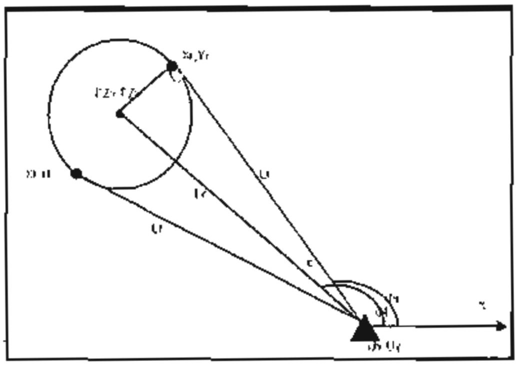

Fig. 2 Tangent Coordinates

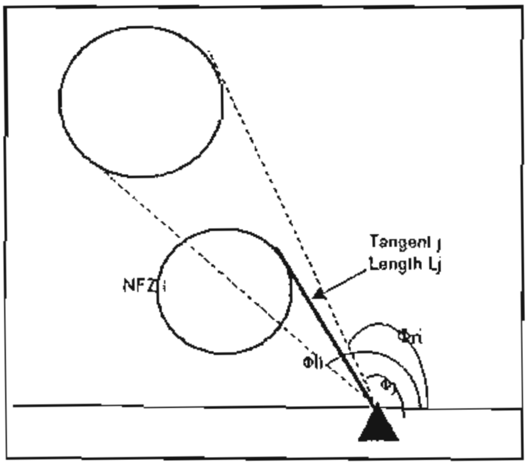

Fig.3 A) Case a Allowed Tangent

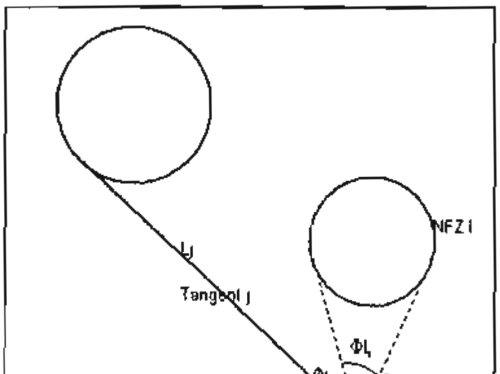

Fig.3 B) Case b Allowed Tangent

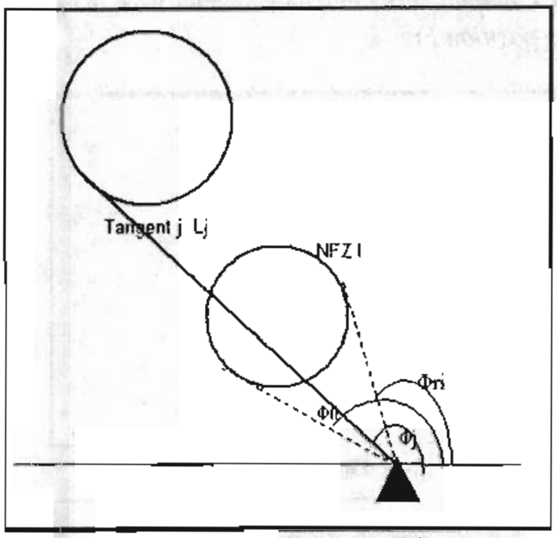

Fig. 3 C) Case c NOT Allowed Tangent

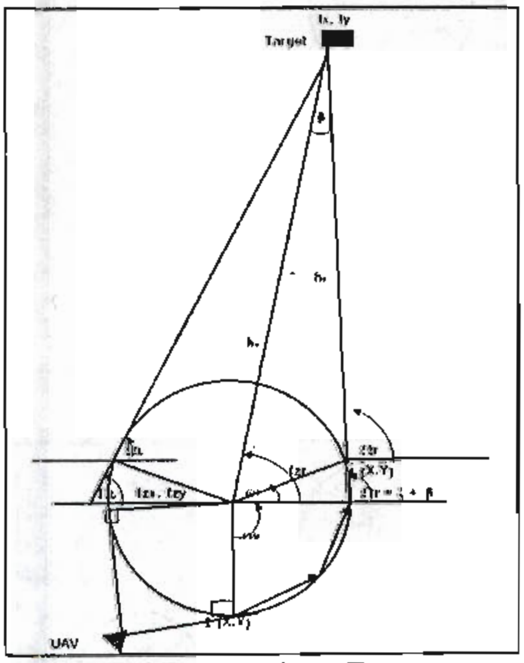

Fig 4 Tangent From Target

The center line length from T to NFZ center is

$$h_c = \sqrt{(fz_y - t_y)^2 + (fz_x - t_x)^2} \quad (14)$$

its angle is :

$$\xi = \arctan\ [\ (fz_y - t_y) / (fz_x - t_x)\ ] \quad (15)$$

$$h_l = \sqrt{h_c^2 - fz_r^2} \quad (16)$$

This tangent has an angle

$$\beta = \arctan\ (fz_r / hc) \quad (17)$$

For R.H.S. the angle Ω_r is computed as

$$\Omega_r = \xi + \beta \quad (18)$$

The tangent point coordinates are :

$$\left.\begin{aligned} \overline{x_r} &= t_x - h_l \cos \Omega r \\ \overline{y_r} &= t_y + h_l \sin \Omega r \end{aligned}\right\} \quad (19)$$

Similarly for L.H.S.

The left tangent (from T) has the angle Ω_L

$$\Omega_L = \xi - \beta \quad (20)$$

And the tangent coordinates are

$$\left.\begin{aligned} \overline{x_L} &= t_x + h_l \cos \Omega_L \\ \overline{y_L} &= t_y - h_l \sin \Omega_L \end{aligned}\right\} \quad (21)$$

3.2 Turning Arround The NFZ

It is clear from Fig. 5 that if point 1 is turned to point $\overline{1}$ by an angle $\omega_u + \omega_t$ the vehicle will be closer to the target and a new group of edges are generated.

* Checking whether the turning is possible (to get the UAV has a face to the target) and compute the turning angle:

From triangle (0-1-B) Fig. 6. For the R.H.S. tangent point #1

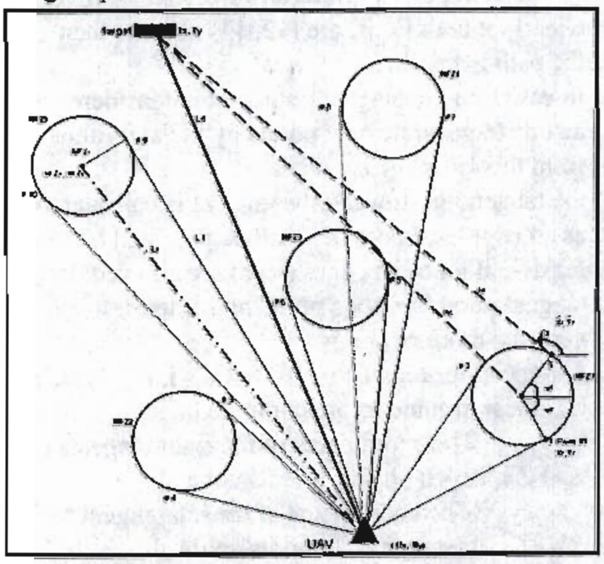

Fig 5 Tangents From the Target And The Turning Angle

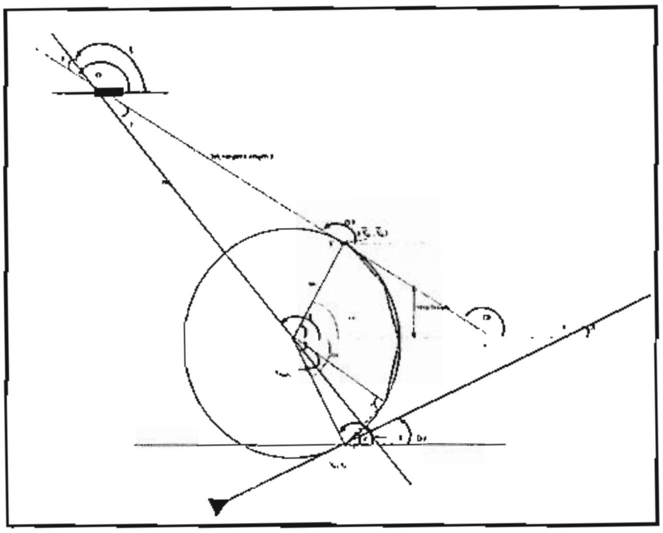

Fig 6 Turning Steps Around a NFZ

$\pi - \Omega = \pi/2 + \omega_u$

then the turning angle from the tangent point to the x – axis

$\omega_u = \pi/2 - \Omega$ (22)

From triangle (0-A-$\bar{I}$)

$\bar{\Omega} = \pi/2 + \omega_l$

the turning angle from the x – axis to the tangent point

$\omega_l = \bar{\Omega} - \pi/2$ (23)

The turning angle from tangent point to the target tangent point is computed as

$\Delta\omega = \omega_u + \omega_l = \bar{\Omega} - \Omega$

These two angles (ω_u, ω_l) determines if it is possible to turn or turning is not necessary as:

A) If $\omega_l < \omega_u$ then point I is in face and closer to the target than point $\bar{I}$ (turning is not necessary)
B) If $\omega_l > \omega_u$ it can turn on the circumference by an angle :

$\Delta\omega = \omega_l + \omega_w$ for R.H.S. (24)

with a similar procedure followed with left hand tangent

$\Delta\omega = \omega l - \acute{\omega} u$ for L.H.S (25)

- *Computing The Turning Steps:*

One of the dynamic constraints of the vehicle is its maximum turning rate θ_{max}, the turning on the NFZ circumference is done through turning steps, the number of steps is computed as

$N_{steps} = \Delta\omega / \theta_{max}$ (26)

Where θ_{max} is the maximum turning angle of the vehicle .

Each step length (h-step) Fig.6 is computed as :

½ h-step = $f_{zr} \sin(\theta_{max}/2)$

h-step =2 $f_{zr} \sin \theta_{max}/2$ (27)

The angle $\gamma = \pi/2 - \theta_{max}/2$ (28)

The angle of the Radius at tangent point

$\delta = \arctan [(fz_y - y_r) / (fz_x - x_r)]$ (29)

the step angle $\varepsilon = \delta - \gamma$ (30)

Getting the step length h_step , step angle ε the step coordinates can be computed as:

$$\left.\begin{aligned} X_{step} &= x_r + \text{h-step} \cos(\varepsilon), \\ Y_{step} &= y_r + \text{h-step} \sin(\varepsilon) \end{aligned}\right\} \quad (31)$$

The coordinates of the second step are

$X_{step\,2} = X_{step\,1} + \text{h-step} \cos(\varepsilon)$,

$Y_{step\,2} = Y_{step\,1} + \text{h-step} \sin(\varepsilon)$,

Generally the i+h step is computed as:

$$\left.\begin{aligned} X_{step\,i} &= X_{step\,i-1} + \text{h-step} \cos(\varepsilon), \\ Y_{step\,i} &= Y_{step\,i-1} + \text{h-step} \sin(\varepsilon), \end{aligned}\right\} \quad (32)$$

Where i = 2 to N_{steps}

The same procedure is repeated for the left hand side tangent with its own number of steps. A remarkable note here is that the turning is in the same flying direction, the vehicle does not have to do sharp turns or unnecessary movements over adversarial territories.

3.3 Generation Of A New Group Of Edges

The allowed tangent points from UAV starting position before and after turning are considered parent nodes .They are used to generate new children point of LOSs and edges.

- *Generating New Tangents From level 0 Nodes (before turning) :* These are the tangent points generated from all level 0 nodes. After the coordinates of all level 0 points are computed in the previous steps (in sections 3.1 and 3.2) and with the same procedure level 1 points are computed they are:

- the direct line from the nodes to the target position.
- tangent points on the NFZs between it and the target.

Fig.8 shows these for a sample point i.e. point number 1. The number of NFZs to be tangented is reduced by one (this one is the NFZ containing the parent node point 1). Considering Ns = 5 these tangents will be :

From points (1,2) on NFZ_2 to NFZ_3, NFZ_4, NFZ_5.
From points (3,4) on NFZ_3 to NFZ_4, NFZ_5
From points (5,6) on NFZ_3 to NFZ_4, NFZ_5
From points (7,8) on NFZ_4 to NFZ_5

The coordinates of the tangents are computed as (Fig. 7) :

$$L_c = \sqrt{(y_1 - fzy_2)^2 + (x_1 - fzx_2)^2} \quad (33)$$

$$\Phi_c = \arctan\left[(y_1 - fzy_2) / (x_1 - fzx_2)\right] \quad (34)$$

$$L_t = \sqrt{L_c^2 - fzr_2^2} \quad (35)$$

The angle $\theta = \arctan(fzr_2 / L_c)$ (36)

The tangent angle of the tangent **at point g** is

$$\Phi_g = \Phi_c - \theta \quad (37)$$

Then the tangent coordinates of point g are:

$$x_g = x_c + L_t \cos\Phi_g \quad (38)$$

$$y_g = y_c + L_t \sin\Phi_g$$

The LOS at point 1 is

$$L_d = \sqrt{(y_1 - t_y)^2 + (x_1 - t_x)^2} \quad (39)$$

With an angle $\Phi_d = \arctan\left(\frac{y_1 - t_y}{x_1 - t_x}\right)$

These L_t tangents are checked for allowance by Equ. 12, 13 and the LOS is checked by using Equ.11.

- *Generating New Tangents From level 0 Nodes (After Turning):* After computing the coordinates of the last turning step i.e. the coordinates of point 1 a LOS from 1 is computed and checked for allowance. Also, a group of tangents are generated to the remaining NFZs and checked for allowance. Fig.8 shows them for a sample point 1. The Computation of coordinate and allowance is analogous to Equ. 33 to 39.

The feasibility check are applied at each point. If the direct line (or the tangent) is allowed, it will be a possible edges and it will form with its previous edges a part of the pathway. Its length is added to the pathway length and its threat value is added to the pathway threat.

Repeating for all points, excluding the not allowed legs and reducing NFZ number by one as we pass one NFZ we complete all possible tangents and direct points from UAV starting position to the target position.

When the node is the vehicle starting point the allowance checks by the Equ. 11 for the direct line and in Equs.12, 13 for the tangent allowance are sufficient . But when the node is a point on the circumference of a NFZ an L_t and/or L_d may be passing over this NFZ for example L_t (in Fig. 8) passes over its own NFZ1. So another check on this NFZ to specify if L_t and/or L_d are allowed or not. This is done as:

The distance between the two NFZ circles is (Fig. 7) :

$$L_0 = \sqrt{(fzy_2 - fzy_1)^2 - (fzx_2 - fzx_1)^2} \quad (40)$$

$$vh = \sqrt{(L_0^2 - f_{zr1}^2)} \quad (41)$$

vh represents the length of a tangent originated from NFZ_1 to NFZ_2 so

IF $L_t <= v_h$ then L_t is allowed . else, if $L_t > v_h$ the Lt is not allowed

3.4 Points Of Other Levels 2 to N_{s-1} :

The tangent points are generated for the other levels as explained in level 1 points. They are generated, turned and checked as explained in sections 3.1 to 3.3.

- *Forming The Paths :*

From UAV starting position to all level points all tangents generated are checked. If it is not passing over any NFZ then it is in the free space and is considered an edge of the path. Its child points (of level 1,2.., N_{s-1}) are checked also. Each children point is connected to its father node to form a path provided all path edges are feasible. This includes the paths generate from the turned points. At any point in the path multi child points are considered including LOS from that point. The maximum number of edges (MNE) is computed as:

$$MNE = 8\left[N_s + 4 \sum_{i=1}^{Ns-1} (N_s - i)^2\right] \quad (42)$$

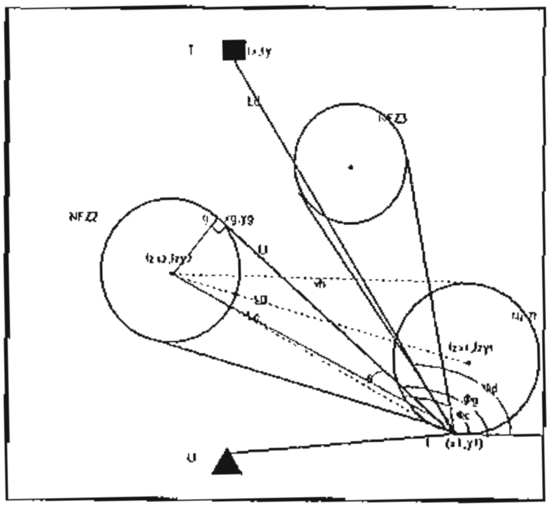

Fig 7 New Tangents Generated From Point 1

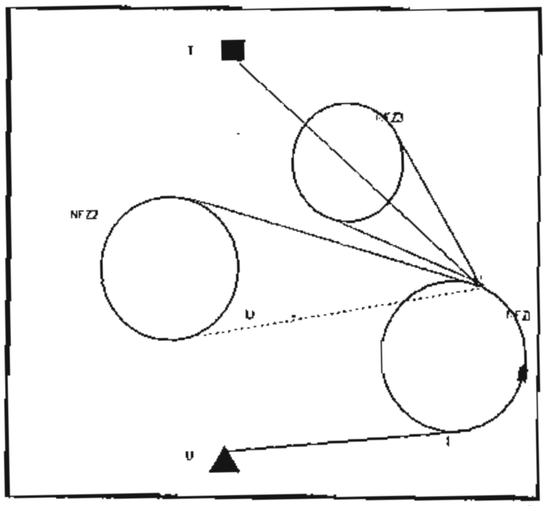

Fig 8 New Tangents Generated From Turned Point 1

4 Multi Vehicles To Multi Targets :

If multi UAVs are assigned to multi targets i.e. ($n > 1$ and $m > 1$), each vehicle will plan all feasible paths from its position to (each) target taking into consideration that the turning angles to face targets are different from target to anthor for the same UAV on the same NFZ. This means that the paths generated from a UAV position to one target are different from paths generated to other targets. Consequently, the number of edges is increased in proportion to the number of both vehicles and targets (m,n). It can be computed as:

$$VNE = 8\, m\, n\, [\, N_s + 4 \sum_{i=1}^{Ns-1} (N_s - i)^2 \,] \qquad (43)$$

The algorithm is shown in Fig. 12

5. Forming The Threat Function :

The vehicle is subjected to two types of threats i.e. detection by radar sites and SAMs sites. It has other counter measures against other threats like jamming and decoying.

5.1 Radar Sites:

Threat cost computation is based on the UAVs radar signature. Since the strength of a UAV's radar signature is proportional to $1/d^4$, where d is the distance between UAV and radar site [17], [18], the threat cost for traveling along an edge is proportional to the inverse of d to the forth power. An exact computation of the threat cost for traveling along an edge would involve the integration of the cost along the edge. A simpler approach involves computing the threat cost at several locations along an edge and taking into account the length of the edge is used [4],[7],[19],[20-23]. It can be employed for the path way legs. The threat cost J_r is computed at three points along each edge: $L_i/6$, $L_i/2$, $5L_i/6$, where L_i is the length of the edge i.

$$J_r = L_i \sum_{j=1}^{N_r} \left(\frac{1}{d^4_{\frac{1}{6}i,j}} + \frac{1}{d^4_{\frac{1}{2}i,j}} + \frac{1}{d^4_{\frac{5}{6}i,j}} \right) \qquad (44)$$

This is depicted graphically in Fig. 10.

Using Eqn. 44 to compute the threat cost for all legs of the allowed trajectories including the turning steps and $d_{1/6\,i,j}$ is the distance from the $1/6^{th}$ point on ith edge to the J_{th} radar site.

5.2 SAM Sites :

Although the algorithm avoids these sites. There is still a considerable threat due to sensor errors time delays, ... etc. The threat on the vehicle is due to the site which it is turning around only. The threat value δsi for turning step i is computed as shown in Fig. 11 as [24]:

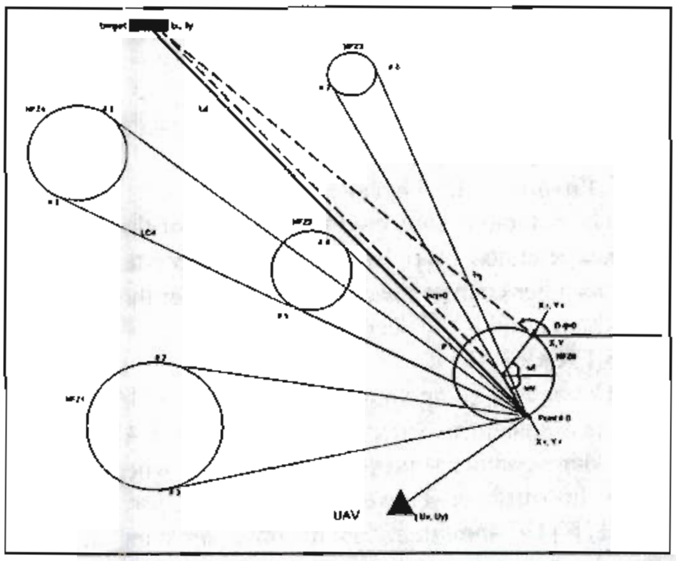

Fig 9. All leg points Generated from point # 0

$$\delta si = (1+ds)^{-2} \quad \text{if } ds <= D \tag{45}$$
$$\delta si = 0 \quad \text{if } ds >= D$$

Where d_s is the distance spacing SAMs site and the specified point on the path.

D is the safe circle diameter (greater than SAMs range) range for this site that the UAVs turning around. By analogy to Eqn. 44 the SAMs threat Js can be computed as :

$$J_s = (h_{-step} * k_s) \sum_{i=1}^{k_s} \left(\frac{1}{(1+d_{s1/6})^2} + \frac{1}{(1+d_{s1/2})^2} + \frac{1}{(1+d_{s5/6})^2} \right) \tag{46}$$

The total SAMs threat is :

$$Js = \sum_{i=1}^{k_s} \delta si \tag{47}$$

Where k_s is the number of steps to be turned around the specified NFZ. And the threat cost from radars & SAMs is :

$$J_{th} = J_r + J_s \tag{48}$$

The trajectory has to minimize another cost, i.e. the length of the trajectory. Minimizing path length leads to minimizing the fuel consumption and the flying time. The length cost is the summation of the lengths of all the edges forming the path. This includes the tangents and the length of the turning steps.

The length cost is :

$$J_L = \sum_{i=1}^{k_L} L_i \tag{49}$$

Where K_L is the number of consecutive tangents forming one path, this includes the turning steps number (if any). If J_L exceeds the constraints for the vehicle, the specified path is excluded.

The trajectory total cost J_{tot} is computed from Eqn. 48 and 49 is :

$$J_{tot} = k\,(J_{th}) + (k-1)\,J_L \tag{50}$$

Where ($0 \leq k \leq 1$) is a weighting factor to be specified by the mission planner.

The same technique can be used to plan trajectories for multi UAVs in different position attacking the same target or multi targets. A lot of additional problems arise in this case, among these problems is the coordinate of the rendezvous time, every UAV has to arrive to its target simultaneously. After planning the optimal trajectory for each, the algorithms specifies the cruising speed for each UAV, assuming the UAV that has the longest trajectory is traveling by its maximum speed.

6 Results :

The planning algorithm is used for different situations to plan trajectory on (3) different machines. Although computation time requires a few seconds it is system dependent. The following results are performed on a (3.2) GHZ processor, (400) MHZ bus speed and (256) MB RAM.

In Fig. 13 a single UAV planned its trajectory on adversary field of 6 NFZs and 5 radar sites. The path avoids passing over any NFZ and keeps exposure to radars as small as possible. The computation time is 11 seconds.

Another result shown in Fig. 14, (2) vehicles are assigned to, (2) targets through (7) NFZ and (4) radar sites the computation time is (12) seconds. A significant advantage of the algorithm is cleared here that is : it considers for the overlapping coverage ranges of two NFZs and planning paths that avoid it. This is not possible in most of the surveyed literatures.

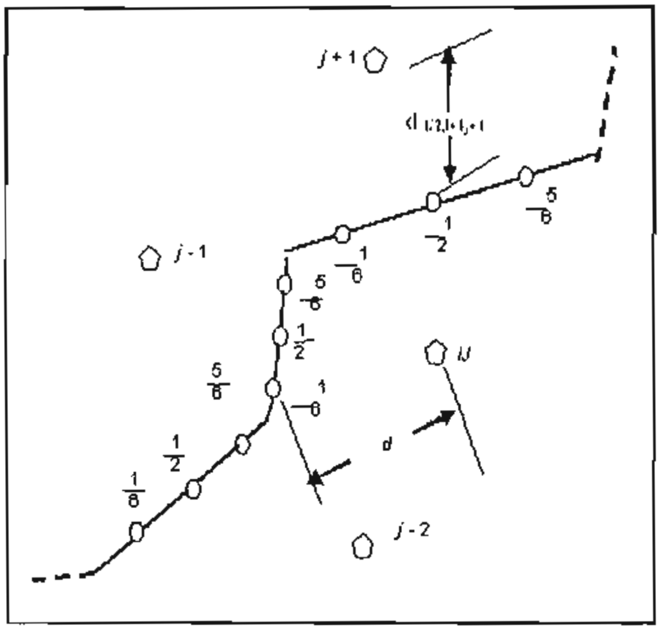

Fig. 10 Radar Threat Cost

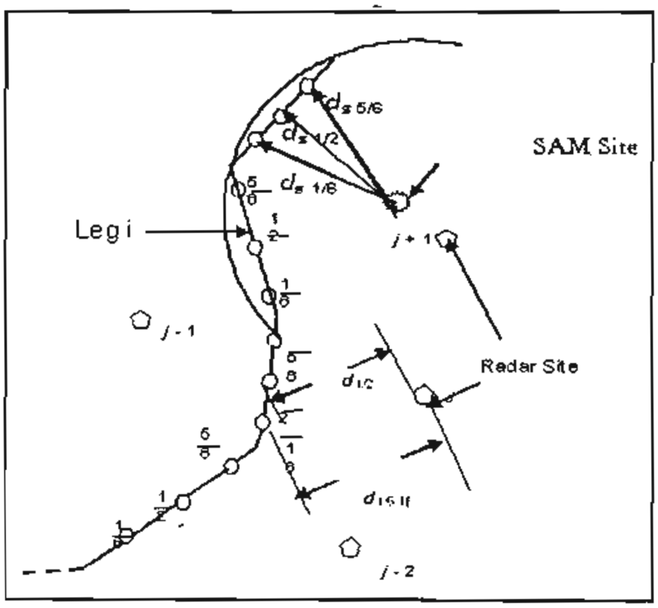

Fig 11 Radar and SAMs Threats

For multi UAVs assigned to multi targets as shown in Fig. 15 (6) vehicles and (6) targets, the algorithms plans the minimum cost path for each UAV to its the specified target. All trajectories avoid all NFZs and be as for as possible from the radar sites. The computation time for this algorithm is 14 seconds which is perfect, and proves that the increase in computation time is no longer NP-hard.

6. Conclusions

The introduced algorithm is a powerful tool to plan an optimal or near optimal trajectory for a single Unmanned Air Vehicle guided to specified target, or multi UAVs specified to multi targets. The primary strengths of the algorithm are :

1) It avoids passing over the range circles of the SAMs sites.
2) It minimizes both the trajectory length and the radar signature. Consequently, minimizes fuel consumption, flying time and the probability of detection by the adversarial radars.
3) It avoids the overlapped areas covered by more than one SAM site.
4) It allows turning around the SAMs sites in its flying direction, no sharp turns are included.
5) There are no unnecessary movements as other algorithms may have.
6) Preserves the dynamic constraints of the vehicle i.e. maximum turning angle and maximum speed.
7) The computation cost is significantly small. Consequently, planning and re-planning can be executed in real time which enables the vehicle to be autonomous
8) The increase in computation cost (due to the increase in number of UAVs, targets, and SAMs sites) is deterministic. So, searching for the optimal trajectory among all possible trajectories cannot be an NP-hard problem, like other techniques.

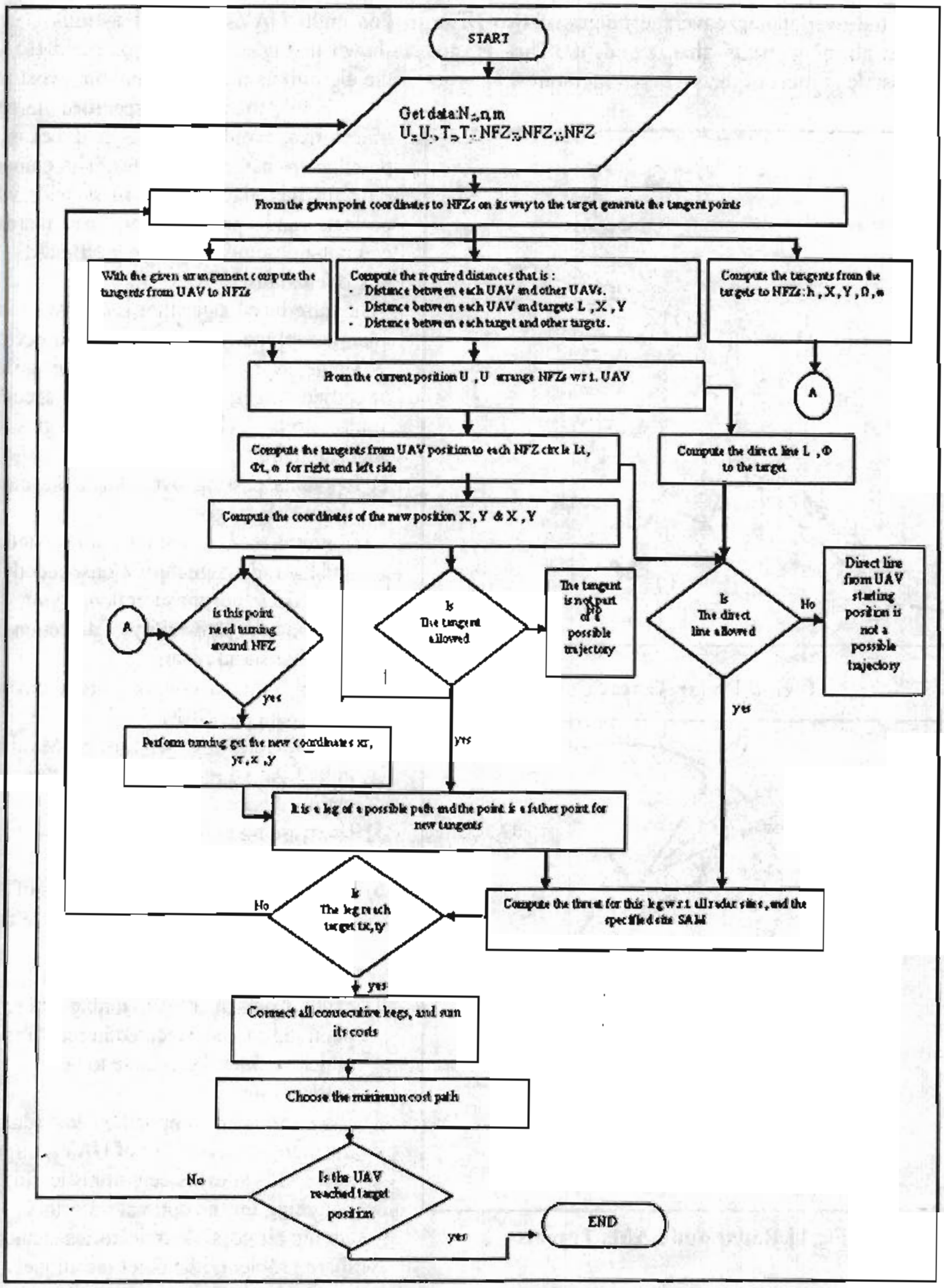

Fig 12 Optimal Trajectories Planned for two Targets with Different Positions

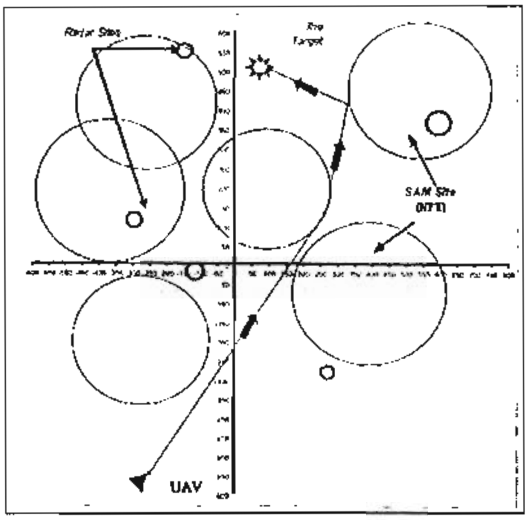

Fig. 13 The Optimal Trajectory For Single UAV To A Single Target

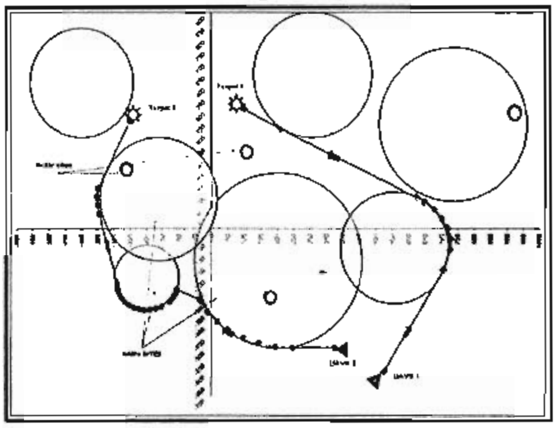

Fig 14 Optimal Trajectories Planned for Two Targets with Different Positions

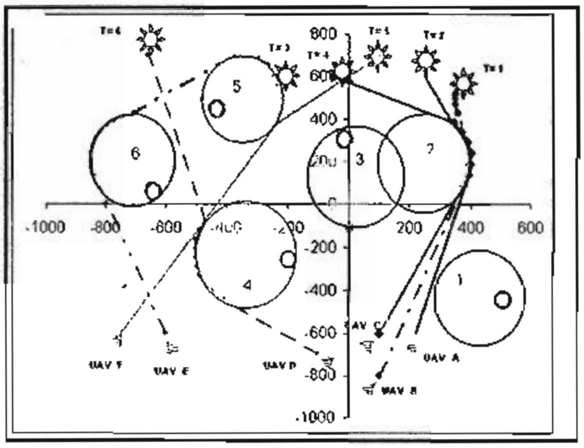

Fig. 15 Multi Vehicles To Multi Targets

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