EFFECT OF UNCERTAINTIES IN FORECASTED LOADS AND FORCED OUTAGE ON POWER SYSTEM RELIABILITY

BY :

M.A. TANTAWY.*

ABSTRACT

The paper presents a comprehensive analysis for the effect of uncertainties inherent in forecasted loads and forced outage rates of generating units on the power system reliability. New equations are derived for the loss of load expectation and variance interms of these uncertainties. The proposed equations are distinguished by simplicity and low computer time required for computations. An application is made on a hypothetical power system for the general case of having both FOR and FPL are uncertain and for the special cases when one of them is considered deterministic. It is demonstrated that consideration of uncertainty in load forecast increases both the mean and variance of the loss of load probability.

1. INTRODUCTION

The power system operational planning has been strongly affected by the uncertainties in the daily forecas as the forced outage rates (FOR) of ger uncertainty in FPL results in uncertaint indices of a power system, it is require reserve margin that should be allocated security constraints.

Recent publications studied the pro

* Asst. Prof., Electrical Power and Mac

REFRENCES

1. Rabiner L.R. and Gold B., "Theory and Applications of digital signal processing", Prentice. Hall. N J., 1975, Chapter III.

2. Temes G.C., and Mitra S.K.ed., "Modern Filter Theory and design", John Wiley N.Y., 1973, Chapter 6.

3. Helms H.D., "Pigital Filters With Equiripple or Minimax Responses," IEEE Trans-on Audio and Electroacoustics, AU-19, No. 1, pp. 87-94, 1971.

4. PArks T.W. and MoClellan J.H., "Chebyshev Approximation for Nonrecursive digital filters with linear phase", IEEE Trans. Circuit Theory, CT-19, pp. 189-194, Mar., 1972.

5. J.R. Rice and K.H. Usow, "Lawson's algorithm and extensions", Math. Comp., Vol. 22, pp. 118-127, Jan. 1968.

6. Bandler J.W., _et al_., "New Results in the least p^{th} Approach to Minimax Design", IEEE Trans. on Microwave Theory and Technique, MTT-24, pp. 116-119, FEB. 1976.

order to reduce the load forecast uncertainty[1-3]. Patton[5,6] analyzed the effect of uncertainty in FOR on the system reliability indices (while the FPL is assumed deterministic). The effect of uncertainty in FPL is studied in [4,7]. An approximate method was described by Billinton[4] for calculating the expected loss of load probability when uncertainties in FPL, but the FOR is fixed. This method[4] does not provide informations on the variance of LOLP. Wang[7] has introduced a method for calculating the mean and variance of the LOLP when uncertainties exist in the FPL only. Also, an approximate method for analyzing the general case when both FPL & FOR are considered random variables is given[7].

2. LOLP as a measure of reliability level

The loss of load probability (LOLP) is generally defined as the long run average number of days in a period of time that load exceeds the available installed capacity and thereby capacity deficiency occurs. During a state of capacity deficiency the load demand is not met by an adequate generation due to either unexpected loss of generation as a result of technical problems or unexpected increase in the daily peak load caused by the uncertainty in load forecast.

The LOLP, for a single area system and for N-day period is:

$$LOLP = \sum_{i=1}^{N} P_c(C_i - L_i) \qquad \ldots\ldots \quad (1)$$

Where:

C_i = total generating capacity scheduled for service on day i.

L_i = forecasted peak load on day i.

$P_c(C_i-L_i)$ = probability that (C_i-L_i) is less or equal to zero.

i = 1, 2, 3, ..., N.

For the ideal case when both the FPL & FOR are deterministics, exact estimation for the mean of LOLP is given by equation(1) with zero variance. $P_c(C_i-L_i)$ is computed by forming the cumulative capacity outage probability table using the recursive expression[4,6]:

$$P_c^j(x) = P_c^{j-1}(x)\cdot(1-R_j) + P_c^{j-1}(x-c_j)\cdot R_j \qquad \ldots. (2)$$

Where:

$P_c^{j-1}(x)$, $P_c^j(x)$ = cumulative probability of x before and after the addition of j^{th} unit to the cumulative capacity outage probability table.

R_j , c_j = forced outage state probability and capacity of j^{th} unit respectively.

If there are uncertainties in FOR and FPL, the values of C_i and L_i are subjected to some uncertainties and must, therefore, be represented by a set of random variables. Then the LOLP is a random variable defined on both C_i and L_i with non zero variance.

3. Mathematical formulation for the mean & variance of LOLP

It is interesting for power system operational planners to have exact formulations for the loss of load expectation (LOLE) and loss of load variance (LOLV) for the general case when both FOR & FPL are subjected to uncertainties. The FOR and FPL are assumed mutually independent of each other[7]. This assumption is reasonable because forced outages of generating units are usually not considered when making load forecast.

The peak loads are reasonably assumed normally distributed around its mean. It may be scattered on a base of small fraction of standard deviation (k=0, 1, 2, ..., ∞). The probability of having the peak load in certain level k can be supplied from the normal probability table. The loss of load expectation is:

$$LOLE = \sum_{i=1}^{N} \int_0^{\infty}\int_0^{\infty} P_c(C_i - L_i) f(R,k) dR\ dk \qquad ...(3)$$

Where:

$f(R,k)$ = joint density function of FOR and FPL.

$= f(R)f(k)$ by the assumption of mutual independence.

Therefore,

$$LOLE = \int_0^{\infty} f(R)\ dR \cdot \sum_{i=1}^{N} \int_0^{\infty} f(k)\ P_c(C_i - L_i)\ dk \quad ..(4)$$

$P_c(C_i - L_i)$ is a monotonously increasing "staircase" function of k with discontinuities at regular intervals equal to S (the step size of the cumulative capacity outage probability table). Because of this property, equation(4) can be rewritten as:

$$\text{LOLE} = \sum_{i=1}^{N} \sum_{k=0}^{\infty} P_{L_i}(k)\, P_c(C_i - L_i) \qquad \ldots. (5)$$

Where $P_{L_i}(k)$ is the probability of having the peak load in certain level k.

The LOLV, as derived in the appendix, is given by:

$$\text{LOLV} = \sum_i \sum_k P_{L_i}(k) \left[P_c^2(C_i - L_i) + \text{VAR}\left[P_c(C_i - L_i) \right] \right]$$

$$+ \sum_{i \neq j} \sum_k \sum_l P_{L_i}(k)\, P_{L_j}(l)\, \text{COV}\left[P_c(C_i - L_i), P_c(C_j - L_j) \right]$$

$$- \sum_i \text{LOLE}_i^{\,2} \qquad \ldots\ldots (6)$$

Where:

$\text{VAR}\ P_c(C_i - L_i)$ = Variance of load loss probability on day i,

$\text{COV}\ P_c(C_i - L_i), P_c(C_j - L_j)$ = Covariance of probabilities $P_c(C_i - L_i)$ on day i and $P_c(C_j - L_j)$ on day j,

LOLE_i = Loss of load expectation on day i.

Special cases

The above expressions of LOLE and LOLV can be simplified for the special cases of having the FOR fixed while FPL is uncertain and the case of fixed FPL while the FOR is uncertain as follows:

i- Uncertainty in FOR only

The FPL may be considered deterministic when having high accuracy in the forecasted loads. Equation(5) is transformed to:

$$LOLE = \sum_{i=1}^{N} P_c(C_i - L_i) \quad \ldots\ldots \quad (7)$$

Also, equation(6) is transformed to:

$$LOLV = \sum_i P_c^2(C_i - L_i) + \sum_i VAR\left[P_c(C_i - L_i)\right] + \sum\sum_{i \neq j} COV\left[P_c(C_i - L_i), P_c(C_i - L_i)\right] - \sum_i LOLE_i^2 \quad \ldots\ldots \quad (8)$$

But, $\sum_i P_c^2(C_i - L_i) = \sum_i LOLE_i^2$

Therefore,

$$LOLV = \sum_i VAR \; P_c(C_i - L_i) + \sum\sum_{i \neq j} COV\left[P_c(C_i - L_i), P_c(C_j - L_j)\right] \quad \ldots\ldots \quad (9)$$

ii Uncertainty in FPL only

This case is suitable for reliability evaluation when the accuracy of estimating the forced outage rates of generating units is highly sufficient to neglect the uncertainty in the forced outages. The LOLE in this special case is once again given by equation(5). Also, the LOLV given by equation(6) is subjected to the following conditions:

$$\text{VAR} \quad P_c(C_i - L_i) = 0$$

$$\& \quad \text{COV} \; P_c(C_i-L_i), P_c(C_j-L_j) = 0$$

Therefore,

$$\text{LOLV} = \sum_i \sum_k P_{L_i}(k)\, P_c^2(C_i - L_i) - \sum_i \text{LOLE}_i^{\,2} \quad \text{.... (10)}$$

4. Application on hypothetical system

To illustrate the effect of uncertainties in FPL and FOR on the reliability indices of a single-area power system, application is made on the hypothetical system presented in [5,7]. Table I gives the generating units of the system and their mean forced outage rates. The application is made for a 5-days period whose daily peak loads are 1800, 1740, 1700, 1600 and 1500 MW.

Table I

Unit	Capacity in MW	Mean FOR
1	60	0.0087
2	60	0.0137
3	100	0.0226
4	100	0.0271
5	220	0.0313
6	220	0.0357
7	300	0.0401
8	300	0.0445
9	500	0.0576
10	500	0.0621

The probability of having the peak load in certain level k can be supplied from the normal probability table if its step-size k is adjusted to be equal exactly to the step-size of the forced outage cumulative probability table. For the present sample system, this step-size is equal to 20 MW.

The LOLE and LOLV are calculated by the equations derived in previous section. In the computations, the forecasted peak loads are approximated by normal distributions with different standard deviation. The forced outage rates are assumed to have one of the following distributions: uniform distribution, weibull distribution and shifted exponential distribution with the same parameters as given in 7 . These distributions are shown in Figure 1. The application of different FOR's distributions in the computation is intended for testing their influence on the resulting LOLE and LOLV.

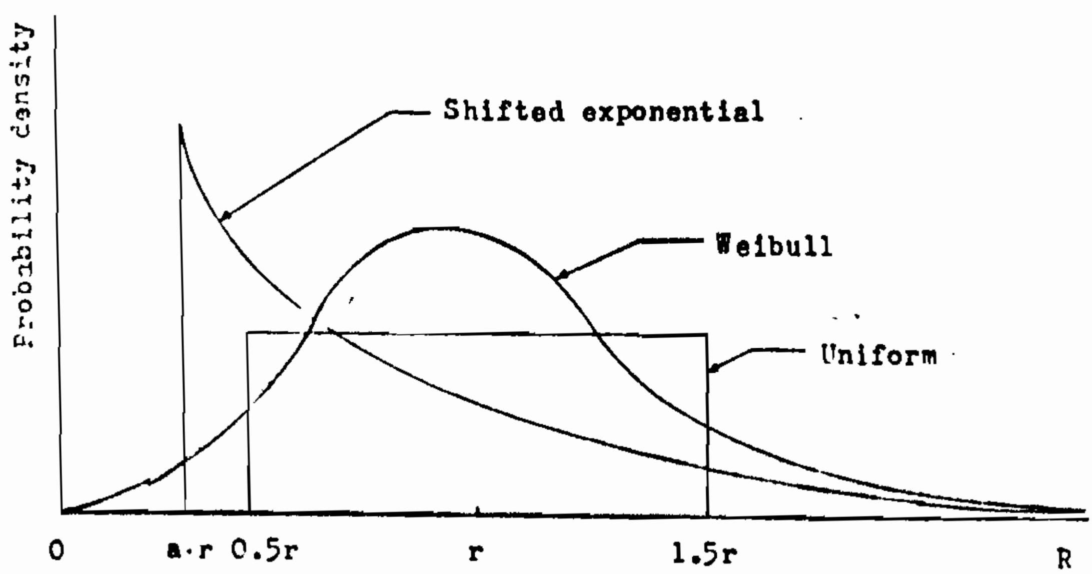

Fig. 1 Sample probability density function of FOR.

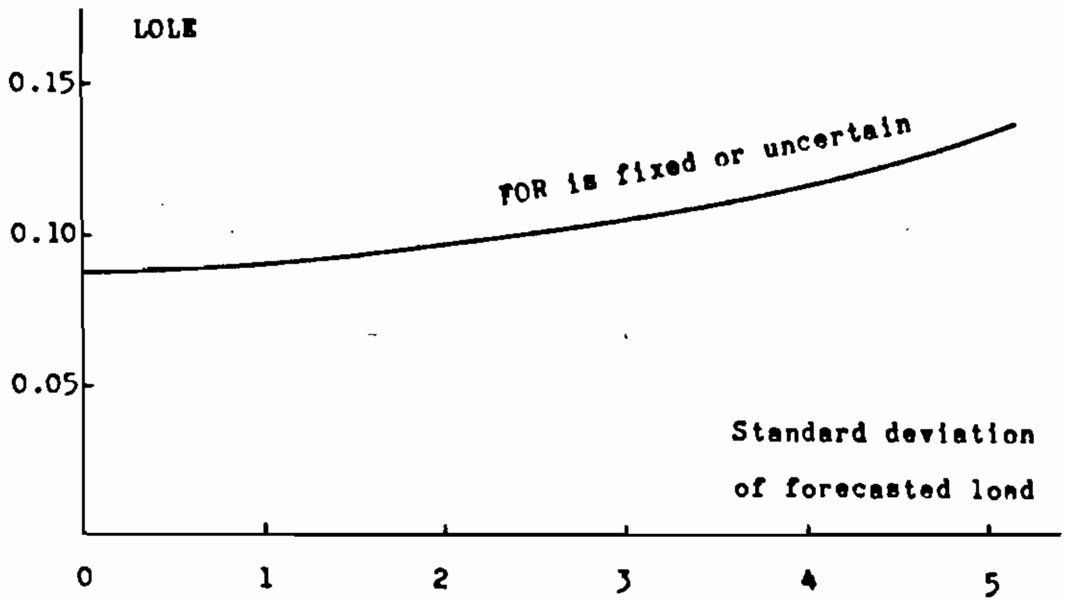

Fig.2 The relation between a 5-days LOLE and FPL standard deviation

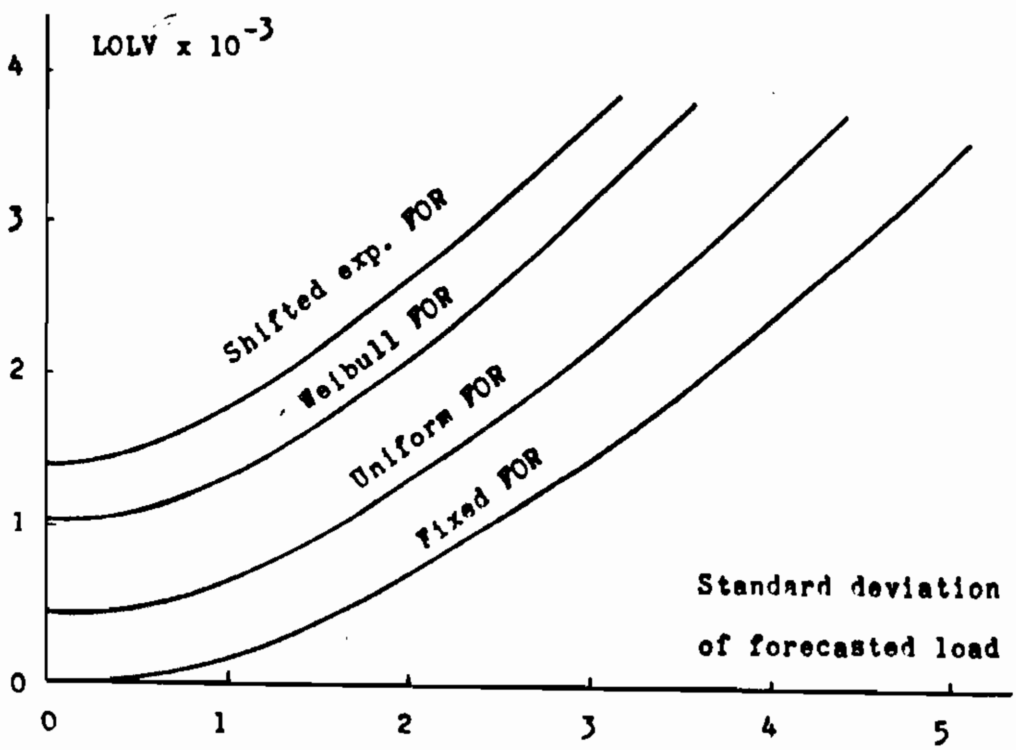

Fig.3 The relation between a 5-days LOLV and FPL standard deviation

Figures 2,3 illustrate the relations of the LOLE & LOLV against the uncertainty in forecasted daily peak loads represented by the corresponding standard deviation and uncertainty in forced outage rates represented by the different distributions given in Figure 1.

CONCLUSIONS

As the power system reliability indices depend on uncertainties in the estimated FOR of generating units and the uncertainty of FPL, equations for LOLE and LOLV are introduced. These equations have the advantage of simplicity and low computer time required for computations. From the application made on a hypothetical power system it could be concluded that:

1) The LOLE is not affected by the uncertainties in FOR while a considerable effect is found in the LOLV.
2) The load forecast uncertainties have a significant effect on the LOLE and LOLV, e.g., assuming Weibull distribution for the FOR, a 2% standard deviation in load forecast would result in a 13% increase in the LOLE and a 100% increase in the LOLV.

APPENDIX

Equation of loss of load variance

Generally, the loss of load variance (LOLV) is:

$$LOLV = E(LOLP - LOLE)^2 = E(LOLP)^2 - LOLE^2$$

Where:

$$E(LOLP)^2 = \sum_i \sum_j \iiint P_c(C_i - L_i) P_c(C_j - L_j) f(R,k,l)\, dR\, dk\, dl$$

Where: $f(R,k,l)$ = joint density of FOR and FPL on days i and j.

EFFECT OF UNCERTAINTIES IN FORECASTED LOADS AND FORCED OUTAGE ON POWER SYSTEM RELIABILITY

BY :

M.A. TANTAWY.*

ABSTRACT

The paper presents a comprehensive analysis for the effect of uncertainties inherent in forecasted loads and forced outage rates of generating units on the power system reliability. New equations are derived for the loss of load expectation and variance interms of these uncertainties. The proposed equations are distinguished by simplicity and low computer time required for computations. An application is made on a hypothetical power system for the general case of having both FOR and FPL are uncertain and for the special cases when one of them is considered deterministic. It is demonstrated that consideration of uncertainty in load forecast increases both the mean and variance of the loss of load probability.

1. INTRODUCTION

The power system operational planning has been strongly affected by the uncertainties in the daily forecas as the forced outage rates (FOR) of ger uncertainty in FPL results in uncertaint indices of a power system, it is require reserve margin that should be allocated security constraints.

Recent publications studied the pro

* Asst. Prof., Electrical Power and Mac

REFRENCES

1. Rabiner L.R. and Gold B., "Theory and Applications of digital signal processing", Prentice. Hall. N J., 1975, Chapter III.

2. Temes G.C., and Mitra S.K.ed., "Modern Filter Theory and design", John Wiley N.Y., 1973, Chapter 6.

3. Helms H.D., "Pigital Filters With Equiripple or Minimax Responses," IEEE Trans-on Audio and Electroacoustics, AU-19, No. 1, pp. 87-94, 1971.

4. PArks T.W. and McClellan J.H., "Chebyshev Approximation for Nonrecursive digital filters with linear phase", IEEE Trans. Circuit Theory, CT-19, pp. 189-194, Mar., 1972.

5. J.R. Rice and K.H. Usow, "Lawson's algorithm and extensions", Math. Comp., Vol. 22, pp. 118-127, Jan. 1968.

6. Bandler J.W., et al., "New Results in the least p^{th} Approach to Minimax Design", IEEE Trans. on Microwave Theory and Technique, MTT-24, pp. 116-119, FEB. 1976.

order to reduce the load forecast uncertainty[1-3]. Patton[5,6] analyzed the effect of uncertainty in FOR on the system reliability indices (while the FPL is assumed deterministic). The effect of uncertainty in FPL is studied in [4,7]. An approximate method was described by Billinton[4] for calculating the expected loss of load probability when uncertainties in FPL, but the FOR is fixed. This method[4] does not provide informations on the variance of LOLP. Wang[7] has introduced a method for calculating the mean and variance of the LOLP when uncertainties exist in the FPL only. Also, an approximate method for analyzing the general case when both FPL & FOR are considered random variables is given[7].

2. LOLP as a measure of reliability level

The loss of load probability (LOLP) is generally defined as the long run average number of days in a period of time that load exceeds the available installed capacity and thereby capacity deficiency occurs. During a state of capacity deficiency the load demand is not met by an adequate generation due to either unexpected loss of generation as a result of technical problems or unexpected increase in the daily peak load caused by the uncertainty in load forecast.

The LOLP, for a single area system and for N-day period is:

$$LOLP = \sum_{i=1}^{N} P_c(C_i - L_i) \qquad \ldots\ldots \quad (1)$$

Where:

C_i = total generating capacity scheduled for service on day i.

L_i = forecasted peak load on day i.

$P_c(C_i - L_i)$ = probability that $(C_i - L_i)$ is less or equal to zero.

i = 1, 2, 3, ..., N.

For the ideal case when both the FPL & FOR are deterministics, exact estimation for the mean of LOLP is given by equation(1) with zero variance. $P_c(C_i - L_i)$ is computed by forming the cumulative capacity outage probability table using the recursive expression[4,6]:

$$P_c^j(x) = P_c^{j-1}(x) \cdot (1 - R_j) + P_c^{j-1}(x - c_j) \cdot R_j \qquad \text{.... (2)}$$

Where:

$P_c^{j-1}(x)$, $P_c^j(x)$ = cumulative probability of x before and after the addition of j^{th} unit to the cumulative capacity outage probability table.

R_j , c_j = forced outage state probability and capacity of j^{th} unit respectively.

If there are uncertainties in FOR and FPL, the values of C_i and L_i are subjected to some uncertainties and must, therefore, be represented by a set of random variables. Then the LOLP is a random variable defined on both C_i and L_i with non zero variance.

3. Mathematical formulation for the mean & variance of LOLP

It is interesting for power system operational planners to have exact formulations for the loss of load expectation (LOLE) and loss of load variance (LOLV) for the general case when both FOR & FPL are subjected to uncertainties. The FOR and FPL are assumed mutually independent of each other[7]. This assumption is reasonable because forced outages of generating units are usually not considered when making load forecast.

The peak loads are reasonably assumed normally distributed around its mean. It may be scattered on a base of small fraction of standard deviation (k=0, 1, 2, ..., ∞). The probability of having the peak load in certain level k can be supplied from the normal probability table. The loss of load expectation is:

$$LOLE = \sum_{i=1}^{N} \int_0^\infty \int_0^\infty P_c(C_i - L_i) f(R,k) dR\ dk \qquad ...(3)$$

Where:

$f(R,k)$ = joint density function of FOR and FPL.

= $f(R)f(k)$ by the assumption of mutual independence.

Therefore,

$$LOLE = \int_0^\infty f(R)\ dR \cdot \sum_{i=1}^{N} \int_0^\infty f(k)\ P_c(C_i - L_i)\ dk \quad ..(4)$$

$P_c(C_i - L_i)$ is a monotonously increasing "staircase" function of k with discontinuities at regular intervals equal to S (the step size of the cumulative capacity outage probability table). Because of this property, equation(4) can be rewritten as:

$$LOLE = \sum_{i=1}^{N} \sum_{k=0}^{\infty} P_{L_i}(k)\, P_c(C_i - L_i) \qquad \ldots\ldots (5)$$

Where $P_{L_i}(k)$ is the probability of having the peak load in certain level k.

The LOLV, as derived in the appendix, is given by:

$$LOLV = \sum_{i} \sum_{k} P_{L_i}(k) \left[P_c^2(C_i - L_i) + VAR\left[P_c(C_i - L_i) \right] \right]$$

$$+ \sum_{i \neq j} \sum_{k} \sum_{l} P_{L_i}(k)\, P_{L_j}(l)\, COV\left[P_c(C_i - L_i), P_c(C_j - L_j) \right]$$

$$- \sum_{i} LOLE_i^{\,2} \qquad \ldots\ldots (6)$$

Where:

$VAR \;\; P_c(C_i - L_i)$ = Variance of load loss probability on day i,

$COV \;\; P_c(C_i - L_i), P_c(C_j - L_j)$ = Covariance of probabilities $P_c(C_i - L_i)$ on day i and $P_c(C_j - L_j)$ on day j,

$LOLE_i$ = Loss of load expectation on day i.

Special cases

The above expressions of LOLE and LOLV can be simplified for the special cases of having the FOR fixed while FPL is uncertain and the case of fixed FPL while the FOR is uncertain as follows:

i- Uncertainty in FOR only

The FPL may be considered deterministic when having high accuracy in the forecasted loads. Equation(5) is transformed to:

$$LOLE = \sum_{i=1}^{N} P_c(C_i - L_i) \qquad \ldots\ldots \quad (7)$$

Also, equation(6) is transformed to:

$$LOLV = \sum_i P_c^2(C_i - L_i) + \sum_i VAR\left[P_c(C_i - L_i)\right] + \sum_{i \neq j}\sum COV\left[P_c(C_i - L_i), P_c(C_i - L_i)\right] - \sum_i LOLE_i^2 \qquad \ldots\ldots \quad (8)$$

But, $\sum_i P_c^2(C_i - L_i) = \sum_i LOLE_i^2$

Therefore,

$$LOLV = \sum_i VAR\ P_c(C_i - L_i) + \sum_{i \neq j}\sum COV\left[P_c(C_i - L_i), P_c(C_j - L_j)\right] \qquad \ldots\ldots \quad (9)$$

ii Uncertainty in FPL only

This case is suitable for reliability evaluation when the accuracy of estimating the forced outage rates of generating units is highly sufficient to neglect the uncertainty in the forced outages. The LOLE in this special case is once again given by equation(5). Also, the LOLV given by equation(6) is subjected to the following conditions:

$$\mathrm{VAR}\quad P_c(C_i - L_i) = 0$$

$$\&\qquad \mathrm{COV}\; P_c(C_i-L_i), P_c(C_j-L_j) = 0$$

Therefore,

$$\mathrm{LOLV} = \sum_i \sum_k P_{L_i}(k)\, P_c^2(C_i - L_i) - \sum_i \mathrm{LOLE}_i^{\,2} \quad \ldots. (10)$$

4. Application on hypothetical system

To illustrate the effect of uncertainties in FPL and FOR on the reliability indices of a single-area power system, application is made on the hypothetical system presented in [5,7]. Table I gives the generating units of the system and their mean forced outage rates. The application is made for a 5-days period whose daily peak loads are 1800, 1740, 1700, 1600 and 1500 MW.

Table I

Unit	Capacity in MW	Mean FOR
1	60	0.0087
2	60	0.0137
3	100	0.0226
4	100	0.0271
5	220	0.0313
6	220	0.0357
7	300	0.0401
8	300	0.0445
9	500	0.0576
10	500	0.0621

The probability of having the peak load in certain level k can be supplied from the normal probability table if its step-size k is adjusted to be equal exactly to the step-size of the forced outage cumulative probability table. For the present sample system, this step-size is equal to 20 MW.

The LOLE and LOLV are calculated by the equations derived in previous section. In the computations, the forecasted peak loads are approximated by normal distributions with different standard deviation. The forced outage rates are assumed to have one of the following distributions: uniform distribution, weibull distribution and shifted exponential distribution with the same parameters as given in 7 . These distributions are shown in Figure 1. The application of different FOR's distributions in the computation is intended for testing their influence on the resulting LOLE and LOLV.

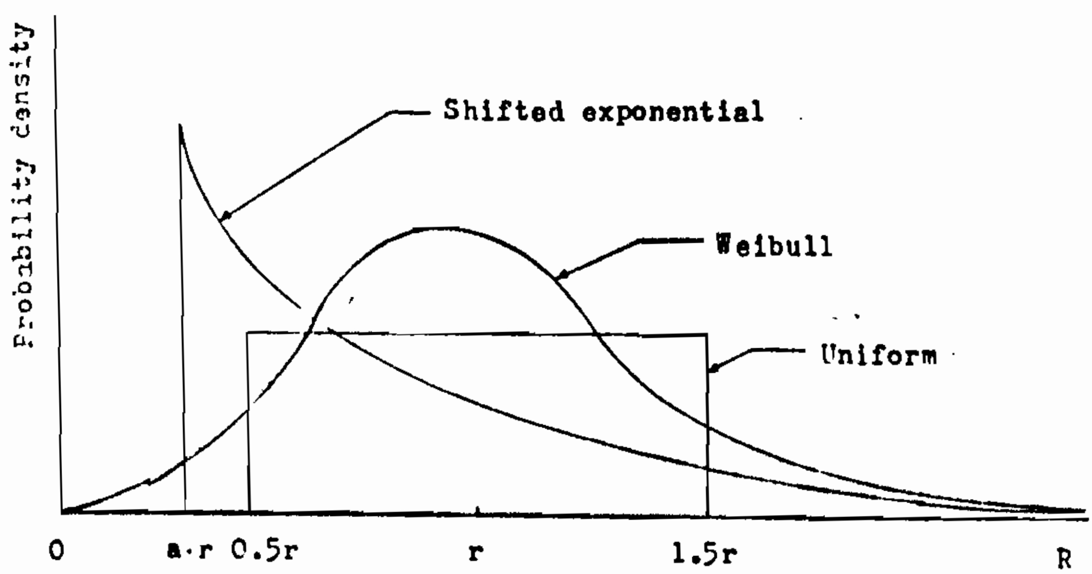

Fig. 1 Sample probability density function of FOR.

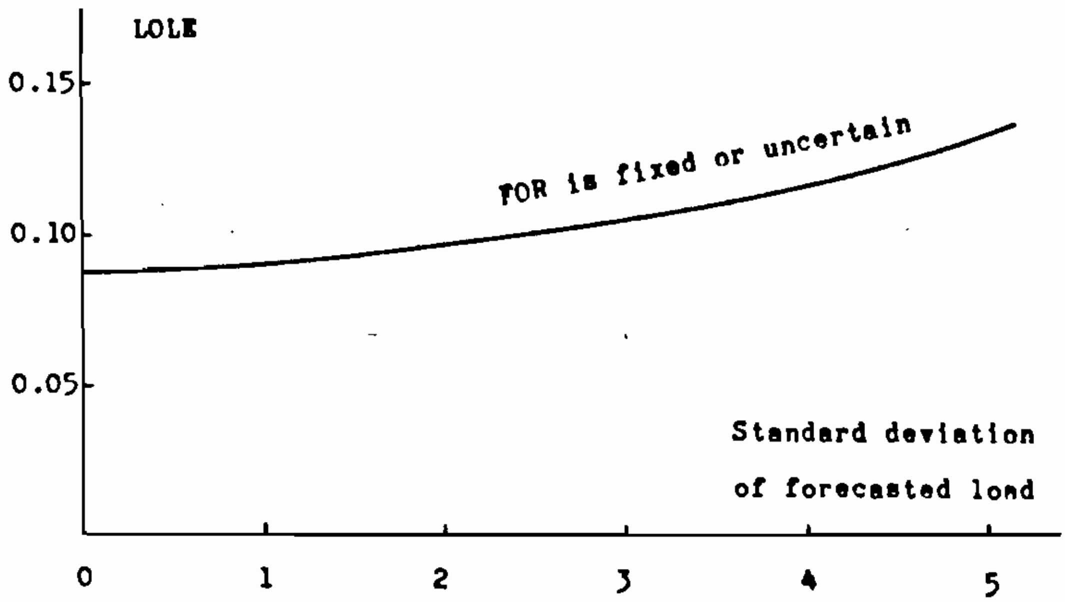

Fig.2 The relation between a 5-days LOLE and FPL standard deviation

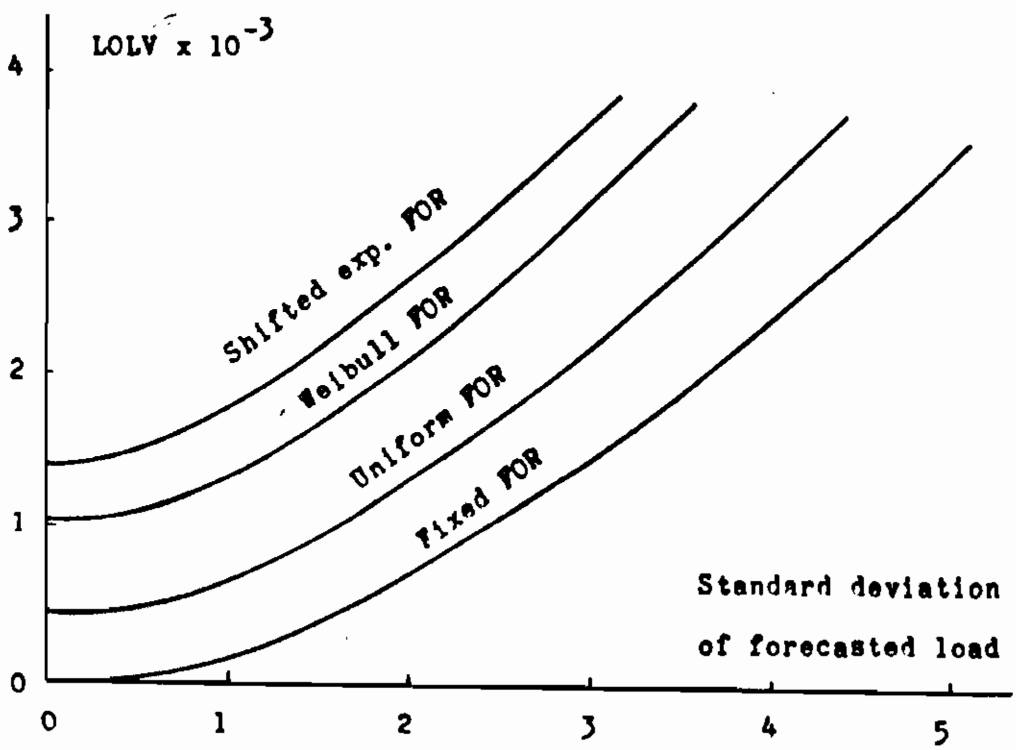

Fig.3 The relation between a 5-days LOLV and FPL standard deviation

Figures 2,3 illustrate the relations of the LOLE & LOLV against the uncertainty in forecasted daily peak loads represented by the corresponding standard deviation and uncertainty in forced outage rates represented by the different distributions given in Figure 1.

CONCLUSIONS

As the power system reliability indices depend on uncertainties in the estimated FOR of generating units and the uncertainty of FPL, equations for LOLE and LOLV are introduced. These equations have the advantage of simplicity and low computer time required for computations. From the application made on a hypothetical power system it could be concluded that:

1) The LOLE is not affected by the uncertainties in FOR while a considerable effect is found in the LOLV.
2) The load forecast uncertainties have a significant effect on the LOLE and LOLV, e.g., assuming Weibull distribution for the FOR, a 2% standard deviation in load forecast would result in a 13% increase in the LOLE and a 100% increase in the LOLV.

APPENDIX

Equation of loss of load variance

Generally, the loss of load variance (LOLV) is:

$$LOLV = E(LOLP - LOLE)^2 = E(LOLP)^2 - LOLE^2$$

Where:

$$E(LOLP)^2 = \sum_i \sum_j \iiint P_c(C_i - L_i) P_c(C_j - L_j) f(R, k, l) \, dR \, dk \, dl$$

Where: $f(R,k,l)$ = joint density of FOR and FPL on days i and j.

By the assumption of mutual independence;

$$f(R,k,l) = f(R)\ f(k)\ f(l)$$

Let $P_c(C_i - L_i) = P_i$ and $P_c(C_j - L_j) = P_j$

$$E(LOLF)^2 = \sum_i \sum_j \iint f(k)\ f(l) \int P_i\ P_j\ f(R)\ dR \quad dk\ dl$$

But $\int P_i\ P_j\ f(R)\ dR = E(P_i\ P_j) = P_i\ P_j + COV(P_i, P_j)$

Therefore,

$$LOLV = \sum_i \sum_j \iint f(k)\ f(l) \left[P_i\ P_j + COV(P_i, P_j) \right] dk\ dl - LOLE^2$$

$$= \sum_i \int f(k) \left[P_i^2 + VAR(P_i) \right] dk$$

$$+ \sum_{i \neq j} \sum \iint f(k)\ f(l) \left[P_i\ P_j + COV(P_i, P_j) \right] dk\ dl$$

$$- \sum_i LOLE_i^2 + \sum_{i \neq j} \sum \iint f(k)\ f(l)\ P_i\ P_j\ dk\ dl$$

Assuming a normal distribution for the forecasted load, the LOLV is:

$$LOLV = \sum_i \sum_k P_{L_i}(k) \left[P_c^2(C_i - L_i) + VAR\left[P_c(C_i - L_i) \right] \right]$$

$$+ \sum_{i \neq j} \sum \sum_k \sum_l P_{L_i}(k)\ P_{L_j}(l)\ COV\left[P_c(C_i - L_i), P_c(C_j - L_j) \right]$$

$$- \sum_i LOLE_i^2$$

E.117.Mansoura Bulletin December 1978.

REFERENCES

1- Gupta, P.C., and Yamada, K., "Adaptive short term forecasting of hourly loads using weather information" IEEE Trans. Power Apparatus and Systems, Vol. PAS-91, 1972, pp. 2085-2094.

2- Srinivasan, K., and Pronovost, R., "Short term load forecasting using multiple correlations model" Ibid Vol. PAS-94, 1975, pp. 1854-1858.

3- Tantawy, M.A., "Short term load forecasting for holidays and special days", Jahresbericht 1976, der Forschungs-gesellschaft Energie an der RWTH Aachen e.v. 1977.

4- Billinton, R., "Power system reliability evaluation", New York, Gordon and Breach, Science Publishers, 1970.

5- Patton, A.D., and Stasinos, A., "Variance and approximate confidence limits on LOLP for a single-area system", IEEE Trans. Vol. PAS-94, July/August, 1975, pp. 1326-1336.

6- Patton, A.D., and Tram, N.H., "Sensitivity of generation reliability indices to generator parameter variations", Ibid, Vol. PAS-97, July/August 1978, pp. 1337-1343.

7- Wang, L., "The effect of uncertainties in forced outage rates and load forecast on the loss-of-load-probability (LOLP)", Ibid Vol. PAS-96, Nov./Dec. 1977, pp. 1920-1927.

By the assumption of mutual independence;

$$f(R,k,l) = f(R)\, f(k)\, f(l)$$

Let $P_c(C_i - L_i) = P_i$ and $P_c(C_j - L_j) = P_j$

$$E(LOLF)^2 = \sum_i \sum_j \iint f(k)\, f(l) \int P_i\, P_j\, f(R)\, dR \quad dk\, dl$$

But $\int P_i\, P_j\, f(R)\, dR = E(P_i\, P_j) = P_i\, P_j + COV(P_i, P_j)$

Therefore,

$$LOLV = \sum_i \sum_j \iint f(k)\, f(l) \left[P_i\, P_j + COV(P_i, P_j) \right] dk\, dl - LOLE^2$$

$$= \sum_i \int f(k) \left[P_i^2 + VAR(P_i) \right] dk$$

$$+ \sum_{i \neq j} \sum \iint f(k)\, f(l) \left[P_i\, P_j + COV(P_i, P_j) \right] dk\, dl$$

$$- \sum_i LOLE_i^{\,2} + \sum_{i \neq j} \sum \iint f(k)\, f(l)\, P_i\, P_j\, dk\, dl$$

Assuming a normal distribution for the forecasted load, the LOLV is:

$$LOLV = \sum_i \sum_k P_{L_i}(k) \left[P_c^2(C_i - L_i) + VAR\left[P_c(C_i - L_i) \right] \right]$$

$$+ \sum_{i \neq j} \sum \sum_k \sum_l P_{L_i}(k)\, P_{L_j}(l)\, COV\left[P_c(C_i - L_i), P_c(C_j - L_j) \right]$$

$$- \sum_i LOLE_i^{\,2}$$

REFERENCES

1- Gupta, P.C., and Yamada, K., "Adaptive short term forecasting of hourly loads using weather information" IEEE Trans. Power Apparatus and Systems, Vol. PAS-91, 1972, pp. 2085-2094.

2- Srinivasan, K., and Pronovost, R., "Short term load forecasting using multiple correlations model" Ibid Vol. PAS-94, 1975, pp. 1854-1858.

3- Tantawy, M.A., "Short term load forecasting for holidays and special days", Jahresbericht 1976, der Forschungs-gesellschaft Energie an der RWTH Aachen e.v. 1977.

4- Billinton, R., "Power system reliability evaluation", New York, Gordon and Breach, Science Publishers, 1970.

5- Patton, A.D., and Stasinos, A., "Variance and approximate confidence limits on LOLP for a single-area system", IEEE Trans. Vol. PAS-94, July/August, 1975, pp. 1326-1336.

6- Patton, A.D., and Tram, N.H., "Sensitivity of generation reliability indices to generator parameter variations", Ibid, Vol. PAS-97, July/August 1978, pp. 1337-1343.

7- Wang, L., "The effect of uncertainties in forced outage rates and load forecast on the loss-of-load-probability (LOLP)", Ibid Vol. PAS-96, Nov./Dec. 1977, pp. 1920-1927.