ERJ
Engineering Research Journal
Faculty of Engineering
Minoufiya University

A SIMPLIFIED NONLINEAR MODEL OF SWITCHED RELUCTANCE MOTOR

H. Abdel-Maksoud, M. M. Khater, and A.M.Oshieba

Department of Electrical Engineering,
Faculty of Engineering, Minoufiya University, Egypt

ABSTRACT

This paper presents a simple nonlinear mathematical model of the switched reluctance motor (SRM) which requires a minimum pre-calculated input data. The proposed model depends on describing the characteristic data obtained from finite element analysis (FEA) in a simple parametric formula. The inductance and torque equations are derived as functions of the rotor position and phase current. Computer simulation based on the proposed model produces accurate current, voltage and torque wave shapes. It provides good flux linkage-current, torque-angle relationships. The model also gives reliable results in both steady state and dynamic operation. Simulation results of the proposed model are compared with those based on FEA to verify its accuracy. The simplicity and accuracy of this model makes it a useful tool for controller design and implementation.

تعتبر الخواص اللاخطية لمحرك المعاوقة المغناطيسية النبضي من أهم التحديات التي تواجه إدراج هذا المحرك داخل نظام تحكم. ويهدف هذا البحث إلى تقديم نموذج رياضي بسيط يقوم بوصف هذه الخواص اللاخطية معتمدا على البيانات المستنتجة من تحليل الدائرة المغناطيسية، وقد تم في هذا البحث عمل نموذج رياضي يعتمد على استنباط معادلتين لكل من المحاثة والعزم كدوال في التيار وزاوية موضع العضو الدوار، وتم استخدام هذا النموذج في استنتاج خواص المحرك الاستاتيكية وخواص أدائه في حالات التشغيل المستقر وتحت التغييرات الديناميكية، وتم مقارنة هذه النتائج مع تلك المستنتجة على نموذج رياضي آخر أكثر تعقيداً يعتمد مباشرة على التحليل العددي للدائرة المغناطيسية بطريقة العنصر المحدود، وقد تبين من المقارنة أن النموذج الرياضي المقترح على قدر كبير من الدقة والبساطة التي يمكن معها الاعتماد عليه في تمثيل خواص المحرك وكذلك تمثيله في دوائر التحكم.

Keywords: Proposed Model, Switched Reluctance Motor, Finite Element Analysis (FEA).

1. INTRODUCTION

The inherent simplicity, ruggedness, and low cost of a switched reluctance motor (SRM) makes it a viable candidate for various general-purpose adjustable-speed and servo-type applications. The SRM drives have the additional attractive features of fault tolerance and the absence of magnets, but these merits are overshadowed by its inherent torque ripple and nonlinearity of its characteristics. The performances improvement of SRM strongly depends on the applied control [1, 2]. The first step to design the control system is to develop the dynamic model but the high degree of its nonlinearity makes it impossible to model the flux linkage or phase inductance by a SR motor phase exactly. Many researchers have addressed the problem of modeling the SRM analytically with various degrees of accuracy [3, 4]. The selection of a SRM model depends on a proper mathematical representation of the static characteristics, the computational facilities and control techniques available. Although accuracy considerations influence the choice of the model, there are additional issues of computation complexity and difficulties in practical implementation should be taken into account [5, 6]. A model based on decomposing the magnetic saliencies due to non-uniform air gap and saturation of laminations at high stator currents is proposed [7]. However, the large number of coefficients which should be calculated limit using this model for online control. A nonlinear model of the SRM uses the equivalent magnetic circuit of the motor as a set of reluctances linked in series and in parallel is presented in reference [8]. However, besides, it needs an accurate geometry data; the B-H curve for each part of the machine as well as the magnetization curve should be defined. Some papers proposed an analytical model derived from the motor geometry and material magnetic property [9, 10]. This approach may be useful for the physical machine model; little guidance is given to model the magnetic structure for the purpose of controller design. A piecewise linear inductance model based on the current and reluctances presented in [11]. The introduction of many approximations reduces the accuracy of the model, besides the large number of coefficients precludes its use in real time controller. The energy conversion principles show that accurate prediction of the developed torque can be obtained from the relationship between the flux linkage (λ), phase current (i) and rotor position angle (θ). These magnetization characteristics can be obtained from direct measurements on an existing motor or alternatively, from sufficiently precise numerical calculations such as finite element analysis (FEA) [12, 13]. Although models based on finite-element analysis and magnetic circuits are suitable for motor designers, they are time consuming and can not be employed for control purposes [14].

Therefore the aim of this paper is to propose a simple and compact model of SRM that could be used for online control and would be based on minimum required data and additional calculations. The validity of this model has been verified through comparing with finite element based model results. The comparison between them shows that the errors are fairly small and that the proposed method is effective and accurate.

2. MODELING OF SRM

Switched reluctance machine is usually operated in the magnetically saturated mode to maximize the energy conversion. The magnetic nonlinearities of an SRM can be taken into account by appropriate modeling of the nonlinear flux–current–angle (λ, i, θ) characteristics of the machine. Figure 1 depicts the cross-section of the 6/4 SRM (six stator teeth, four rotor teeth) used to test the proposed model [15]. The finite element method FEM is used, firstly, to analyze the magnetic circuit of the motor under study. After that a simplified model is developed based on the results of the FE analysis.

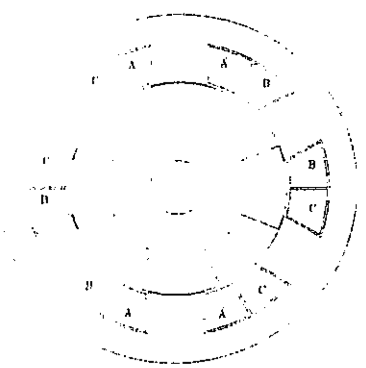

Fig.1. Cross section of 6/4 SRM

2.1. FE Analysis

A two-dimensional FE is used for which magnetic field governing equation of SRM for magnetic vector potential A_z is given by

$$\frac{\partial}{\partial x}\left(v\frac{\partial A_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(v\frac{\partial A_z}{\partial y}\right)+J=0 \qquad (1)$$

Where v, J are the magnetic reluctivity of the material and the induced current density respectively. Based on the above equation the finite element numerical solution is used to analyze the magnetic circuit. The solution started from the unaligned (θ=0) up to the aligned position (θ=45^0) with different values of phase current. The flux distributions as well as the flux linkage-current curves at different rotor position angles are obtained as shown in Figs. 2, 3.

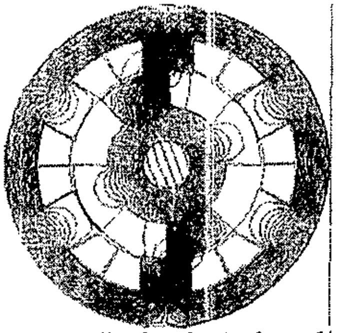

Fig.2. Flux distribution for 3-phase 6/4 SRM

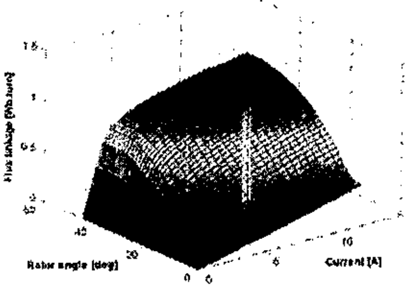

Fig.3. A 3-dimension plot for λ-i-θ curves of 3-phase 6/4 SRM.

2.2. Proposed Model

The flux linkage for phase j can be described as:

$$\lambda_j(i_j,\theta)=L_j(i_j,\theta)i_j \qquad (2)$$

Where L_j is the self inductance of phase j. Hence, the self inductance can be derived from this equation:

$$L_j(i_j,\theta)=\frac{\lambda(i_j,\theta)}{i_j} \qquad (3)$$

The self inductance equation is the key input to the proposed model. Based on the flux linkage and phase current data obtained by FEA, the computer program is built to obtain the self inductance data as a function i_j and θ. This data is programmed and simulated to obtain inductance-angle curves at different values of phase current as well as inductance-current curves at different rotor positions as shown in Fig. 4, 5.

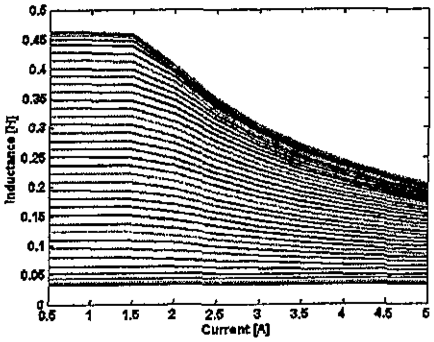

Fig.4. Inductance-current curves at different rotor positions

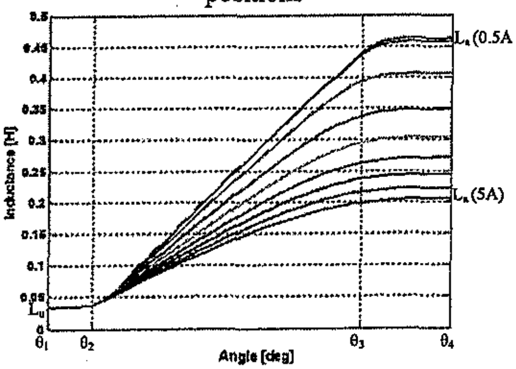

Fig.5. Inductance-angle curves at different values of phase current.

Figure 5 shows the variation of phase inductance versus rotor position angle, from which it can be observed that:

1. The phase inductance is constant from θ_1 to θ_2 and equal the unaligned inductance value L_u.
2. The phase inductance varies nearly linear from θ_2 to θ_3 and changes with phase current and rotor position angle.
3. The phase inductance depends only on the phase current from θ_3 to θ_4 and equal the aligned inductance value L_a for each value of current.

On the other hand, the phase inductance can be represented by a group of trapezoidal curves; its bottom value is constant at L_u and the top value L_a which changes with phase current. So, it can be described as follows:

$$L_j(i_j,\theta_j)=\begin{cases} L_u & \theta_1 \le \theta \le \theta_2 \\ L_u + k\,\theta_j & \theta_2 \le \theta \le \theta_3 \\ L_a & \theta_3 \le \theta \le \theta_4 \end{cases} \quad (4)$$

$$k = \frac{L_a - L_u}{\beta_S} \quad (5)$$

Where β_S is the stator pole arc, the variation of the aligned phase inductance with the phase current is represented by a second order polynomial equation as

$$L_a = a_0 i_j^2 + a_1 i_j + a_2 \quad (6)$$

The coefficients a_0, a_1, a_2 are determined by the curve fitting method.

3. MODEL VERIFICATION

The results based on the proposed model should be compared with those obtained from FEA to validate its accuracy. In the following sections the flux linkage characteristics, steady state and dynamic performances are presented and compared.

3.1 Flux Linkage Verification

The flux linkage-current characteristics (at aligned and unaligned positions) obtained from the proposed model and from the FEA method are compared. The comparison insures that the curves are typically very close as shown in Fig.6.

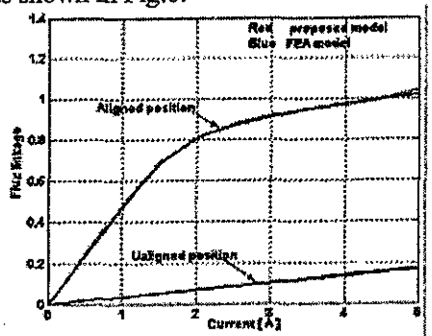

Fig.6. Comparison of flux linkage characteristics for proposed model and FEA

3.2. Steady State Verification

In steady state, all voltages and currents are periodic, while the speed is constant. The voltage applied to any phase j at the rotor position θ_j can be expressed as;

$$V_j(\theta_j) = R_j i_j + \frac{d\lambda_j(\theta_j, i_j)}{dt} \qquad j = 1, 2, 3 \quad (7)$$

where R_j is the phase resistance, $V_j(\theta_j)$ is the voltage applied to the phase winding j by the power electronic converter and is expressed as follows

$$V_j(\theta_j)=\begin{cases} +V_{dc} & \theta_{on} \le \theta_j \le \theta_{off} \\ -V_{dc} & \theta_{off} \le \theta_j \le \theta_{ext} \\ 0 & \theta_j > \theta_{ext} \end{cases} \quad (8)$$

Where $\theta_j = \theta + (j-1).\delta$, $\theta = \omega t$ and the step angle is given by; $\delta = 2\pi / qN_r$, where q , Nr, θ_{ext} are the number of phases , number of rotor poles and extension angle respectively.

3.2.1. *FEA Method*

The steady state performance for the motor can be obtained using the family of the corresponding flux linkage-current curves $\lambda(\theta,i)$ at different rotor positions which were obtained by the FE analysis. These curves are rearranged and stored in the form of $i(\theta,\lambda)$ look-up tables. Using a time stepping numerical integration procedure applied to Eqn. (7) the flux linkage of each phase is obtained. At each integration step the phase current is obtained with the aid of the stored $i(\theta,\lambda)$ data. The developed torque is also obtained by a time stepping procedure. For each step the phase co-energy can be obtained by numerical integration of the $\lambda(\theta,i)$ with respect to phase current:

$$W_j(\theta_j,i_j)=\int_0^{i}\lambda_j(\theta_j,i_j)di_j \qquad (9)$$

From which the developed torque can be obtained by numerical differentiation of the $W(\theta,i)$ with respect to the corresponding rotor position θ_j

$$T_j(\theta_j,i_j)=\frac{\partial W_j(\theta_j,i_j)}{\partial\theta_j} \qquad (10)$$

The total developed torque is obtained as the summation of the instantaneous torque developed by all phases.

$$T=\sum_{j=1}^{q}T_j(\theta_j,i_j) \qquad (11)$$

3.2.2. Proposed Model

In the proposed model, to calculate the phase current, the Runge-Kutta method can be applied to solve the differential equation:

$$\frac{di_j}{dt}=\frac{1}{L_j}=(V_j-i_jR_j-\frac{\partial\lambda_j}{\partial\theta}\omega) \qquad (12)$$

Based on the formula of the self inductance L_j, the torque production can be obtained from the basic torque equation:

$$T=\frac{1}{2}i^2\frac{dL}{d\theta} \qquad (13)$$

Substituting Eqn. (4) into Eqn. (13) one obtains the phase torque as:

$$T_j=\begin{cases}0 & \theta_1\le\theta\le\theta_2\\ \frac{1}{2}i_j^2\,k & \theta_2\le\theta\le\theta_3\\ 0 & \theta_3\le\theta\le\theta_4\end{cases} \qquad (14)$$

The summation of phases torque can be obtained as the same way used with FEA.

3.2.3. The Verified Results

The steady state simulation results (current wave forms, voltage waveforms and phase torque waveforms), of the proposed model and FEA method are compared in Figs.7, 8, 9. The results are taken when the motor was operated at 1000 r/min. While the torque -speed and current-speed characteristics, are shown in Figs. 10 and 11 are taken with change the speed from 100 up to 1500 r/min by 10 r/min step. The results show a good agreement for one phase operations cases, which strengthen our confidence in propose model.

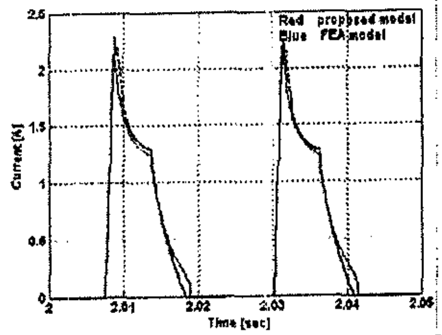

Fig.7. Comparison of current waveforms for proposed model and FEA

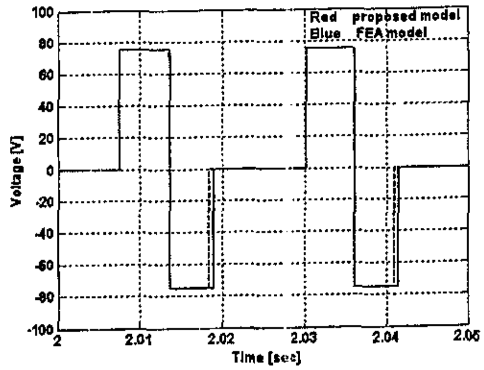

Fig.8. Comparison of voltage waveforms for proposed model and FEA

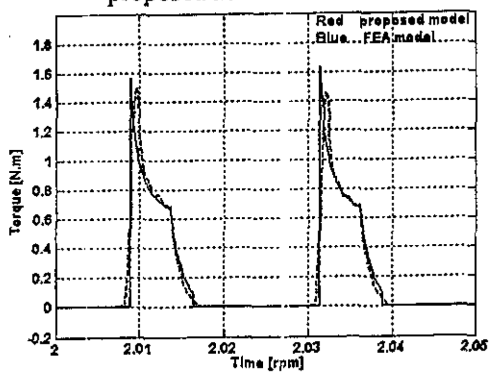

Fig.9. Comparison of torque waveforms for proposed model and FEA

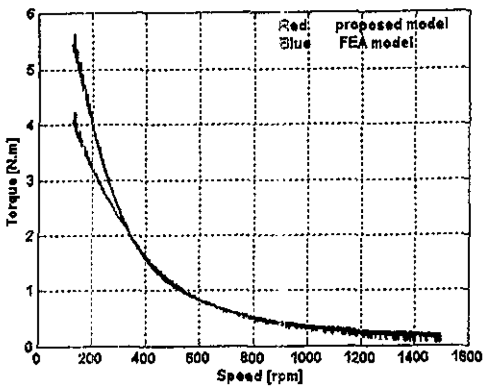

Fig.10. Comparison of torque-speed characteristics for proposed model and FEA

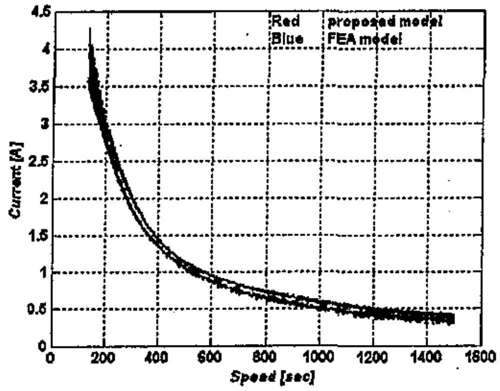

Fig.11. Comparison of current-speed characteristics for proposed model and FEA

3.3. Dynamic Verification

Because the dynamic model can be considered as the backbone for any control system in the SRM drives, so, it is essential to study the dynamic responses (speed, torque and current) under the most commonly conditions such as startup, load change and voltage change. The dynamic model is built based on the following equations:

$$\frac{di_j}{dt} = \frac{1}{L_j}(V_j - i_j R_j - i_j \omega \frac{dL_j}{d\theta}) \quad (15)$$

$$\frac{d\theta}{dt} = \omega \quad (16)$$

$$\frac{d\omega}{dt} = \frac{1}{M}(T - T_L - B\omega) \quad (17)$$

Where ω, M, B are rotor speed, moment of inertia and viscous damping constant.

3.3.1. Start Up

Figures 12, 13, 14 show the dynamic responses during the motor starting. The motor was supplied by rated voltage with a load torque of 0.2 N.m. The speed and torque responses of the proposed model are exactly as FEA model. The current response does not match exactly the one from FEA, however, it is still good enough for a fast analysis. It can be also observed that the developed torque is greater than the load torque by friction and viscous torque.

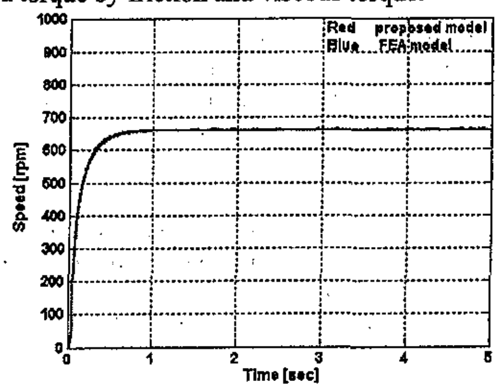

Fig.12. Comparison of the motor speed in the start up for proposed model and FEA.

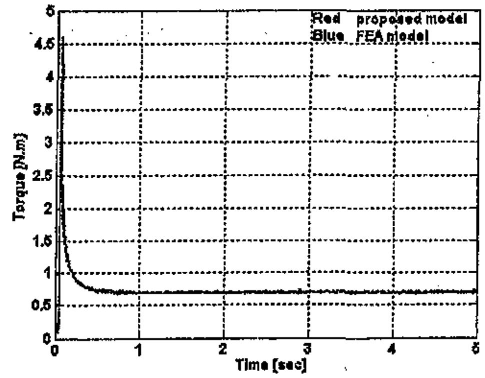

Fig.13. Comparison of the developed torque in the start up for proposed model and FEA.

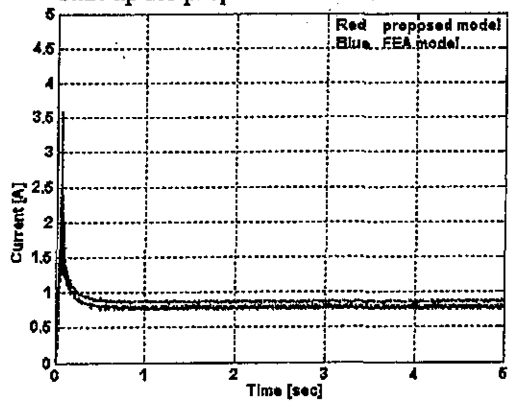

Fig.14. Comparison of the dc link current in the start up for proposed model and FEA.

3.3.2. *Load Change*

Firstly, the motor starts with a load of 0.2 N.m and after that the load torque is increased to 0.5 Nm. Figs. 15, 16, 17 show the change in speed, torque and current with load increase. It can be seen that the speed decreases with increasing the load while the developed torque and the current increase with the same change.

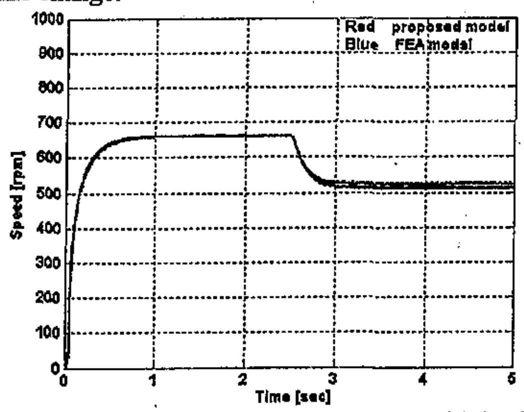

Fig.15. Comparison of the motor speed with load change for proposed model and FEA.

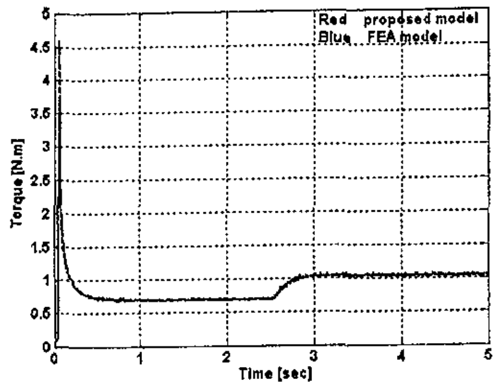

Fig.16. Comparison of the developed torque with load change for proposed model and FEA.

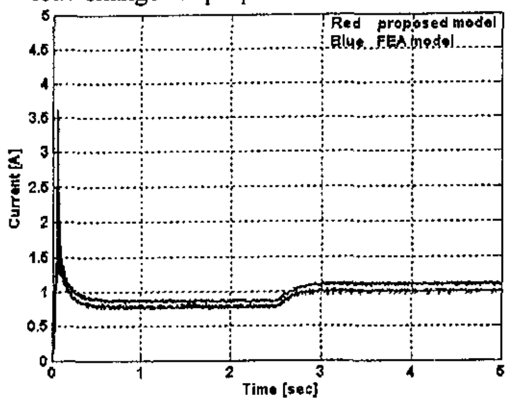

Fig.17. Comparison of the dc link current with load change for proposed model and FEA.

3.3.3. *Voltage Change*

The motor starts with 80% of the rated voltage with a load torque of 0.2 Nm and after 2.5 sec; the voltage simultaneously is increased to the rated voltage. The responses of speed, torque and current with variation of voltage for both models are shown in Fig. 18, 19, 20.

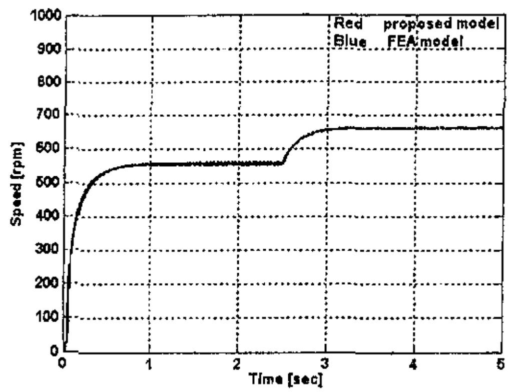

Fig.18. Comparison of the motor speed with voltage change for proposed model and FEA.

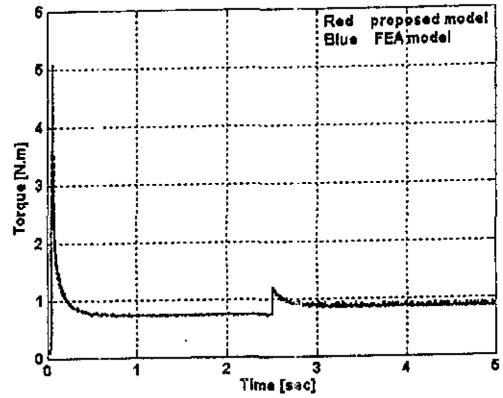

Fig.19. Comparison of the developed torque with voltage change for proposed model and FEA.

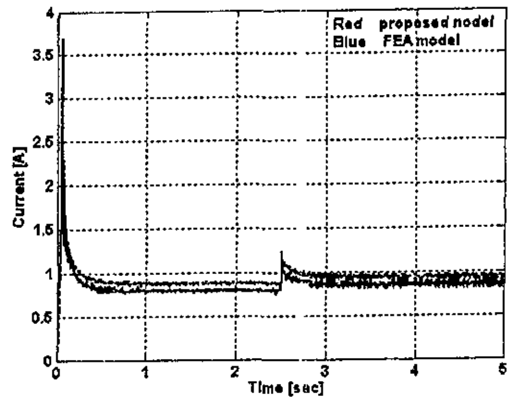

Fig.20. Comparison of the dc link current with load change for proposed model and FEA

4. CONCLUSION

This paper has presented a simplified mathematical model of the SRM by which the steady state and dynamic behaviors can be successfully predicated. The phase inductance and developed torque are expressed as functions of phase current and rotor position angle incorporating the motor nonlinearity. The static characteristics, steady state as well as dynamic performances based on both the proposed model and FEA are compared. The comparison insures that the proposed model has a good accuracy in representing SRM at different conditions. Hence, the analytical model developed here would be useful for representing the SRM and it is more suitable for implementing controllers and observers due to its quick, simple and accurate features.

5. REFERENCES

[1] Jae-Hak Choi, Sol Kim, Yong-Su Kim, Sang-Don Lee and Ju Lee, "Multi-Object Optimization of the Switched Reluctance Motor", KIEE International Transactions on EMECS, Vol. 4-B No. 4, 2004, PP. 184-189.

[2] K.N.Srinivas and R. Arumugam, "Dynamic Characterization of Switched Reluctance Motor

by Computer-Aided Design and Electromagnetic Transient Simulation", IEEE Transaction on Magnetics, Vol. 39, No. 3, May 2003, PP 1806-1812.

[3] X. D. Xue, K. W. E. Cheng, and S. L. Ho" A Self-Training Numerical Method to Calculate the Magnetic Characteristics for Switched Reluctance Motor Drives" IEEE Transactions on Magnetics, Vol. 40, No. 2, MARCH 2004. PP. 734-737.

[4] Ge Baoming, Wang Xiangheng, Su Pengsheng, and Jing Jingping, "Nonlinear Internal Model Control for Switched Reluctance Drives", IEEE Transactions on Power Electronics, Vol. 17, No. 3, May 2002. PP. 379-388.

[5] Christophe Roux and Medhat M. Morcos, "On the Use of a Simplified Model for Switched Reluctance Motors", IEEE Transactions on Energy Conversion, Vol. 17, No. 3, September 2002. PP. 400-405.

[6] Benjamin Loop, D. N. Essah, and Scott Sudhoff, "A Basis Function Approach to the Nonlinear Average Value Modeling of Switched Reluctance Machines", IEEE Transactions on Energy Conversion, Vol. 21, No. 1, March 2006, PP 60-68.

[7] Farzad R. Salmasi and Babak Fahimi, "Modeling Switched-Reluctance Machines by Decomposition of Double Magnetic Saliencies", IEEE Transactions on Magnetics, Vol. 40, No. 3, May 2004, PP 1556-1561.

[8] Vladan Vujiˇcic´ and Slobodan N. Vukosavic,"A Simple Nonlinear Model of the Switched Reluctance Motor", IEEE Transactions on Energy Conversion, Vol. 15, No. 4, December 2000.PP. 395-400.

[9] Syed A.Hossain, and Iqbal Husain, "A Geometry Based Simplified Analytical Model of Switched Reluctance Machines for Real-Time Controller Implementation" IEEE Transactions on Power Electronics, Vol. 18, No. 6, November 2003. PP 1384-1389.

[10] Iqbal Husain, and Syed A. Hossain, " Modeling, Simulation, and Control of Switched Reluctance Motor Drives", IEEE Transactions on Industrial Electronics, Vol. 52, No. 6, December 2005, PP 1625-1634.

[11] Nikolay Radimov, Natan Ben-Hail, and Raul Rabinovici, "Simple Model of Switched-Reluctance Machine Based Only on Aligned and Unaligned Position Data" IEEE Transactions on Magnetics, Vol. 40, No. 3, May 2004. PP. 1562-1572.

[12] Xie Dexin, Yan Xiuke, and Zhang Yihuang," A Direct Field-Circuit-Motion Coupled Modeling of Switched Reluctance Motor" IEEE Transactions on Magnetics, Vol. 40, No. 2, March 2004, PP 573-576.

[13] Jinhui Zhang and Arthur V. Radun, " A New Method to Measure the Switched Reluctance Motor's Flux", IEEE Transactions on Industry Applications, Vol. 42, No. 5, September/October 2006,PP 1171-1176.

[14] D. Rodger, H. C. Lai, R. J. Hill Cottingham, and P. C. Coles" Some Tricks for Modeling Rotating Electrical Machines Using Finite Elements", IEEE Transactions on Magnetics, Vol. 40, No. 2, March 2004. PP. 802-805.

[15] M. M. Khater, H. Abdel-Maksoud, and S. El-Doheimy, "Performance of Three-Phase Switched Reluctance Motors with Different Structures: Comparative Study", 10th International Middle East Power Systems Conference (MEPCON), 12-15 December 2005, Port Said, Egypt, PP 239-245.