Sci. J. Fac. Sci. Menoufia Univ. Vol. VIII (1994)

QUASILINEARIZATION OF THE MIXED CONVECTION IN A VERTICAL SLOT OF FERROMAGNETIC FLUID

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ABSTRACT

Combined forced and free steady convection in a vertical slot of ferromagnetic fluid in presence of transverse magnetic field is studied numerically by using the quasilinearization technique [1]. We have obtained the velocity and the temperature distribution for both free and mixed convection with both small and large values of the magnetization parameter "A". The present results for velocity and temperature distributions are compared with those of numerical and analytic results obtained by previous authors in the case of small "A", and good improvement has been found. The results obtained are discussed and explained in detail.

INTRODUCTION

During the last few decades there has been an increased interest in studying problems originating from magnetic fluids in relation to the development of practical engineering applications such as magnetic fluid seal, levitation and energy conversion systems .Magnetic fluids are synthetic fluids in the form of colloidal . Suspension of small ferrite particles of 50 - 100 A° in diameter stably dispersed in a carrier fluid such as kerosene, heptane or water [2]. Surface coating of each particle by a surfactant such as oleic acid prevents particles from agglomeration. Particle concentration does not take place even in the presence of a magnetic field. Hysterisis is unlikely in the fluid . Now it has been admittable that the magnetic fluid behaves as continuum and can be treated theoretically as a Newtonian fluid interactive with the external magnetic field . The magnetic force experienced in magnetic fluids is due to magnetic polarization of the fluid itself in contrast to the Lorentz force in MHD - The magnetization of magnetic fluids is, in general, a finction of the magnetic field, the temperature and the density of the fluid . Any variation of these quantities can induce the corresponding spatial distribution of the body force . The temperature gradient is, in particular, of importance in the phenomena where the buoyancy due to the thermal expansion is essential . Depending on the gradient of the magnitude of magnetic field, the magnetic force reinforces or weakens the buoyancy effects . The heat generation in many practical problems cited earlier, for example, in magnetic fluid seal under higher rotating speed and also heating and cooling of a magnetic fluid in the energy conversion systems, induce the interaction between temperature and fluid velocity distributions

The most familiar example of thermo- mechanical interaction is the buoyancy induced convection. Convection can also occur in ferromagnetic fluids in the presence of a magnetic field and a temperature gradient the onset of convection in a horizontal layer of magnetic fluid heated from below, analogous to that of the Rayleigh-Benard problem has been investigated by many authors [3-5]. But the study of the combined forced and free convection in a magnetic fluid has not been given much attention inspite of its applications in the practical problems mentioned above.

Therefore the main aim of this paper is to study the mixed convective flow in a vertical slot of ferromagnetic fluid in the presence of a vertical magnetic field considering the two cases: first, the boundaries are maintained at the same temperature and second, the boundaries are maintained at different temperatures with the aim of understanding the velocity and temperature distributions of thermomagnetic fluids. The governing equations for the velocity and temperature are highly nonlinear and are solved using the quasilinearization technique. The results obtained have been compared with corresponding results obtained earlier by analytical technique, and the Rung - Kutta-Gill method [6]. The quasilinearization technique an efficient analytical numerical procedure for solving complicated boundary value problems has been demonstrated by many authors [7-9] .

This study investigates the different values of the magnetization parameter "A" , and the suitable values of R_a, R_m and P .

2 - BASIC EQUATIONS :

The momentum equation for an incompressible ferromagnetic Boussinesq fluid with the constant viscosity [1] is :

$$\rho \frac{d\vec{u}}{dt} = -\nabla P + \rho \vec{g} + \mu_o M \nabla H + \mu \nabla^2 u \qquad (1)$$

where $\rho, u, t, P, g, \mu_0, \mu, M$ and H denote the density, velocity, time, pressure, acceleration due to gravity, magnetic permeability of vacuum, viscosity , magnetization and magnetic field respectively .

The energy equation for an incompressible fluid [3] is:

$$\rho c \frac{DT}{Dt} + \mu_0 T \frac{\partial M}{\partial T} \frac{DH}{Dt} = K_1 \nabla^2 T + \phi \qquad (2)$$

where c is the specific heat, T is the temperature, K_1 is the thermal conductivity and ϕ is the viscous dissipation .The second term on the left hand side of (2) expresses the heating due to the magnetocaloric effect of a magnetic substance in the presence of a magnetic field. Maxwell's equations, simplified for a non-conducting fluid with no displacement currents, are:

$$\nabla . \vec{B} = 0 \quad , \quad \nabla x \vec{B} = 0 \qquad (3)$$

Where the magnetic flux density $\vec{B}$ is expressed as

$$\vec{B} = \mu_0 (\vec{M} + \vec{H}). \qquad (4)$$

We assume that the magnetization is aligned with the magnetic field, but allows a dependence on the magnitude of the magnetic field as well as the temperature and it is of the form

$$\vec{M}=\frac{\vec{H}}{H}M(H,T) \qquad (5)$$

The magnetic equation of state is linearized about the magnetic field H_o and an average temperature to T_o, to become .

$$M = M_0 + \chi(H - H_0) - \alpha(T - T_0) \qquad (6)$$

where M_o is the constant mean value of the magnetization ,χ is the susceptibility and α is the pyromagnetic coefficient defined by:

$$\chi=\left(\frac{\partial M}{\partial H}\right)_{H_o} \quad \text{and} \quad \alpha=-\left(\frac{\partial M}{\partial T}\right)_{T_o}$$

The equation of state for a Boussinesq fluid is

$$\rho=\rho_0\left[1-\beta(T-T_0)\right] \qquad (7)$$

where β is the thermal expansion coefficient.

Consider a layer of steady, Boussinesq thermomagnetic fluid flow confined between the two vertical rigid plates in the x - direction and the physical quantities vary with respect to y.

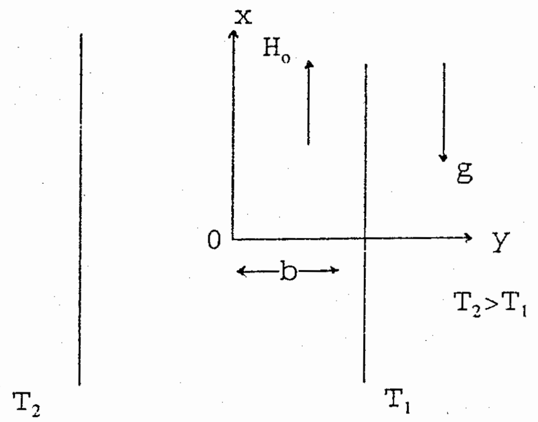

Figure (1): Physical Configration of flow

The basic equations (1) and (2), neglecting the viscous dissipation and with the above assumptions, take the form

$$\upsilon\frac{d^2u}{dy^2}+\beta g(T-T_\circ)-\frac{\mu_\circ\alpha}{\rho_0}(T-T_\circ)\frac{\partial H}{\partial x}=\frac{1}{\rho_0}\frac{\partial p'}{\partial x} \qquad (8)$$

where

$$p^{\backprime} = p + \mu_0 M_\circ H - \frac{1}{2}\mu_0 \chi (H - H_\circ)^2, \text{ and } \frac{\mu_\circ}{\rho_0 c}\left(\frac{\partial M}{\partial T}\right)\left(\frac{\partial H}{\partial x}\right) uT = K\frac{d^2T}{dy^2}$$

.... (9)

where K is the thermal diffusivity = $K/\rho_0 c$. These equations are solved using the boundary conditions

$$\left.\begin{array}{lll} u = 0 & \text{at} & y = \pm b \\ T = T_1 & \text{at} & y = b \\ T = T_2 & \text{at} & y = -b \end{array}\right\} \qquad (10)$$

The boundary conditions on the velocity represent the no-slip conditions and that on the temperature points to the fact that the plates are isothermally maintained at different temperatures T_1 and T_2 ($T_2 > T_1$).

Equations (8) and (9) using the dimensionless quantities

$$\overset{*}{x} = x/b, \quad \overset{*}{y} = y/b, \quad \overset{*}{U} = bu/K, \ \bar{\theta} = (T - T_0)/(T_1 - T_0),$$

$$p^{*\backprime} = b^2 p^{\backprime} / \rho_0 k^2$$

and for simplicity neglecting the asterisks (*) become,

$$\frac{d^2U}{dy^2} + R\theta + p = 0 \qquad (11)$$

$$\frac{d^2\theta}{dy^2} = AU\theta + \lambda A\theta \qquad (12)$$

where

$R = R_a + R_m$,

$R_a = \dfrac{\beta g(T_1 - T_2)b^3}{\upsilon K}$ is the thermal Rayleigh number

$R_m = \dfrac{\mu_o \alpha (T_1 - T_0)b^3 - \left(\dfrac{\partial H}{\partial x}\right)}{\rho_0 \, \upsilon K}$ is the magnetization Rayleigh number

$P = \dfrac{-\partial p^{\backslash}}{\partial x}$, $P^{\backslash} = p / p_r = kp / \upsilon$, $\lambda = T_0 / (T_1 - T_0)$ and

$A = \dfrac{\mu_0 \alpha M_0}{\rho_0 c}\left(-\dfrac{\partial H}{\partial x}\right)$ is the magnetization parameter.

The corresponding boundary conditions are:

$$U = 0 \quad \text{at} \quad y = \pm 1$$
$$\theta = 1 \quad \text{at} \quad y = 1 \qquad (13)$$
$$\theta = 1 + \bar{\theta} \quad \text{at} \quad y = -1$$

where

$$\bar{\theta} = (T_2 - T_1)/(T_1 - T_0) \qquad (14)$$

Thus equations (11) and (12) must be solved subject to the boundary conditions (13) to have the desired velocity and temperature profiles. Due to non linearity in equation (12), analytical solutions of these equations are difficult. However, the problem solved analytically for small "A" in [6] by utilizing a regular perturbation technique following [10-11]. To know the validity of these solutions and to find the effects of large "A" on the flow, the equations (11) and (12) are solved numerically using the quasilinearization method, also a comparison between the present solution and the corresponding numerical solution using the Rung-Kutta-Gill method [6].

NUMERICAL SOLUTIONS:

The method of quasilinearization is most valuable technique for solving non-linear two-point boundary value problems [8,9]. We have herein endeavoured to test the efficacy and reliability of this technique to find the effect of large "A" on the flow. The velocity and temperature distributions are obtained for different values of "A". We have drawn the velocity profiles for different R_m and P to see the effect of magnetization: Rayleigh number and pressure.

To facilitate the application of the method of quasilinearization [7,8] to the present problem, we rewrite Eq.s (11) and (12) in the form.

$$\left.\begin{array}{ll} \dot{X}_1 = X_2, & \dot{X}_2 = -B_0X_3 - B_1 \\ \dot{X}_3 = X_4, & \dot{X}_4 = B_2X_1X_3 + B_3X_3 \end{array}\right\} \quad (15)$$

where

$$(X_1, X_2, X_3, X_4) = (u, u', \theta, \theta')$$

and

$$B_0 = R_a + R_m$$

$$B_1 = P$$

$$B_2 = A$$

$$B_3 = \lambda A$$

and the dot denotes differentiation with respect to y .

The boundary conditions are .

$$X_1(-1) = 0, \quad X_3(-1) = 1 + \bar{\theta}, \quad X_1(1) = 0, \quad X_3(1) = 1 \quad (16)$$

Now, the quasilinearized version of Eq.s (15) can be written as the matrix equation .

$$\dot{X}^{n+1} = AX^{n+1} + B \quad (17)$$

where

$$X = (X_1, X_2, X_3, X_4)^T$$

$$A = (a_{ij}), (i, j = 1,2,3,4), B = (b_1, b_2, b_3, b_4)^T$$

$a_{11} = a_{13} = a_{14} = a_{21} = a_{22} = a_{24} = a_{31} = a_{32} = a_{33} = a_{42} = a_{44} = o,$

$a_{12}=a_{34}=1,$

$a_{23}=-\beta_0$

$a_{41}=\beta_2 x_3^n$

$a_{43} = \beta_2 X_1^n + \beta_3$

$b_1 = b_3 = o,$

$b2 = - \beta_1 ,$

$b_4=-\beta_3 x_1^n x_3^n$

and $n = o, 1, 2, \ldots\ldots$

The boundary conditions for Eq. (17) are

$x_1^{n+1}(-1)=0,\ x_3^{n+1}(-1)=1+\bar{\theta},x_1^{n+1}(1)=0,\ x_3^{n+1}(1)$ (18)

To obtain the solution of Eq. (17) starting with the assumed initial values, we generate a particular solution P with .

$$p^{n+1}(0)=\left(0,0,1+\bar{\theta},0\right)^T \quad (19)$$

and two homogeneous solutions H_2 and H_4 with

$$H_2^{n+1}(0)=\left(0100\right)^T,\ H_4^{n+1}(0)=\left(0001\right)^T \quad (20)$$

As in [10] all these solutions particular and homogeneous, are obtained by the fourth order Rung-Kutta method and then linearly combined to give the general solution of equations (17) subject to conditions (18) in the form .

$$X^{n+1}(y)= P^{n+1}(y)+C_1H_2{}^{n+1}(y)+C_3H_4{}^{n+1}(y), \qquad (21)$$

where C_1 and C_3 are evaluated using the conditions at $y = 1$. thus, one is led to the linear algebraic system .

$$\overline{H}\,\overline{C}=\overline{P} \qquad (22)$$

where the square matrix $\overline{H}$ and $\overline{C}$, $\overline{P}$, the column vectors of order 2, obtainable from Eqs. (21) satisfy the boundary conditions at $y = 1$, are given by

$$\overline{C}=\begin{bmatrix} C_1 \\ C_3 \end{bmatrix}; \qquad \overline{P}=\begin{bmatrix} 0-P_1^{n+1}(y) \\ 1-P_3^{n+1}(y) \end{bmatrix}_{y=1} \quad \text{and}$$

$$\overline{H}=\begin{bmatrix} H_{21}^{n+1} & H_{41}^{n+1} \\ H_{23}^{n+1} & H_{43}^{n+1} \end{bmatrix}_{y=1} \qquad (23)$$

In Eq.s (23). $P_i^{n+1}(y)$, $H_{2i}^{n+1}(y)$, and $H_{4i}^{n+1}(y)$, are the i^{th} components, respectively, of $P^{n+1}(y)$, $H_2^{n+1}(y)$, and $H_4^{n+1}(y)$. For the applicability of quasilinearization technique the square matrix $\overline{H}$ must be non-singular and this also turns out to be the condition for the existence of the solution of the system (22) . The solution of system (22) yields the constants C_1, C_3 which in turn, determines the next approximation to the solution of Eq.s (11)-(12) subject to the conditions

(13) if one knows the initial profile X^0. The initial approximation adheres to the initial consistency of the field variables are assumed to be.

$$\left.\begin{aligned} X_1^0(y) &= 0 \\ X_2^0(y) &= 0 \\ X_3^0(y) &= -\frac{\bar{\theta}}{2}\frac{y}{L} + \left(1 + \frac{\bar{\theta}}{2}\right) \\ X_4^0(y) &= -\frac{\bar{\theta}}{2L} \end{aligned}\right\} \qquad (24)$$

where L is the parameter that controls the duration of integration and is to be chosen in such a way that the boundary conditions at y = 1 are smoothly satisfied .

DISCUSSION AND RESULTS

The present study utilized to obtain the velocity and temperature distribution for magnetic fluids in the vertical channel .

The analysis lead to the following four cases :

(1) $\frac{\partial H}{\partial x} \prec 0$ and $(T_1 - T_0) \succ 0$

(2) $\frac{\partial H}{\partial x} \succ 0$ and $(T_1 - T_0) \succ 0$

(3) $\frac{\partial H}{\partial x} \prec 0$ and $(T_1 - T_0) \prec 0$

(4) $\frac{\partial H}{\partial x} \succ 0$ and $(T_1 - T_0) \prec 0$

The study under consideration concentrate more on the first case. R_m , which defines the thermomagnetic mechanism of convective motion may be positive or negative depending on the direction of the constant gradient of the magnetic field. It is positive when it acts in the direction of gravity and is negative when it acts opposite to gravity. The increase or decrease in the velocity will depend respectively on $R_m > 0$ or $R_m < 0$. The situation $R_m < 0$ stabilizes the system. The velocity and temperature distributions are computed for P=0 and P≠ o . The case P = 0, corresponds to the free convection and P ≠0, corresponds to mixed convection . These are discussed below .

In the case of small "A" we obtained solutions in a few iterations (2 to 4) while for large "A" we obtained solutions in more iterations (4-18), its more large specially for P=0 , $\bar{\theta}>0$ and $P\neq0$, $\bar{\theta}>0$. It is found also that in most cases "A" did not exceed 12 specially when $P\neq0$. When $R_a = -5$, $R_m = -7$, P=0, A=1 or 10, $\lambda = -2$ and $R_a = -5$, $R_m = 7$, P=0 , A= -10 , $\lambda = -2$, it is found that the solution is not convergent when 35 iterations utilized. For the purposes of comparison, the obtained results for small "A" are compared with those analytic and numerical study in [6] as depicted in the figures (2-6). It is found from the figures that there is improvement in most cases and a good agreement all solutions when A is very small. The velocity distribution obtained for P=0 and A= -1, -10 or 10 for different values of $\bar{\theta}$, Ra and Rm are depicted in figures (7-9).

We see that the velocity of flow increases with an increase in the temperature difference between the boundaries when the gradient of the magnetic field is in the direction of gravity and decreases with the increase of temperature difference between the boundaries when the gradient of the magnetic field is in the opposite direction of gravity .

From figures (10-13), we observe in the case P=0, that the velocity increases and temperature decreases with an increase in A, for

A=1 and A= 10, the temperature profiles is depicted in figures (12-13) where in both cases the temperature profiles increase with the increase of temperature difference between the boundaries .

Most values of the velocity and temperature distributions obtained in the case of mixed convection (P≠0) are similar to those of free convection (see figures (14-18)) .But in figures (16) and (18) it is found that when P=10, with $\bar{\theta}$= 1 & $\bar{\theta}$ = 0.5 and R_a = 5 , R_m=7, the order of curves of both velocity and temperature are different, which means that the parametric values chosen here do not have a negligible effect on mixed convection .

Finally, we have depicted the velocity profile for R_m and P to see the effect of magnetization Rayleigh number and pressure when A is large, these are shown in figures (19-20) .

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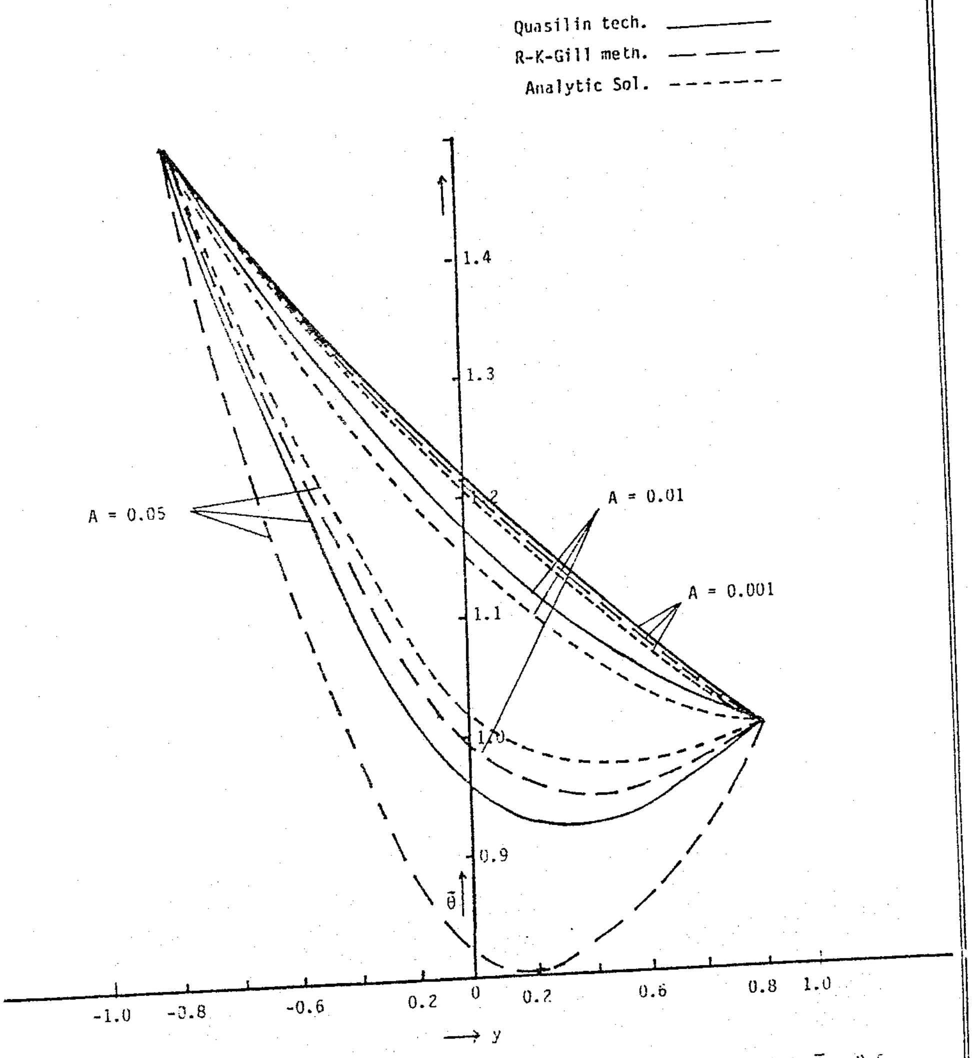

Fig.(2): Temperature profiles for Ra = 5.0, Rm = 7.0, P = -10.0, λ = 1.0, $\bar{\xi}$ = 0.5

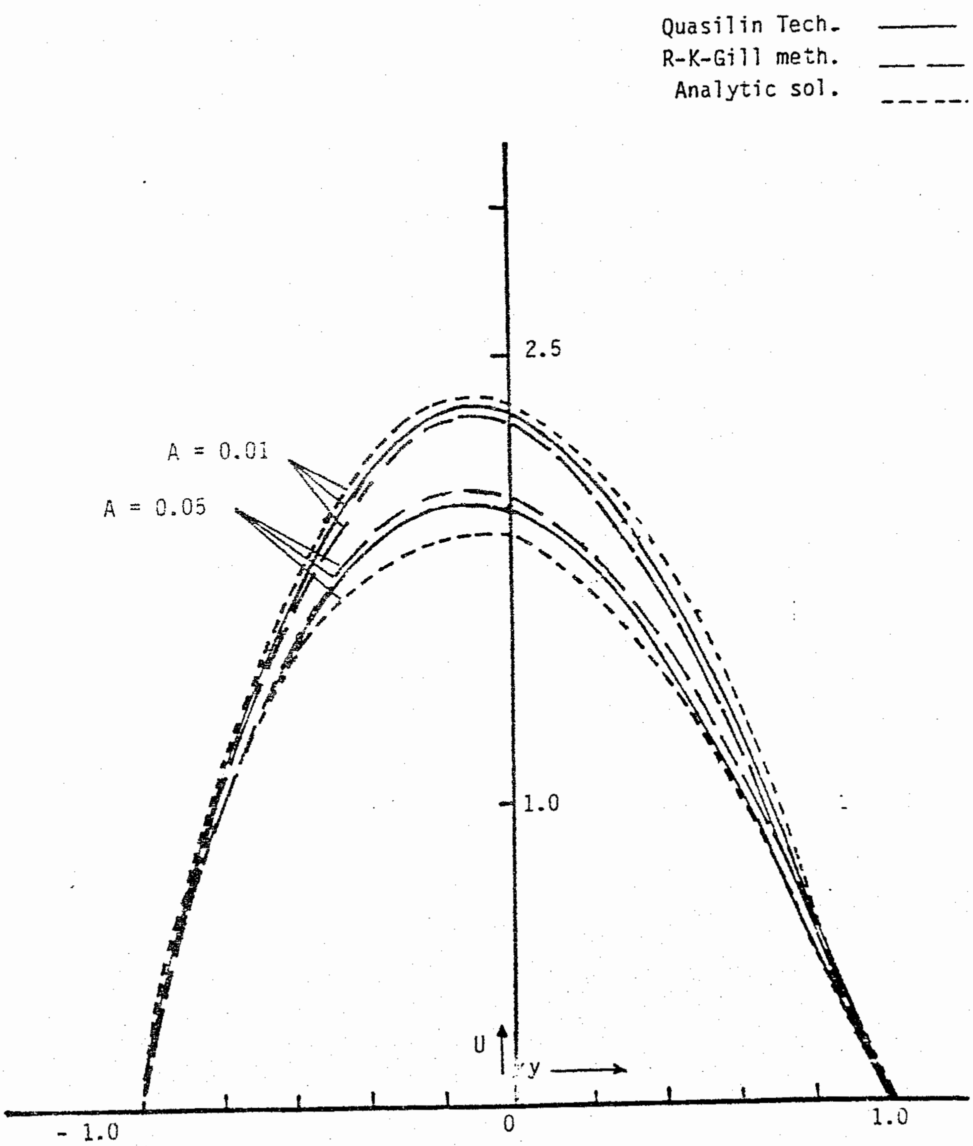

Fig.(4): Velocity profiles for Ra = 5.0, Rm = 7.0, P =- 10 , λ= 1.0, $\bar{\theta}$ = 0.5.

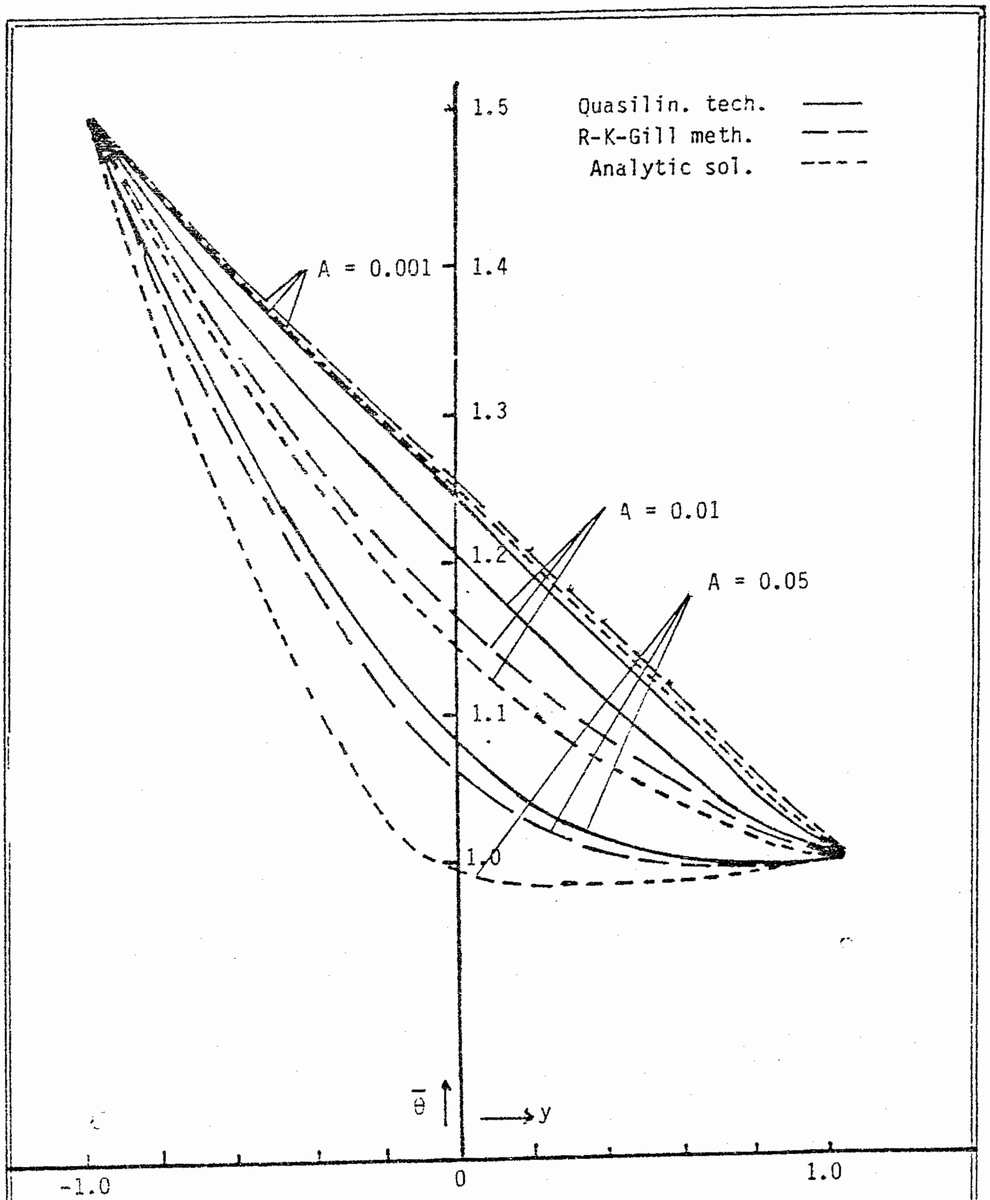

Fig.(3): Temperature profiles for Ra = 5.0, Rm = 7.0, P = 0.0, λ = 1.0 , $\bar{\theta}$ = 0.5

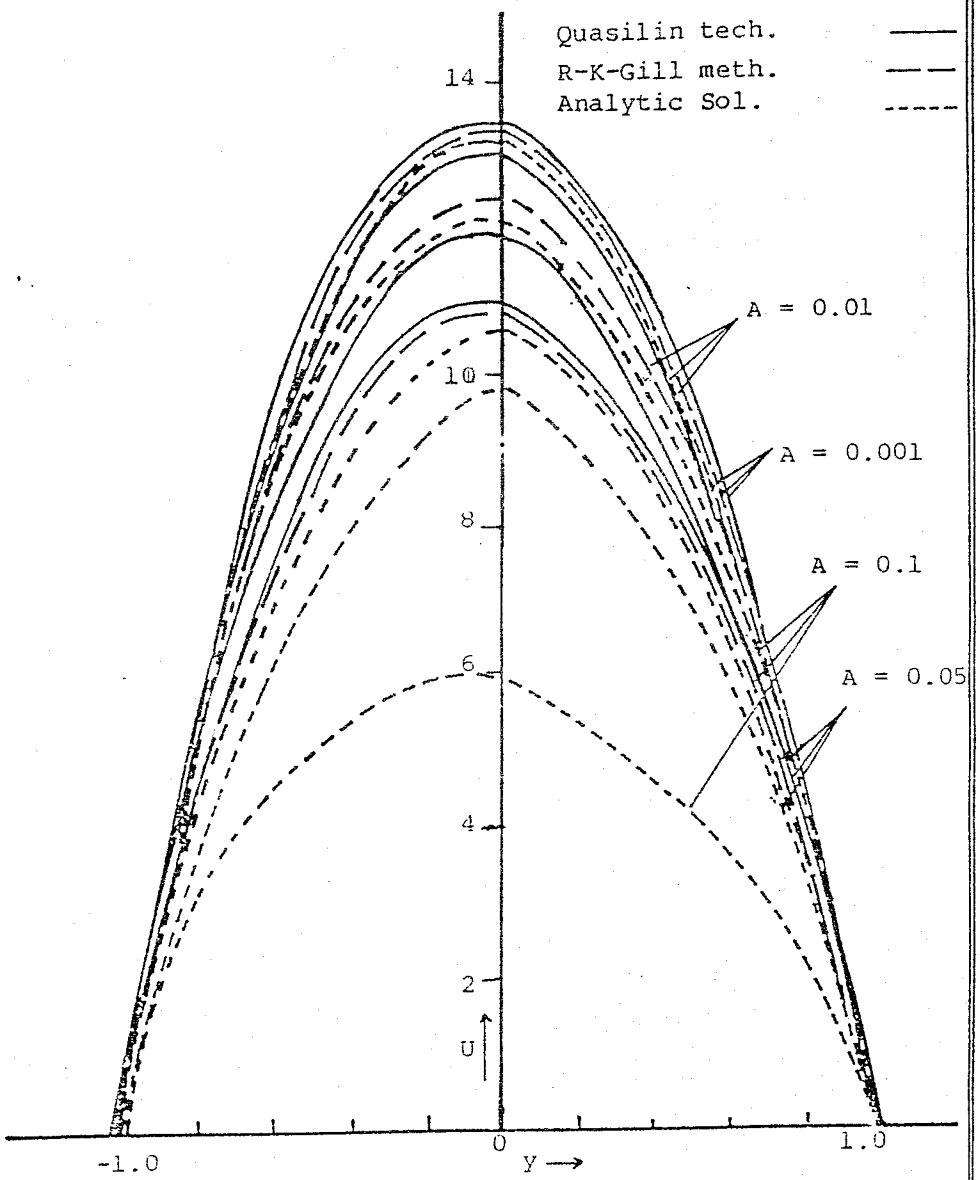

Fig.(5): Velocity profiles for Ra = 5.0, Rm = 7.0, P = 10.0 , λ = 1.0 , $\bar{\theta}$ = 1.0

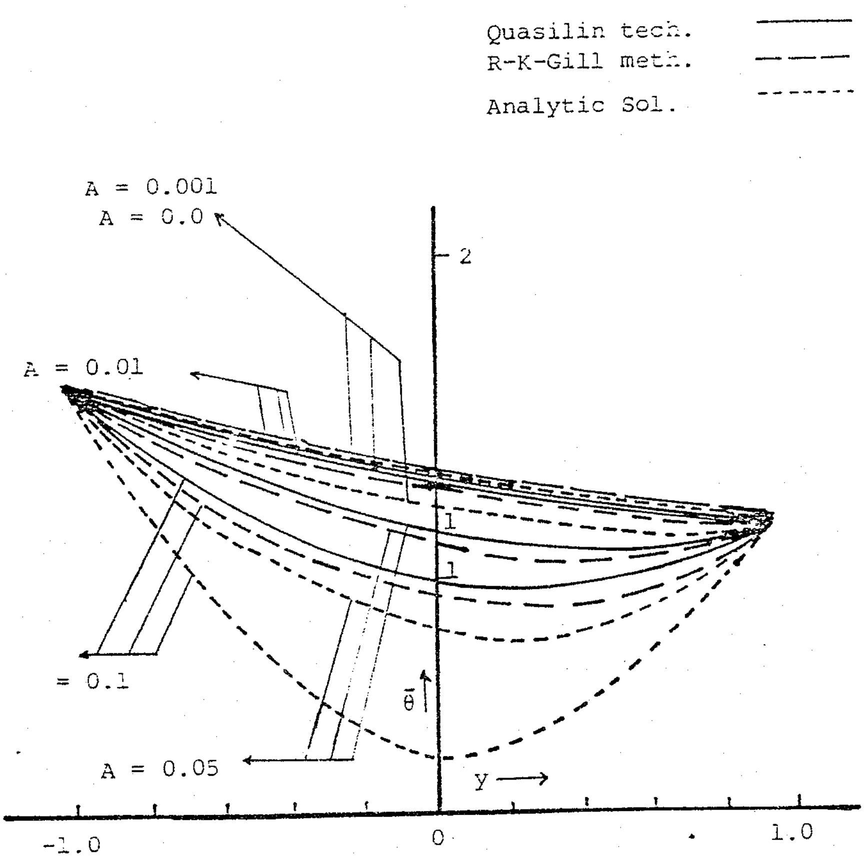

Fig. (6): Temperature profiles for Ra = 5.0, Rm = 7.0, P = 10.0, λ = 1.0, $\bar{\theta}$ = 0.5

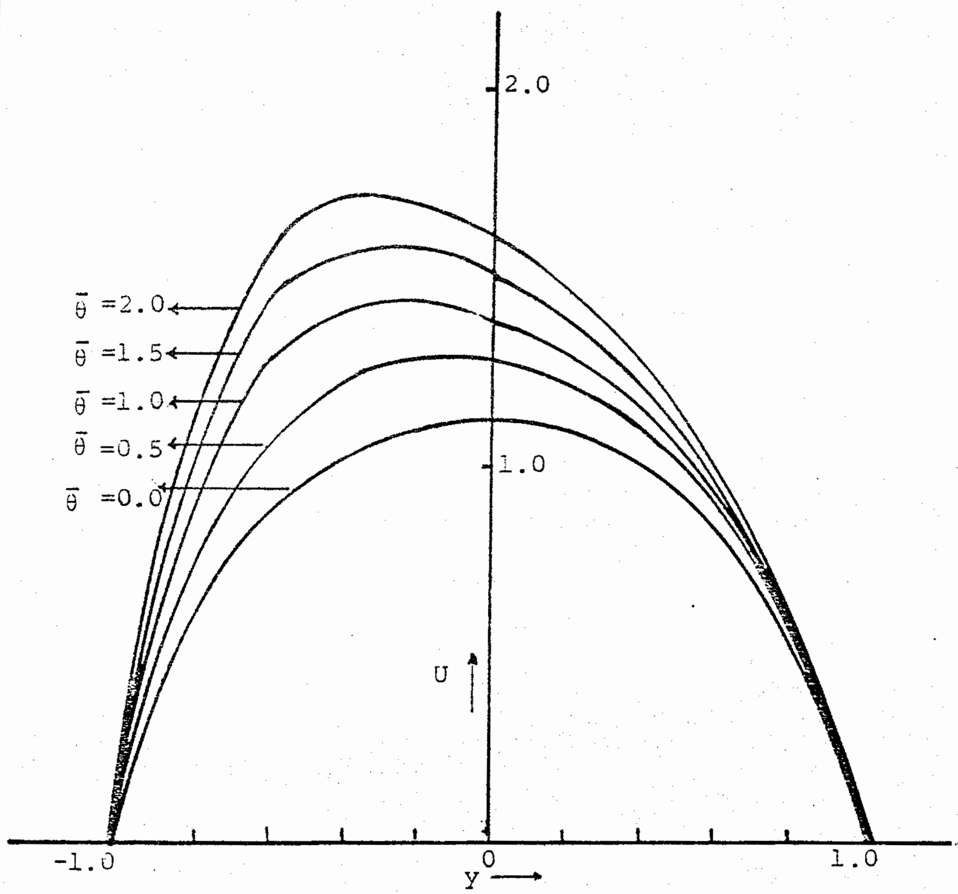

Fig.(7): Velocity profiles for A = 10.0, Rm = 7.0 , Ra = 0.5, P = 0.0, λ = 1.0

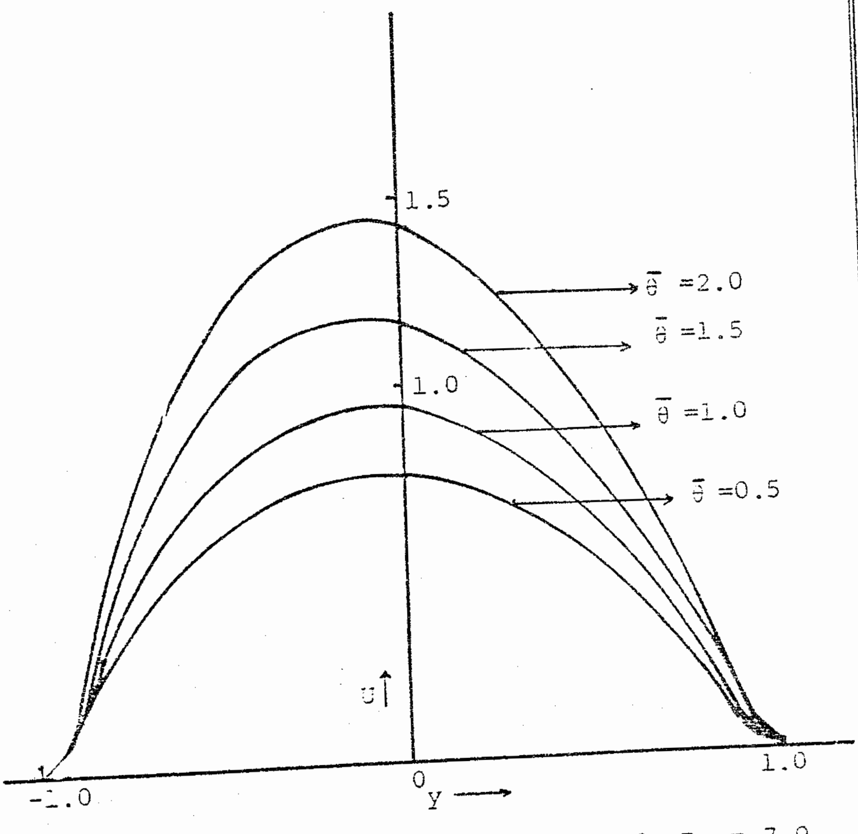

Fig.(8): Velocity profiles for A = -1.0, Rm = 7.0, Ra = -5.0, P = 0.0, λ = -2.

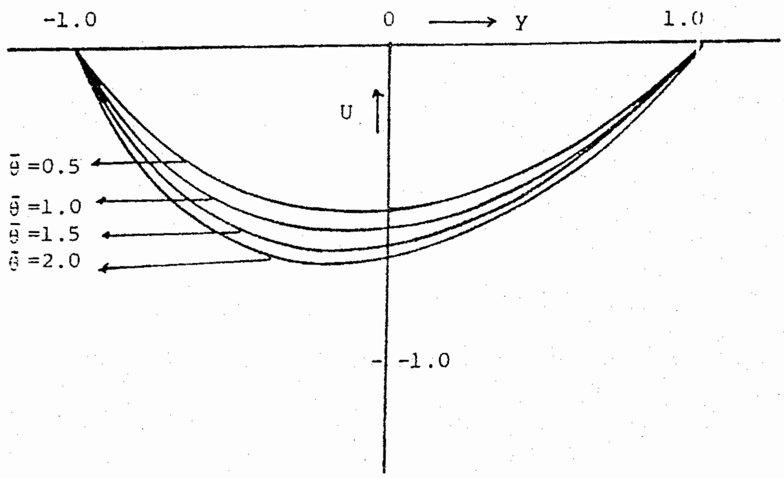

Fig.(9): Velocity profiles for A = -10.0, Rm = -7.0, Ra = 0.5, P = 0.0, λ = 1.0

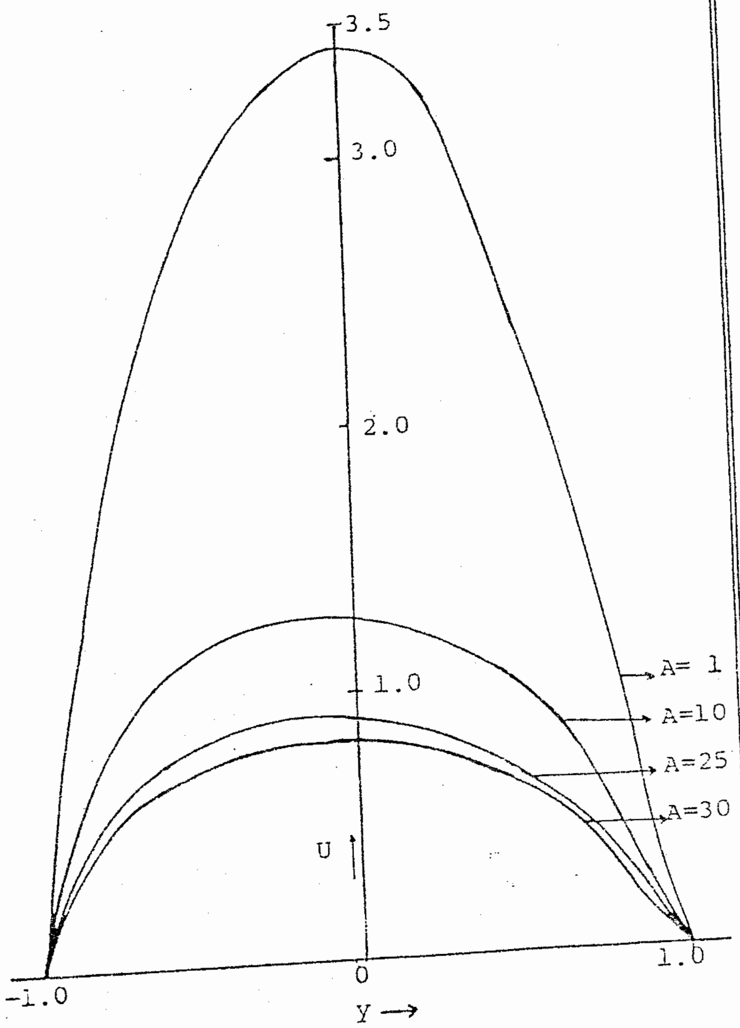

Fig. (10): Velocity profiles for Ra = 5.0, Rm = 7.0, P = 0.0, λ = 1.0, $\tilde{\theta}$ = 0.5

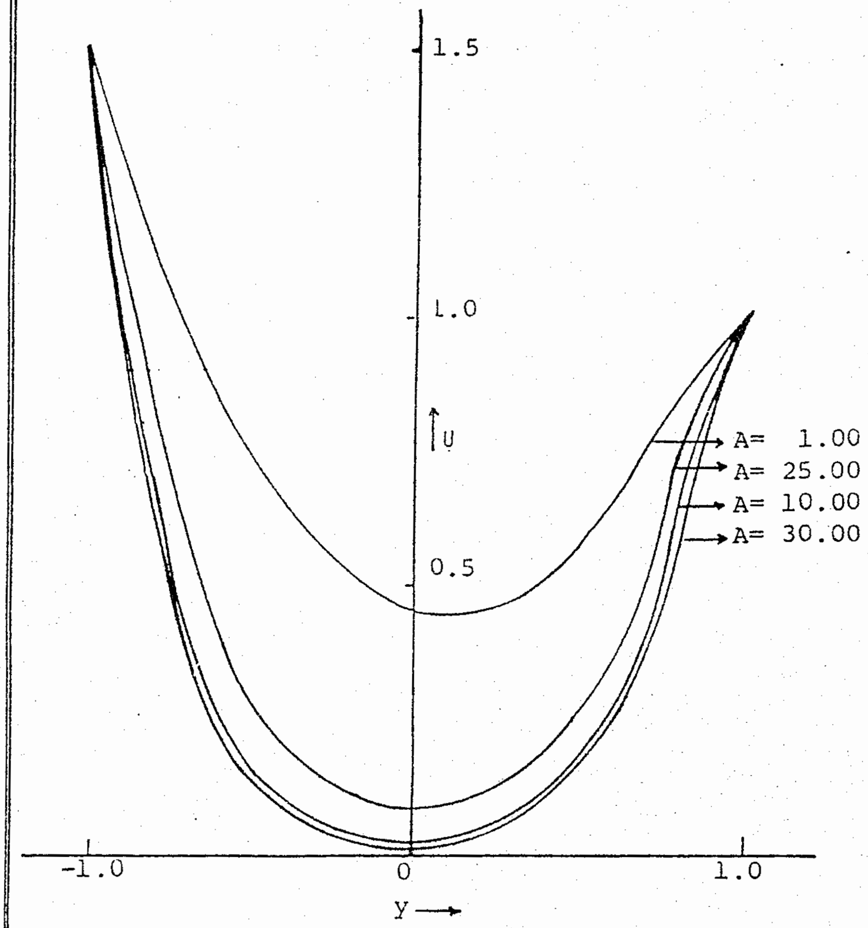

Fig.(11): Velocity profiles for Ra = 5.0, Rm = 7.0,
P = 0.0, λ = 1.0, $\bar{\theta}$ = 0.5

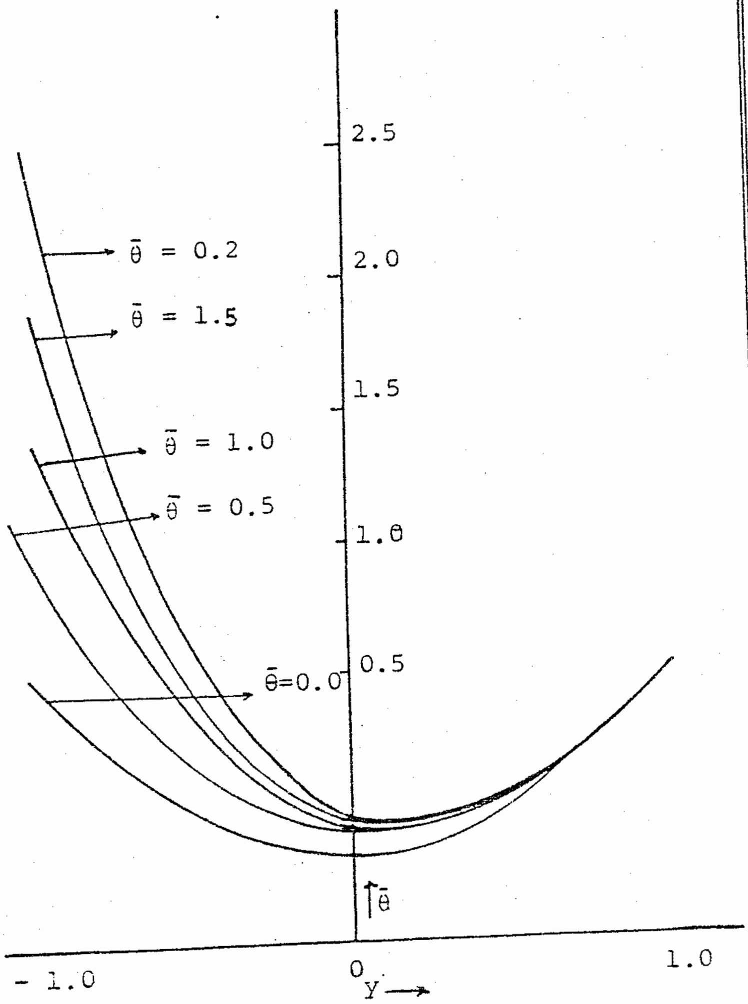

Fig.(12) : Temperature profiles for
$A = 1.0$, $Rm = 7.0$, $Ra = 5.0$,
$P = 0.0$, $\lambda = 1.0$

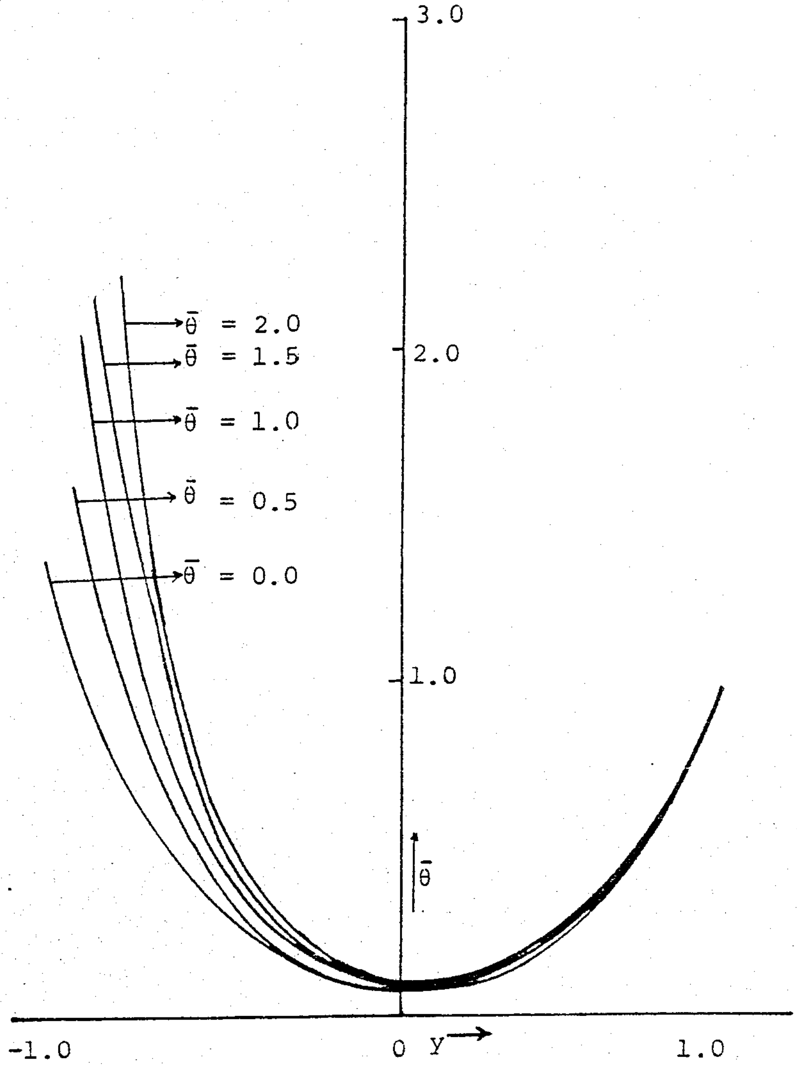

Fig.(13): Temperature profiles for
A = 10.0 , Ra = 5.0, Rm = 7.0, P = 0.0
λ = 1.0

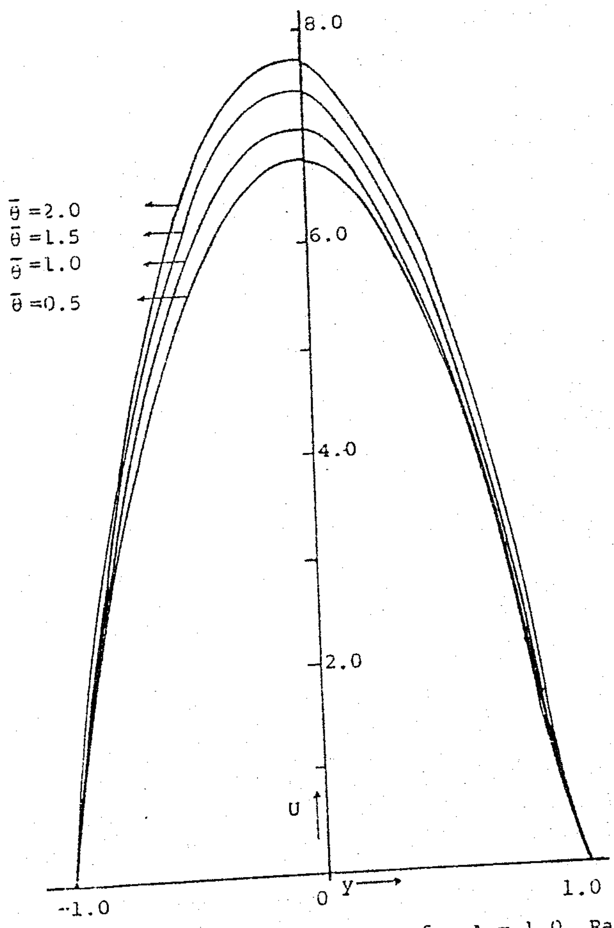

Fig.(14): Velocity profiles for A = 1.0, Ra=5.0
Rm = 7.0, P = 10.0 , λ = 1.0

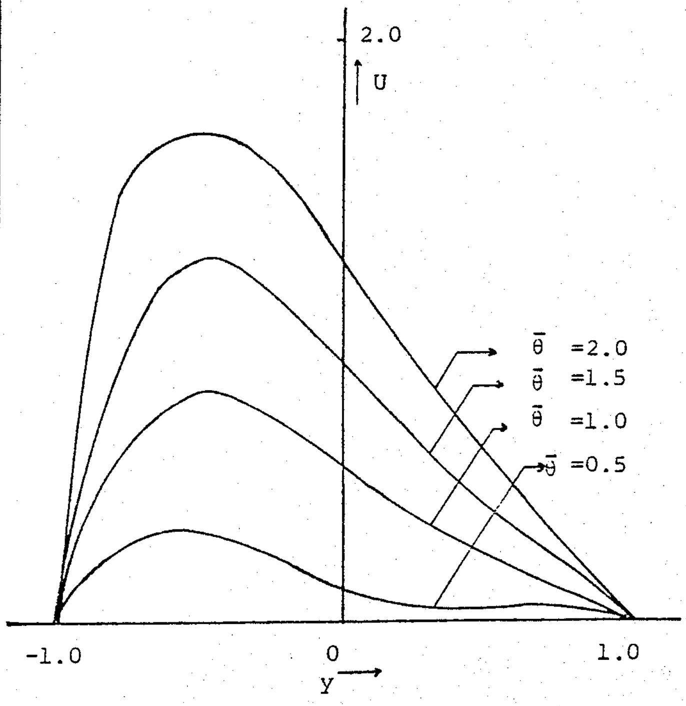

Fig. (15): Velocity profiles for A = 1.0, Ra = 5.0, Rm = 7.0, P = 10.0, λ = 1.0 .

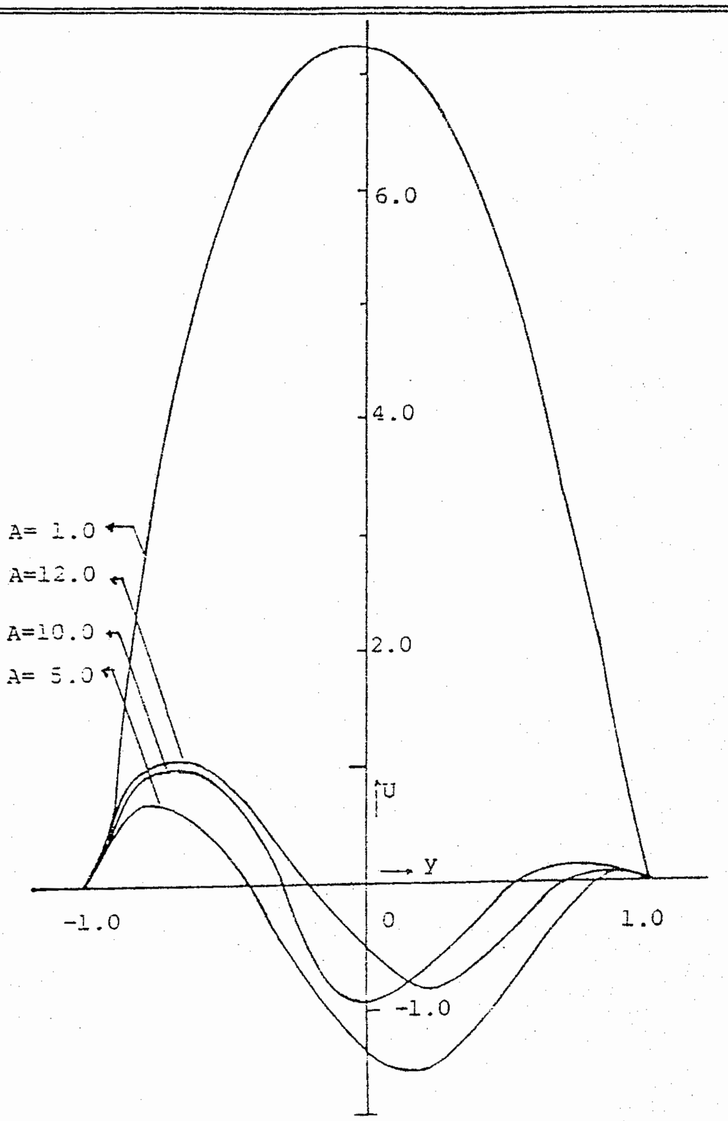

Fig.(16): Velocity profiles for Ra = 5.0, Rm = 7.0
λ = 0.1, P = 10.0 , $\bar{\theta}$ = 1.0

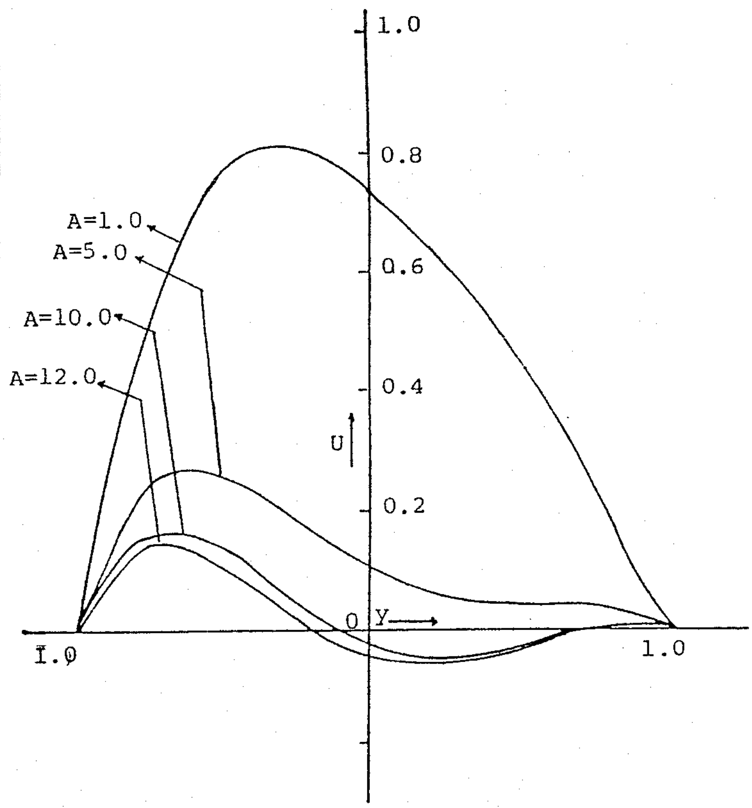

Fig.(17): Velocity profiles for Ra = 5.0
Rm = 7.0, P = -10.0, λ = 10.0,
$\bar{\theta}$ = 0.5 .

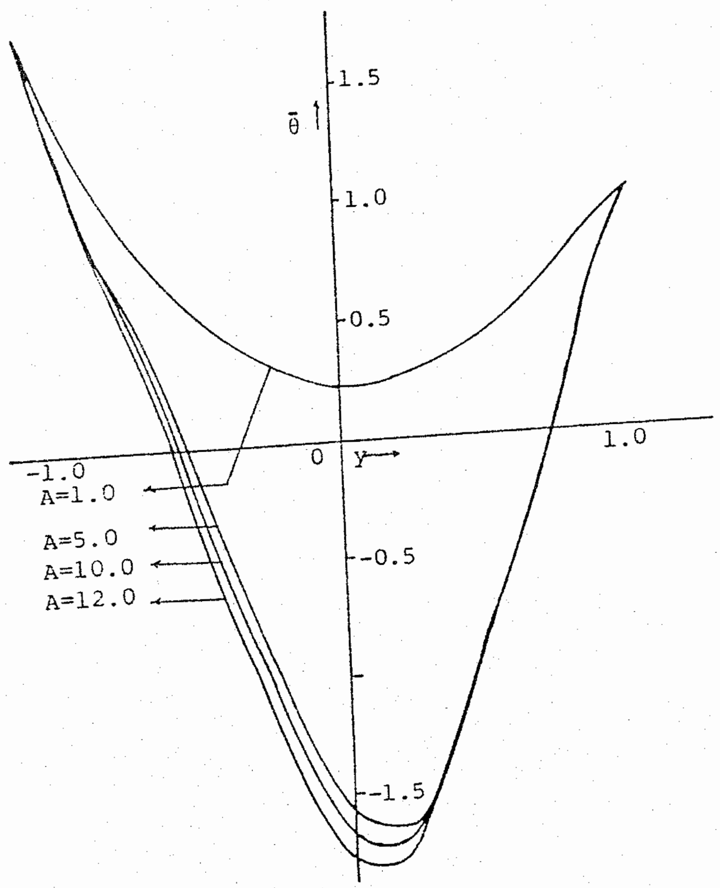

Fig.(18): Temperature profiles for Ra = 5.0, Rm = 7.0, P = 10.0, $\bar{\theta}$ = 0.5, λ = 1.0

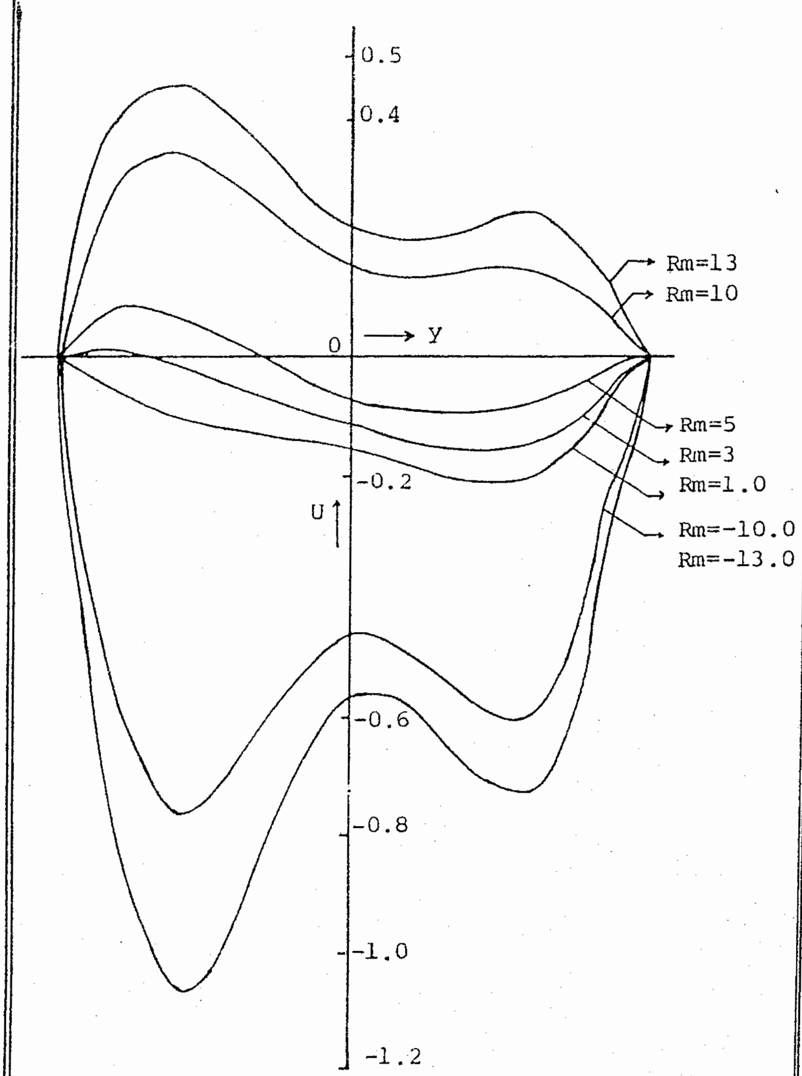

Fig. (19) : Velocity profiles for A = 10.0, Ra = 5.0, λ = 1.0, P = -10.0

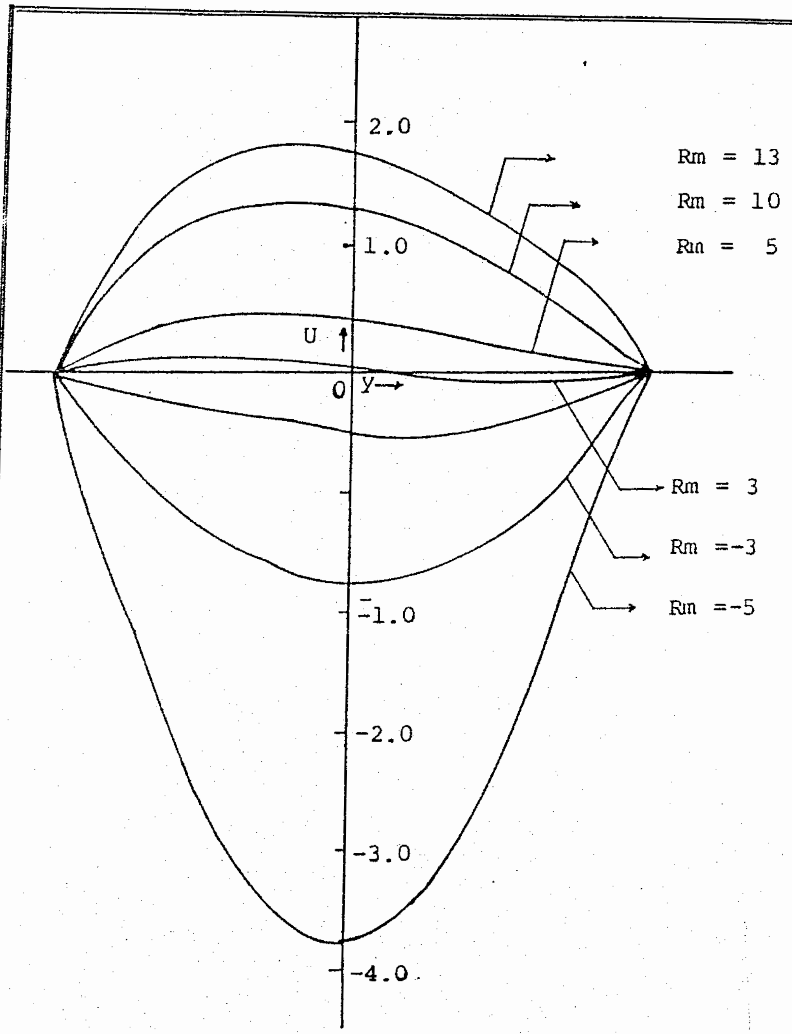

Fig. (20) : Velocity profiles for A = 1.0, Ra = 5.0, λ = 1.0, P = -10.0 , $\bar{\theta}$ = 0.5

"طريقة شبه الخطيات لمسألة الحمل الحراري المختلط خلال ثقب رأسي لمائع عالي اللا نفاذيه المغناطيسيه"

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الحمل الحرارى المختلط وغير المختلط لمسأله سريان مائع عالى اللانفاذيه المغناطيسيه خلال ثقب رأس فى وجود مجال مغناطيسى مستعرض قد تم دراستها عددياً بواسطة طريقة شبه الخطيات وقد أمكن الحصول على توزيعات السرعه والحرارة لكلا الحالتين المذكورتين عند قيم صغيرة وكبيرة للباراميتر المغناطيسى "A".

النتائج الحاليه لتوزيعات السرعة والحراره قورنت بمثيلتها لدراسة عددية وتحليلية لنفس المسأله وقد وجد تحسن فى النتائج التى تم الحصول عليها كما أنه قد نوقشت هذه النتائج تفصيلياً.