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# Effect of Air Recirculation on Compressor Performance

# تأثير إعادة تدوير المانع على أداء الضاغط

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**ملخص البحث**

تتعرض الضواغط لظاهرتي عدم الاستقرار(تدهور الأداء والتموج ) والتى تحد من تشغيلها فى حدود معينة. يستخدم النموذج الرياضي لنظام الانضغاط بإستخدام تقنية رجوع المانع بعد وضعة في صورة مبسطة للحصول على الصورة المثلى للقانون المستخدم للتحكم في تلك المشاكل والتغلب على مشاكل عدم الاستقرار . بإستخدام تلك التقنية تم تشغيل الضاغط فى حالة مستقرة.

عند تشغيل الضاغط عند سرعة حوالى 100% من السرعة التصميمية ونسبة رجوع المانع حوالى11% فإن نسبة المستفاد حوالى 13% بينما عند تشغيلة عند سرعة فى حدود ( 60-80) % ونسبة رجوع المانع حوالي(7-9) % فأنة يكون في حدود( 22-25) %. كذلك وجد أن أعلى نسبة المستفاد حوالى40% عند سرعة حوالى 90% من السرعة التصميمية ونسبة رجوع المانع حوالى8%.

## Abstract

Rotating stall and surge are two aerodynamic instabilities in compressors. The instabilities limit the flow range in which the compressor can operate "surge margin". Surge and rotating stall also restrict the performance (pressure rise) and efficiency of the compressor. Compression system modeling is very important because development a suitable control system requires a deep understanding of the transient behavior of the compressor to be controlled. The model of compression system is used for capturing the dynamics of rotating stall and surge. Recirculation technique is used to eliminate the instabilities of compression system, where bled air from downstream compressor via bleed valves re-circulates to injectors at compressor inlet. The form of the modeling takes the four-state low order model which applied to design the nonlinear controller. By using recirculation technique the compressor works in stable regime.

At the rotor speed ratio ($N/N_{design}$) of 100% and recirculation flow approximately of 11% the gain is 13%. At ratio from (60-80) % with recirculation flow approximately (7-9) % the gain in the range (22-25%).The higher gain becomes 40% at rotor speed ratio approximately 90% at recirculation flow approximately 8%.

**Keywords:** Compressor performance, Compression system modeling with re-circulation, Surge control, Improvement stall margin.

## 1. Introduction

Compressors are exposed to Rotating stall and surge which are two aerodynamic instabilities. It is clear that both phenomena can affect each other which give the simple fact that they occur in the same operating region of turbo-compressor. Rotating stall is an aerodynamic instability confined to the compressor internals. It is characterized by a distortion of the circumferential flow

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pattern. One or more regions of stagnant flow, so-called stall cells, travel around the circumference of the compressor at (10 – 90) % of the shaft speed. On the other hand, surge is an aerodynamic instability that affects the entire compression system. Surge is characterized by a limit cycle oscillation that results in large amplitude fluctuations of both the pressure and the flow rate [1].

The compression system model is used for capturing the dynamics of rotating stall and surge. Moore and Greitzer [2] introduced the model which captures the dynamics of rotating stall and surge. Behnken [3] and Yeung [4] modified the model of Moore and Greitzer [2] using air injection, by putting the nonlinear terms in their models. Fontaine et. al. [5] modified Moore and Greitzer model [2] using bleed valves, by putting the nonlinear terms in their models.

Elzahaby, et. al.[6],used air recirculation technique for the following reasons: the first is that source of air injection doesn't depend on external source, secondly, instead of loosing the bled air, it is used in air injection, the third is that to keep the environment from pollution in case of using the compressor in pressurizing natural gas, and the fourth reason is that air injection doesn't need auxiliary system and consequently it needs low cost. They modified the Moore and Greitzer model [2] using air recirculation and applied Lyapunov function to introduce linear and nonlinear optimal control to stabilize the compressor when it undergoes surge. Their simulation results show that re-circulation technique is powerful for stabilizing the compressor at higher Greitzer parameter (B), and the operating point lies in the unstable regime.

In this paper, the effect of recirculation technique on the compressor performance at different speeds is introduced, whereas the effect of air recirculation on compressor performance is discussed.

## 2. Model with recirculation

Air is re-circulated from the compressor downstream to injector at compressor inlet as shown in Figure (1). Elzahaby, et. al. [6], introduced the low order model for air recirculation in the form:

$$\dot{\Phi}=\frac{1}{l_c}\Big[\Psi_c(\Phi)+\frac{1}{4}\Psi_c''(\Phi)(a^2+b^2)-\Psi +\Big(\Psi_c(\Phi)+\frac{1}{4}\Psi_c''(\Phi)(a^2+b^2)+\Phi_b\Big)\Phi_b+\frac{1}{2}(aC_a+bC_b)\Big] \quad (1)$$

$$\dot{\Psi}=\frac{1}{4l_cB^2}(\Phi-\Phi_T), \quad (2)$$

$$\dot{a}=\frac{1}{m+\mu}\Big[-\lambda b +\Big(\Psi_c'(\Phi)+\frac{1}{8}\Psi_c'''(\Phi)(a^2+b^2)+2\Phi_b+\Phi_b\Psi_c''(\Phi)\Big)a +\Phi C_a\Big], \quad (3)$$

$$\dot{b}=\frac{1}{m+\mu}\Big[\lambda a +\Big(\Psi_c'(\Phi)+\frac{1}{8}\Psi_c'''(\Phi)(a^2+b^2)+2\Phi_b+\Phi_b\Psi_c''(\Phi)\Big)b +\Phi C_b\Big] \quad (4)$$

Where:

$\Phi,\Phi_T,\Phi_b$: The non-dimensional mean axial, throttle, and bleed flow coefficient.

$\Psi$ : The plenum pressure rise coefficient.

$\Psi'_c(\Phi),\Psi''_c(\Phi),\Psi'''(\Phi)$: The first, second, and third derivative of $\Psi$ with respect to $\Phi$.

$l_c$: The effective length of setup.

$\mu,m,\lambda$: The inertia, duct, and rotor flow parameters respectively.

$a,b,C_a,C_b$: The first spatial mode of axial, and bleed flow respectively.

$\dot{\Phi},\dot{\Psi},\dot{a},\dot{b}$: The derivative of $\Phi,\Psi,a,b$ with respect to time.

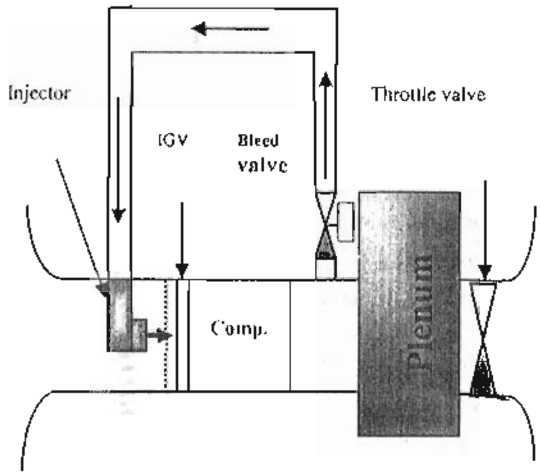


**Figure (1): Scheme of compression system with recirculation**

## 3. Control design

### 3.1. System linearization

Consider the linear system, it takes the following form:-

$$\dot{x} = Ax + Bu \tag{5}$$

Where A and B are the matrices of the linearization as the following:-

$$A = \left.\frac{\partial f(x,u)}{\partial x}\right|_{x^*}, \quad B = \left.\frac{\partial f(x,u)}{\partial u}\right|_{x^*} \tag{6}$$

- If all eigenvalues of A are strictly in the left-half complex plane, then the equilibrium point is asymptotically stable.
- If at least one eigenvalues of A are strictly in the right-half complex plane, then the equilibrium point is unstable.
- If all eigenvalues of A are strictly in the left-half complex plane, but at least one of them is on the $j\omega$ axis, then one cannot conclude anything from the linearization approximation (the equilibrium point may be stable, asymptotically stable, or unstable for the nonlinear system ).

The present work analyzes operating regions at and around equilibrium points on the characteristic $\Psi_c(\Phi)$ . At these points the A and B matrices of the linearization of (1-4) appear in a decoupled block diagonal structure, where for linearized surge subsystem, it has the form:

$$A_1 = \begin{bmatrix} \frac{\Psi_c'(\Phi^*)}{L_c} & \frac{-1}{L_c} \\ \frac{1}{4L_cB^2} & \frac{\gamma}{8L_cB^2\sqrt{\Psi_c(\Phi^*)}} \end{bmatrix}, B_1 = \begin{bmatrix} \frac{\Psi_c'(\Phi^*)}{L_c} & 0 \\ 0 & 0 \end{bmatrix} \tag{7}$$

Surge subsystem alone is controllable at all equilibrium points on the compressor characteristic.

While for linearized stall subsystem, it has the form:

$$A_2 = \begin{bmatrix} \frac{\Psi_c'(\Phi^*)}{m+\mu} & \frac{-\lambda}{m+\mu} \\ \frac{\lambda}{m+\mu} & \frac{\Psi_c'(\Phi^*)}{m+\mu} \end{bmatrix}, B_2 = \begin{bmatrix} \frac{\Phi^*}{m+\mu} & 0 \\ 0 & \frac{\Phi^*}{m+\mu} \end{bmatrix} \tag{8}$$

This system is controlled by $C_a, C_B$ .

### 3.2. Linear control design

The control that minimizes the cost functional ( $J$ ), [7]:

$$J = \int_0^\infty (x^T Q x + u^T R u)dt \tag{9}$$

Is given by linear Quadratic Regulator (LQR) design:

$$u = -R^{-1}B^T P\tilde{x} \tag{10}$$

Where P is the positive definite solution of Algebra Ricatti Equation (ARE):

$$A^T P + PA - PBR^{1}B^T P + Q = 0 \tag{11}$$

The solution P exists because [A B] is controllable for all operating points on the compressor characteristics. Thus, the LQR design results in local asymptotic stability of the surge subsystem.

### 3.3. Nonlinear control design

Nonlinear control is very important for the following reasons:-

- Improvement existing control systems when the required operation range is large.
- Analysis of hard nonlinearities.
- Dealing with model uncertainties.

- Design simplicity.

Many nonlinear control designs are introduced in [8,9]. Because of simplicity of a nonlinear controller design, similar to [5], it is applied to produce the stall/surge control with recirculation.

If nonlinear system under consideration is given by:

$$\dot{x} = f(x) + g(x) = f(x) + \sum_{i=1}^{m} g_i(x) u_i \tag{12}$$

With $x \in \Re^n$, and $u_i = [u_1, u_2, \ldots, u_m]^T \in \Re^m$.

The design model of compression system with recirculation is thus written as:

$$\dot{x} = f(x) + \left[ g_\phi(x) g_a(x) g_b(x) \right] \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{13}$$

Where:

$$f(x) = \begin{bmatrix} \frac{1}{l_c}\left[\Psi_c(\Phi) + \frac{1}{4}\Psi_c''(\Phi)(a^2+b^2) - \Psi\right] \\ \frac{1}{4 l_c B^2}(\Phi - \Phi_T) \\ \frac{1}{m+\mu}\left\{-\lambda b + \left[\Psi_c'(\Phi) + \frac{1}{8}\Psi_c'''(\Phi)(a^2+b^2)\right] a\right\} \\ \frac{1}{m+\mu}\left\{\lambda a + \left[\Psi_c'(\Phi) + \frac{1}{8}\Psi_c'''(\Phi)(a^2+b^2)\right] b\right\} \end{bmatrix}, \tag{14}$$

And

$$g_\phi(x) = \begin{bmatrix} \frac{1}{l_c}\left[\Psi_c'(\Phi)\right] \\ \frac{-1}{4 l_c B^2} \\ \frac{a}{m+\mu} \\ \frac{b}{m+\mu} \end{bmatrix}, g_a(x) = \begin{bmatrix} \frac{a}{2l_c} \\ 0 \\ \frac{\Phi}{m+\mu} \\ 0 \end{bmatrix}, g_b(x) = \begin{bmatrix} \frac{b}{2l_c} \\ 0 \\ 0 \\ \frac{\Phi}{m+\mu} \end{bmatrix}, \tag{15}$$

$$u_1 = \Phi_r, u_2 = C_a, u_3 = C_b \tag{16}$$

To increase the negativity of $\dot{V}(x)$ for the same Lyapunov function $V(x) = x^T px$ we design the simple nonlinear controller, where the region of $\dot{V}(x) < 0$ is increased as follows:

$$u_1 = -R^{-1} g_\phi^T px \tag{17}$$

$$u_2 = -k_r^{-1} g_a^T px \tag{18}$$

$$u_3 = -k_r^{-1} g_b^T px \tag{19}$$

## 4. Experimental set-up

### 4.1Test-rig

Experimental investigation is carried out on a low-pressure ratio compressor stage. The impeller of radial blower is driven by variable speed, 3-phase, AC, electrical motor (1.1kw). The impeller is of shroud type with single eye. It has five blade sets, where every set consists of one blade and one splitter. The impeller blades are of back –ward swept type. The blower volute casing works as a diffuser and ended with a discharge outlet. The parameters of test rig as in table (1) which is used in simulation.

The test rig shown in figure (2) consists of the following:

- Inlet and outlet pipes.
- Orifice meter to measure the blower air flow rate with uncertainty in the range (0.02-5) %.
- Throttle valve, which control the airflow rate through the blower.
- Water manometer to measure the static pressure head of blower Δp= (p2 -p1), where p2, and p1 (static pressure head at outlet and inlet of the blower respectively).
- Water manometer to measure the static pressure difference of the orifice with uncertainty in the range (0.016-0.4) %.
- Plenum (surge tank).
- Bleed system which consists of (pipes and valve).
- Rotameter which measure the recirculation flow rate with uncertainty in the range (0.16-2) %.
- Injection system which consists of (pipes and injectors).

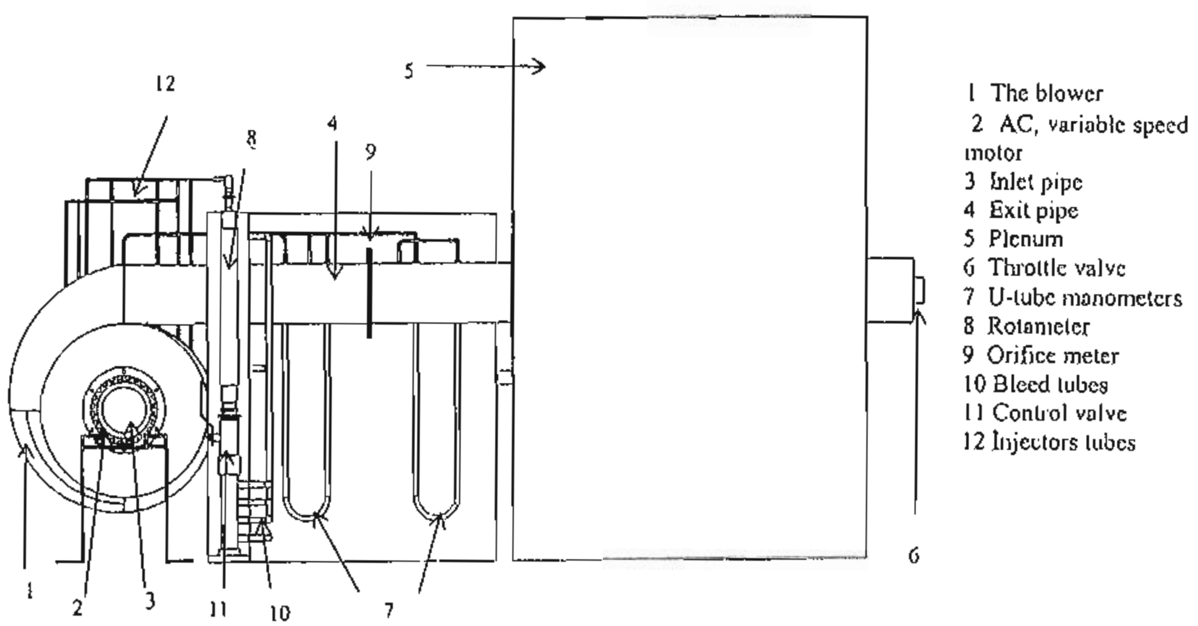


Figure (2) Scheme of test rig

Table (1) Parameters of test rig

| Element | Parameter | Value | Unit |
|---|---|---|---|
| Impeller | No. of blades | 5sets | - |
| | Inducer diameter at hub di,h | 0.11 | m |
| | Inducer diam. at shroud di | 0.13 | m |
| | Exit diam. de | 0.39 | m |
| System | Plenum volume | 0.6 | $m^3$ |
| | Compressor duct area Ac | 0.003 | $m^2$ |
| | Compressor duct length Lc | 2.4 | $m^2$ |
| | Bleed valve Size | 0.025 | m |
| | No. of bleed ports | 4 | - |
| | No. of injectors | 7 | - |
| | Injector diam. | 0.008 | m |

### 4.2. Performance of compressor (blower)

To measure the performance of compressor (blower), which the design rotor revolution is ($N_{design}$=3000rpm), the static pressure head ($\Delta p$), in (cm $H_2O$), as function of the airflow rate in ($m^3/s$), are presented at different speeds. In this paper, the performance is measured at speeds (3000, 2700, 2400, 2100, 1800, rpm). Speed control device is used to variable the speed of the electric motor.

## 5. Result and analysis

### 5.1. Measured compressor (blower) performance

The measured performance of blower is shown in Figure (3), the performance is measured at rotor speeds (3000, 2700, 2400, 2100, 1800, rpm).

As flow rate decrease the static pressure head increase till a point the pressure rise is maximum is called (stall/surge point),then with further decrease flow rate ,the static pressure head decrease. By connecting the stall/surge points we obtain Stall limit line or surge line.

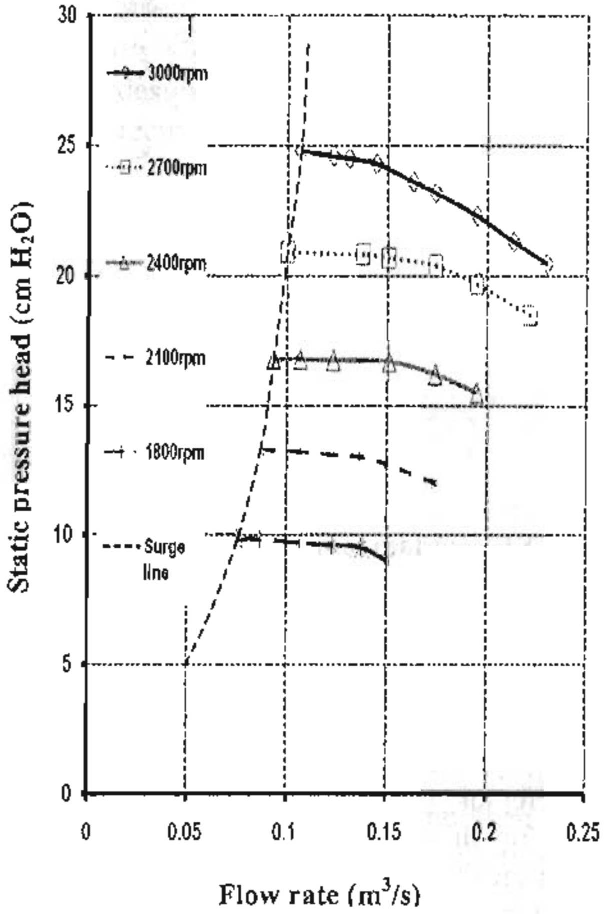


**Figure (3): The measured blower performance curves.**

The right region from surge line is the steady working regimes but the region left the surge line is the unsteady regimes.

## 5.2 Stall margin improvement

The operating range improvement in terms of the change in stalling mass flow referenced to the stall points without control is evaluated. The increase in the NASA standard stall margin (SM) as defined by Reid and Moore [10]:

$$SM = \left[ \frac{\text{Total pressuer ratio (stall)}}{\text{Total pressuer ratio(ref. )}} \frac{\text{Mass flow (ref.)}}{\text{Mass flow(stall )}} - 1 \right] \times 100$$

When the pressure head is nearly flat in the neighborhood of the stall/surge points, the stall margin improvement ($SM_i$) becomes:

$$SM_i = \left[ \frac{\text{Mass flow (ref.)}}{\text{Mass flow(stall)}} - 1 \right] \times 100$$

Where:

Mass flow (ref.): the mass flow rate at stall point without recirculation (at this point the pressure head is maximum).

Mass flow (stall): The stalled mass flow rate with recirculation.

For different speeds as shown in figure (4), the increase in the recirculation flow leads to the increase in stall margin improvement ($SM_i$). While, the increase in the rotor revolution lowers stall margin improvement ($SM_i$) for the same range of the recirculation flow.

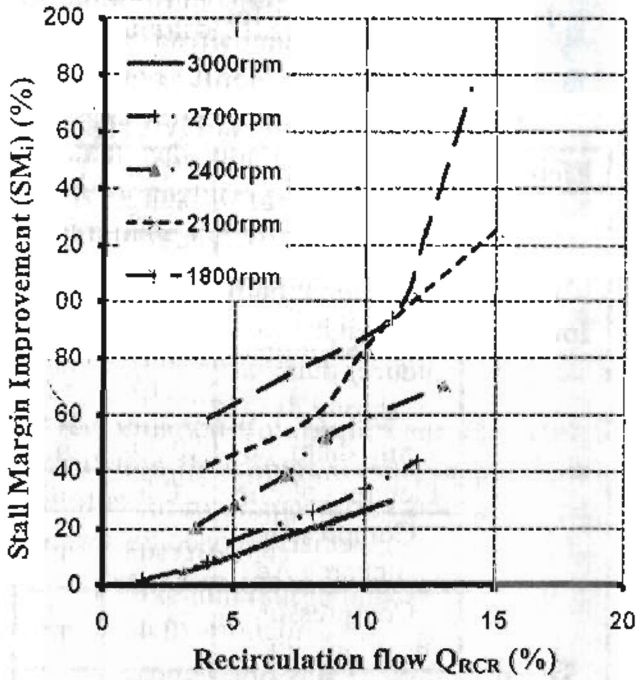


**Figure (4): Effect of recirculation flow on stall margin improvement**

For obtaining the optimum range of recirculation flow, recirculation ratio (RCR) or gain parameter can be introduced and defined as follows:

$$RCR \text{ or gain } = \left[\frac{Q_{RCR}}{Q_s - Q}\right] \times 100$$

And

$$\text{Recirculation flow percentage (\%)} = \frac{Q_{RCR}}{Q_S}$$

Where:

$Q_{RCR}$ : The recirculation mass flow rate.

$Q_S$ : The stall mass flow rate

$Q$ : The stalled mass flow rate with recirculation

From figures (5, 6), as recirculation flow percentage increases the gain decreases till a point at which the gain is approximately constant. For the rotor speed ratio equal to 100%, the gain nearly is constant at value 13% with 11% recirculation flow. On the other side, for the rotor speed ratio equal to 90% the gain is approximately constant at 40% with recirculation flow percentage8%. The rotor ratio percentages in the range (60-80) % the gain range (22-25) % are approximately constant with recirculation flow (7-9) %.

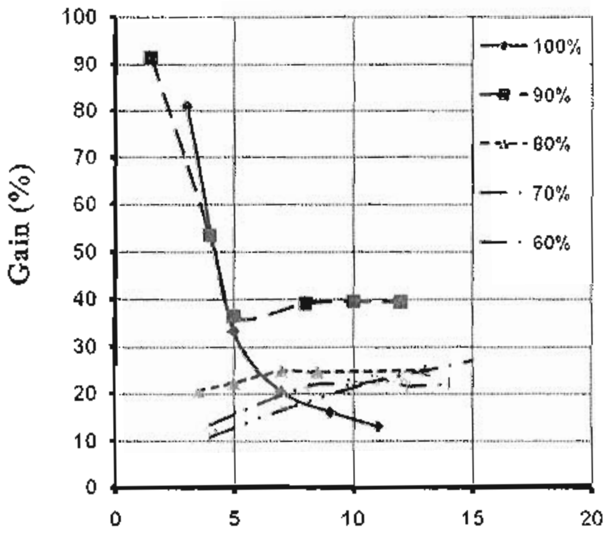


**Figure (5): The effect of recirculation flow on the obtained gain.**

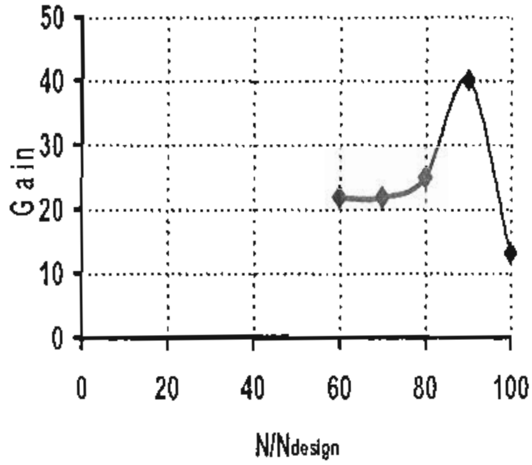


**Figure (6): Relation between rotor speed ratio and the gain**

## 5.3 Performance with recirculation

The surge line with recirculation is determined by connecting the stalled point with recirculation at the recirculation ratio which the gain to be constant as in figure (7).

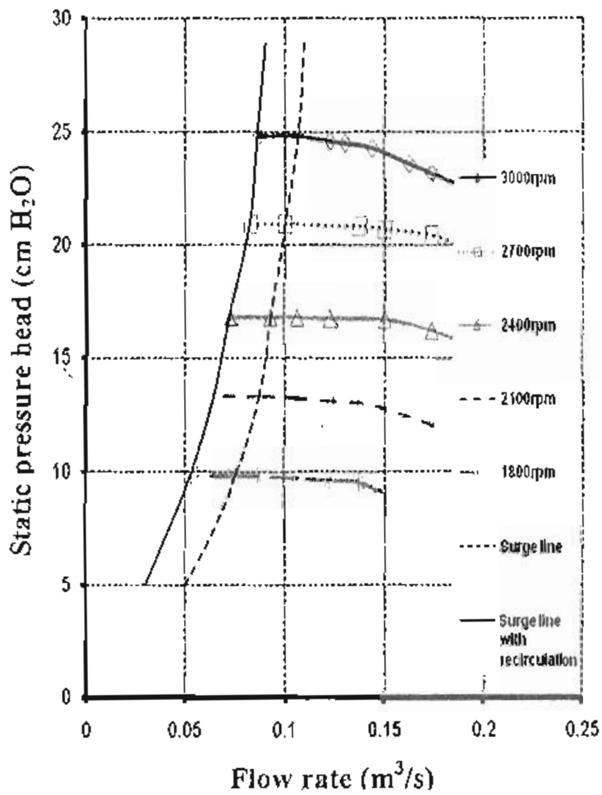


**Figure (7): Compressor (Blower) performance curves with recirculation.**

### 5.4. Simulation analysis

As example, simulation analysis for the point of maximum gain is applied. Results are presented from simulations of the compressor system when driven into surge by a drop in mass flow at throttle coefficient $\gamma = \frac{\Phi}{\sqrt{\Psi}} = 0.5$ and B=0.66 and rotor speed ratio 90% which move the throttle characteristic to the left of the top of steady state characteristic as indicated in figure (8) where surge cycle exist.

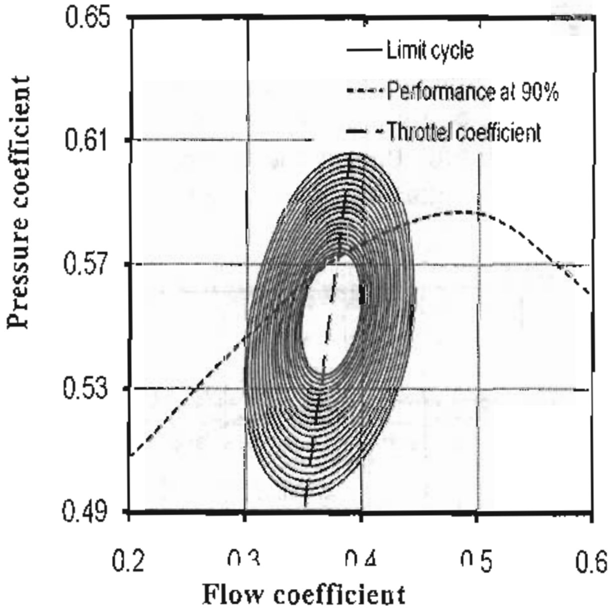


**Figure: (8) Compressor characteristics at B=0.66**

The compressor (blower) under this condition undergoes classic surge with oscillations in mass flow, and pressure rise, as shown in Figure (9). The surge has oscillation with a period 3.5ms. This is corresponding to surge frequency of about 2.8 Hz. By using the nonlinear control law, the bled air from downstream compressor via bleed valves re-circulates to injectors at compressor inlet by using 8% recirculation flow. The compressor can operate in a stable mode even to the left of the surge line where the operating point converges to equilibrium point, as in figure (10). In figure (11) mass flow drops as in the surge simulation is introduced at t = 1.7s, after that the compressor remains stable.

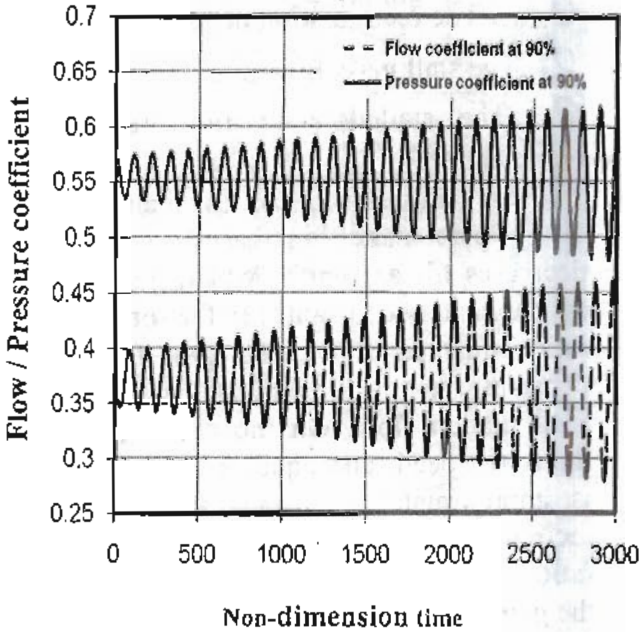


**Figure (9): Simulation of sub-surge system explain classic surge, at B=0.66**

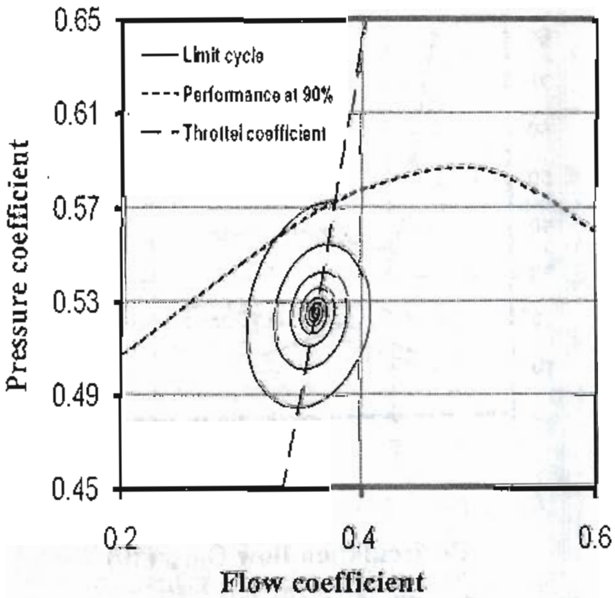


**Figure (10): Stabilization of compression system, at B=0.66 using 8% recirculation flow**

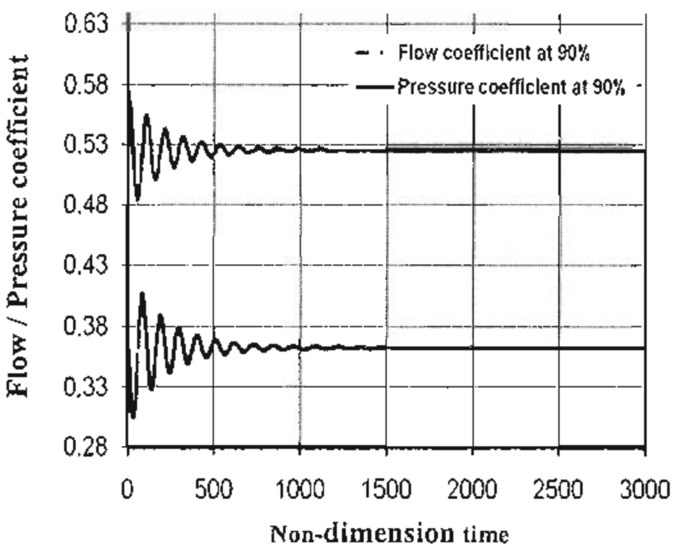


**Figure (11): Stabilization of flow and pressure coefficient, at B=0.66using 8% recirculation flow**

## 6. Conclusions and future work

By using the modelling of compression system with recirculation technique and Lyapunov function, linear and nonlinear optimal control for stabilizing the compressor which undergoes surge are obtained.

Results showed that:

1. Recirculation technique is powerful for stabilizing the compressor at different Greitzer parameter (B), where the operating point lies in the unstable regime.
2. The higher gain becomes 40% at rotor speed ratio approximately 90% and recirculation flow approximately 8%.

## References


1. Helvoirt, J., Centrifugal Compressor Surge Modeling and Identification for Control, PhD Thesis, Technische Universiteit Eindhoven, Eindhoven, 2007.
2. Moore, F. K. and Greitzer, E. M., A theory of post-stall Transients in axial compression systems, part I-Development of equations, Journal of Engineering for Gas Turbines and Power, Vol. 108, pp. 68-76, Jan. 1986.
3. Behnken, R. L., Nonlinear control and modeling of rotating stall in an axial compressor, PhD. Thesis, California Institute of Technology, Pasadena, California, 1997.
4. Yeung, S., Nonlinear control of rotating stall and surge with axisymmetric bleed valve and air injection, PhD Thesis, California Institute of Technology, Pasadena, California, 1999.
5. Fontaine, D., Liao, S., Paduano, J. and Kokotovic, V., Nonlinear control experiments on an axial flow compressor, IEEE Transactions on Control Systems Technology, Vol. 12, No. 5, pp.683 -693, Sep. 2004.
6. El-zahaby,A., El-Shenawy, A., Zyada, z., Sad Eldin, Kh. and Salem, M., Nonlinear Modeling of Compression System and Stability with Recirculation, Mansoura Engineering Journal, Vol.34, NO.1, March 2009.
7. Kalman, R. E., When is a linear control system optimal?, J. Basic Eng., Vol. 86, pp. 51–60, Mar. 1964.
8. Sontag, E. D., A universal construction of artstein's theorem on nonlinear stabilization, Syst. Control Lett., Vol. 13, pp. 117–123, 1989.
9. Sepulchre, R., Jankovic', M. and Kokotovic', P., Constructive Nonlinear Control, Springer-Verlag, New York, 1997.
10. Reid, L. and Moore, R. D. , Performance of Single-Stage Axial-Flow Transonic Compressor With Design Pressure Ratio of 1.82, NASA TP-TP-1338, 1978b.