Coarse Segmentation of Textured Images Using Variance Analysis

التجزئ البنيوى للصور باستخدام تحليل التباين

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ملخص البحث

يتناول البحث تجزئة منظر يحتوى على عده بنيات الى بنيات متمايزة مع عدم وجود معلومات مسبقة عن ١-عددالبنيات التى يحتوى عليها المنظر. ٢- خصائص وأتجاهات هذه البنيات٣- .أماكن المناطق التى تحتلها هذه البنيات . وأستخدمنا فى هذا البحث نماذج ماركوف للحقول العشوائيه الجاوسيه GMRF لنمذجه الصور وقد أبتكرنا طريقة جديدة لتقدير عدد البنيات وأماكنها.فنقسم الصوره إلى عدد من الكتل المربعه غير المتداخله مع إفتراض تجانس كل كتله منها (أى تكوينها من بنيه واحده) ويستخدم مقياس جديد للمسافات بناء على تحليل التباين بفرض تجميع الكتل المتشابهه

وبمقارنه الطريقة الجديدة للتجزى التقريبى بالطرق المعروفه والمنشوره تبين تفوق الطريقه الجديدة من حيث السرعة النسبيه وتحسين عمليه التجزئ التقريبى ومن ثم زيادة دقة تقدير بارمترات النموذج المستخدم مما ينعكس على دقه التجزئ النهائى

ABSTRACT

This paper presents a novel approach for the segmentation of a textured scene. The algorithm is image-based, no specific model is assumed for the image. Also, no *a priori* knowledge about the different texture regions, neither their number, nor their behavior is assumed. The algorithm partitions the image into small disjoint square windows, and the variance for each window data is caleulated. Then the *K-means* clustering algorithm is applied upon these windows. Together with the *K-means* algorithm, a new distance measure has been defined. This new distance measure was deduced from a statistical test known as *Bartlett's* test based on the variance of the windows data. The same statistical test has also been applied but in a different fashion to determinc the number of diffcrent texturcs in the image. The new image-based distance measure has been tested and compared to a model-based *Euclidean* distance measure, with each window modeled by a *non-causal Gaussian Markov Rondom Field* (GMRF). The results of the eomparison have shown that the new distance measure is mueh simpler and faster, while yielding to a still robust and effective segmentation.

Accepted July 30, 1997

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1. Introduction

The objective of texture segmentation is to separate an image into different regions of homogeneous behavior. It is an early processing task necessary to many applications such as: segmentation of scenes into distinct objects or regions, classification or recognition of surface materials, computation of surface shapes and orientation, motion analysis, and image data compression.

Textures are generally defined according to one of two different aspects, namely structural and statistical. From the structural point of view, textures are formed by micro structures or grains, which are primitives known as *texels*, and an accompanying grammar for the placement and reproduction of these texels. Obviously, this approach restricts to quite regular patterns, as it is highly difficult to determine either the texels, or the grammar for non-regular textures. However, from the statistical point of view, texture is defined as a pattern represented by a set of statistical measurement features. Because of the random nature of natural textures, many researchers focused on the latter approach and applied the stochastic models for the image representation. Special interest was directed towards the *Simultaneous Auto Regressive models* (*SAR*) [1-7] and the *non-causal Markov Random Field models* (*MRF*) [2- 13]. Also, the *Gibbs Distribution* (*GD*) received much attention [8].

This interest towards random field models encouraged many researchers to base their texture segmentation algorithms upon these models. *Derin and Elliott* [14], and [15] made use of the Gibbs distribution, while [5, 16-19] used the MRF models and [5, 20, 21] exploited the SAR models.

To achieve good segmentation using the above models, the segmentation should be initiated with sufficient information about the number of textures in the image, and their model parameters. This problem can be solved if an algorithm is defined to coarsely segment the image (segmentation at a low resolution). With a coarsely segmented image, we can define the number of different textures in the image, their occupied locations and therefore the model parameters. Obviously, this algorithm should be simple and fast to initiate a fine segmentation algorithm with a minimized overhead.

Fung et al. [16] proposed an algorithm that performs coarse segmentation. Their algorithm is constructed within the framework of a split and paradigm. It is based on the GMRF together with a likelihood ratio test to guide the split and merge process. The algorithm proposed by *Fung et al.* leads to good results, but is too exhaustive to be used prior to a fine segmenter.

Since many problems of image analysis are related to human visual perception, it will be quite useful to review a model of human texture perception. Texture research has been mostly driven by *Julesz'* conjecture that *the human visual system cannot discriminate between textures differing only in third and higher-order statistics* [22]. Although *Gagalowicz* [1] presented several counterexamples to the *Julezs'* conjecture, he pointed-out that these counterexamples are very difficult to generate, and that they are only faintly discriminable, and thus concluded that the *Julezs'* conjecture seems to provide a useful bound for human visual texture discrimination. As a direct consequence of this conjecture, second-order statistics are frequently used as effective features for texture classification and discrimination. The second order moment (or simply the sample variance) is obviously one of the simplest second order statistics that can be used.

This paper presents a novel approach to coarsely segment a textured seene in an unsnpervised fashion. The algorithm starts by partitioning the image into a number of relatively small disjoint windows, and the variance is ealeulated for thc window data. Becanse of their small size, most of the windows are homogeneous. We apply a well-known vector quantization algorithm known as the *K-means* elustering algorithm upon the windows to cluster them in K different classes (or textures). Together with the *K-means* algorithm, a new distance measure has been defined. This new distance measure was deduced from a statistieal test known as *Bartlett's* test [23] based on the variance of the windows data. The same statistical test has been applied but in a different fashion to determine the number of different clusters formed.

The algorithm results in a coarse segmented image, where the smoothness of the boundaries depends on the size of the used windows.

The coarse segmentation can be used in applications where it is required to estimate the texture locations, which can also be used to model the textures according to some appropriate models. The algorithm also provides a good initiation for finer segmentation algorithms with a minimized overhead.

This paper involves (1)The coarse segmentation of textured scenes in an unsupervised environment, and (2)The definition of a new distance measure based on a statistical test to be used to discriminate between textures.

The organization of the paper is as follows : Section II introduces the new distance measure. Section III defines the proposed segmentation algorithm. Section IV presents a comparison of the time eomplexity of the propused algorithm with other similar algorithms. Section V gives the segmentation and comparison results. Seetion VI is the conclusion with the recommended perspectives.

II. The New Distance Measure

To be useful, a distortion measure must be tractable, so that it can be analyzed and computed, and subjectively relevant, so that differences in distortion values can be visually discriminative. From [24], any *distance function* $d(x, y)$ can be accepted as having the qualities of a distance measure if it satisfies the following rules:

$$\begin{aligned} &d(x,x) = 0, \\ &d(x,y) > 0 \quad \text{for} \quad x \neq y, \\ &d(x,y) = d(y,x), \\ &d(x,y) + d(y,z) \geq d(x,z) \end{aligned} \qquad (1)$$

Our new distance measure is applied on scalar data. It is based on the varianees of the compared data sets. It is ofteu important to know with some degree of certainty whether the variances of two data sets are the same, i.e. they do not differ significantly; such variances are said to be *homogeneous*. We made use of a statistical test known as *Bartlett's* test [23]. This test is applied to test the humogeneity of two or more variances.

Bartlett's test is a special application of the well-known χ^2 test, in which we compare the difference between the total number of degrees of freedom times the natural logarithm of the pooled estimate of variauce and the sum, extended over all samples, of the product of the degrees of freedom and the natural logarithm of the estimate of variance. Thus, if n_i is the sample size, s_i^2 is the estimate of variance from sample i, k is the number of samples, and $\bar{s}^2$ is the pooled estimate of variance, then the *Bartlett's* test requires the calculation of

$$x^2 = \frac{1}{c}\left[v \ln \overline{s^2} - \sum_{i=0}^{k-1} \left(v_i \ln s_i^2 \right) \right] \qquad (2)$$

where

$$s_i^2 = \frac{1}{n_i - 1} \sum_{0}^{n_i - 1} (x - \bar{x}_i)^2 \quad \text{and} \quad \bar{x}_i = \frac{1}{n_i} \sum_{0}^{n_i - 1} x \qquad (3)$$

$$\nu = \sum_{i=0}^{k-1} \nu_i \qquad (4)$$

where ν_i is the number of degrees of freedom of sample i ($\nu_i = n_i - 1$)

$$\bar{s}^2 = \frac{1}{\nu} \sum_{i=0}^{k-1} \nu_i s_i^2 \qquad (5)$$

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$$c = 1 + \frac{1}{3(k-1)} \left[\sum_{i=0}^{k-1} \left(\frac{1}{v_i} \right) - \frac{1}{v} \right] \qquad (6)$$

The distance measure we are proposing will be the χ^2 test, calculated for two variances (k=2) such that

$$d(x, y) \equiv \chi^2 \Big|_{s_0^2 = s_x^2,\ s_1^2 = s_y^2,\ \nu_0 = n_x - 1,\ \nu_1 = n_y - 1,\ k=2} \qquad (7)$$

It can be easily shown that this new distance measure complies well with the required properties in (1).

III. The Segmentation Procedure

The proposed segmentation algorithm begins by partitioning the image into a number of relatively small disjoint windows. The varianee for eaeh window data is calculated. Then, we apply the *K-means algorithm*, with the new distance measure defined in (7).

As previously diseussed, the proposed algorithm works in an unsupervised environment, in which the number of textures in the image is unknown. Therefore, when applying the *K-means algorithm*, we are faced with the problem of determining the number of clusters *K* we want to be formed.

To determine the number of clusters (textures) in the image, we begin the segmentation process by assuming the image to be composed of only two textures, and apply the *K-means algorithm*. Then, we apply a statistical test based on the χ^2 test defined in (2) to determine whether the formed elusters are homogeneous or not to some *pre-defined level of significance*. If the clusters are found to be not homogeneous, we repeat the *K-means algorithm* while enabling it to form one more cluster. This procedure is repeated until we first find that two (or more) of the formed clusters are homogeneous. Finding that some of the formed clusters are homogeneous gives an indication that the number of formed clusters is greater than the actual number of textures in the image. Thus we can determine that the actual number of textures in the image is equal to the number of clusters formed at the previous stage.

The Statistical Test Used to Guide the K-Means Algorithm

A. *Determining The Homogeneity of Two Clusters*

To determine whether two clusters *i* and *j* are homogeneous or not, we are going to test the homogeneity of their variances using the *Bartlett's test* by computing the χ^2 measure defined in (2). We adopt the classical procedure for testing statistical hypotheses [23]:

1- Define the null hypothesis H_0 and the alternative hypothesis H_a

H_0 : the two variances are not significantly different (homogeneous).

H_a : the two variances are signifieantly different.

2- Calculate the test statistie (χ^2 measure) for the pre-defined number of degrees of freedom (k=2, then degrees of freedom = 1)

3- Define the significance level α for the test ($P[\chi^2 > \chi^2_\alpha] = \alpha$), and accordingly determine from the χ^2 tables the value χ^2_α for the specified number of degrees of freedom (in our case, number of degrees of freedom =1).

The significance level α is a measure of the probability that the difference between the two variances is due to chance alone.

4- Perform the test by comparing χ^2 and χ^2_α.

If the caleulated value of χ^2 exceeds χ^2_α , then, the probability that the difference between the two variances is due to ehance alone is even lower than the significance level α, and we are justified in rejecting the null hypothesis (and accepting the alternate hypothesis that the two variances are significantly different).

B. *The Proposed Algorithm*

1- Partition the image into a number of relatively small disjoint square windows, and calculate the variance for each window data.

2- Initialize the *number of clusters* to 2.

3- Calculate the *cluster centers.*

4- Apply the *K-means* algorithm using the distance measure defined in (7).

5- Test the homogeneity of the variances for the formed clusters for a pre-defined significance level. If they are not significantly different go to 7.

6- Store the clustering results (windows classes), increment the current *number of clusters* by 1 and go to 3.

7- Actual *number of clusters* = current *number of clusters* - 1, retrieve the last clustering results.

8- *Segmentation* is done.

IV. Time Complexity and Storage of the Proposed Algorithm:

In this section, we are going to compare our image-based distance measure to a model-based distance measure based on the *Gaussian Markov Random Field Model* (GMRF), from the point of view of the *required computations* and *storage*. We have chosen the GMRF model as it is extensively used in the recent literature. The 4th order GMRF has shown to be robust for the synthesis of natural textures, and therefore we are going to use it in our comparison.

As previously stated, the image is partitioned into small $M \times M$ disjoint square windows.

Let Ω denote the set of grid points of the $M \times M$ square window sueh that:

$$\Omega = \{s = (i, j) : 0 \le i, j \le M - 1\}$$

and let $\{y(s)\}$ denote a random field with $y(s)$ the field at point s representing the zero-mean gray level.

Using the 4th order GMRF, it is needed to calculate for each window, a feature vector $\Phi = \left(\hat{\Theta}, \hat{\sigma}^2, \hat{\mu}\right.$ where $\hat{\mu}$ is the sample mean, and $\hat{\Theta}$ and $\hat{\sigma}^2$ are respectively the GMRF parameters and the mean-square error and are defined as follows

Let $Q(s) = \mathrm{col}[y(s+r) + y(s-r),\ \ r \in N_s]$ and

$N_s = \{(0, 1), (1, 0), (1, 1), (-1, 1), (0, 2),$
$(2, 0), (1, 2), (2, 1), (-1, 2), (-2, 1)\}$

then the *least-square estimates* $\hat{\Theta}$ of the parameters are defined as :

$$\hat{\Theta} = \left[\sum_{s\in\Omega} Q(s)Q^T(s)\right]^{-1}\left[\sum_{s\in\Omega} Q(s)y(s)\right]$$

and the mean square error σ^2 due to the model parameters is defined as

$$\sigma^2 = \frac{1}{M^2}\sum_{s\in\Omega}\left[y(s) - \hat{\Theta}^T Q(s)\right]^2$$

A. **Required Computatiuns**

Factor	Computations Needed (*m*= multiplication, *d*= division, *a*= addition)	
	New Method	**Traditional Method**
Features Calculation	$W\,\{M^2\,a + M^2\,m + 1\,d\}$	$W\,\{(232\,M^2 + 100)\,a$ $+ (211\,M^2 + 100)\,m + 2\,d$ + 10×10 matrix inversion}
n	*0*	$W\,\{12\,a + 12\,d\}$
K-means	$I\,\{3\,a + 4\,d$ + *WK* (distance measure)} $= I\,\{3\,a + 4\,d$ $+ WK\,(3\,a + 2\,m + 1\,d)\}$	$I\,\{14\,a + 15\,d$ + *WK* (distance measure)} $= I\,\{14\,a + 15\,d$ $+ WK\,(19\,a + 10\,m)\}$
Total Com	**$\{(6\,M^2 + 6)W$ $+ (27 + 19\,WK)\,I\}\,a$**	**$\{(1287\,M^2 + 696$ + 10×10 matrix inversion) W $+ (104 + 69\,WK)\,I\}\,a$**

Table 1. Computations needed to perform segmentation

The proposed algorithm achieves a very large speedup over a model-based segmenter using a 4th order GMRF. This speedup is due to several factors. These factors are the features calculation, the features normalization, and the K-means algorithm.

$$segmentation\ time = features\ calculation + features\ normalization$$
$$+ K\text{-}means\ (using\ some\ distance\ measure)$$

If the image is partitioned into W disjoint windows, and composed of K textures, then the computations needed for the segmentation are as shown in Table 1. In the table, I represents the number of iterations needed to perform *K-means*. The last row of the table shows the total number of computations needed in terms of *additions* by considering that typically

Time of one multiplication = at least Time of 5 additions

and Time of one division = at least Time of 6 additions

$$speedup = \frac{(1287M^2 + 696)W + (104 + 69WK)I + 10 \times 10 \text{ matrix inversion}}{(6M^2 + 6)W + (27 + 55WK)I}$$

For typical values of a 512×512 image, with 6 textures, and choosing a 16×16 window, assuming the worst value of I = 20*, the speedup is found to be 88.54! And for a 256×256 image, with 4 textures,* I = 10*, the speedup is 144.56!*

B. *Required Storage*

For the proposed distance measure, we only need to store the variance while for the GMRF, we need to store 12 parameters for each window.

V. Results

In this section, we present the segmentation results performed on some images of real textures. All test images were digitized into 64×64 sample images of 256 gray levels. The digitized images have been used with no pre-processing. Each image was subdivided into non-overlapping windows . We used 16×16 and 8×8 windows. The sample variance was computed for each window. Also, the 4th order GMRF parameters were calculated, to help perform a comparison between the proposed distance measure and a model-based distance measure.

The *K-means* algorithm was applied in both, the image-based and the model-based cases. For the GMRF model, we applied the Euclidean distance measure with normalized parameters.

To determine the homogeneity of the clusters formed by the *K-means*, we apply the statistical test defined in section IV.A. We used for this test a significance level α = 5%, which equivalently corresponds to χ^2 values of $\chi^2_\alpha = 3.841$. Thus if the computed value of χ^2 for any two clusters is found to be larger than 3.841, these two clusters can be considered to be significantly different. Figures 1 through 4 show the original image followed by the segmentation performed using the GMRF parameters and finally by that based on the proposed distance measure. The first row shows the experiments carried out using 16×16 windows , while the second row shows the experiments carried out using 8× 8 windows.

The results have shown to be consistent for values of α ranging from 0.5% to 5%. But if the textures are very similar a value of α = 10% is more appropriate.

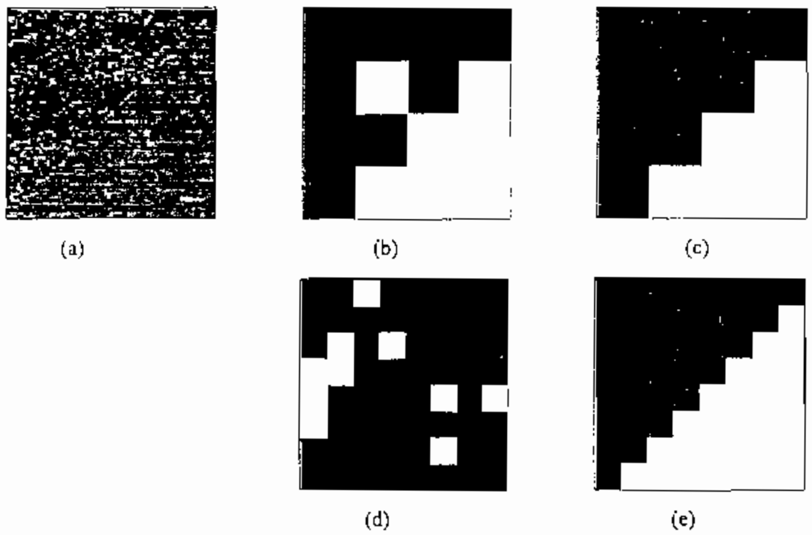

Fig. 1. (a) Original image composed of "Flowers and Tiles" (b) Coarse seg. using GMRF, Euclidean distance, and K-means, 16×16 window (c) Coarse seg, using new algorithm, 16×16 window (d) Coarse seg. using GMRF, Euclidean distance, and K-means, 8×8 window

(e) Coarse seg, using new algorithm, 8×8 window

VI. Conclusion

In this paper, we have presented a new algorithm for the coarse segmentation of a textured scene using an image-hased approach. The algorithm works in an unsupervised fashion, with no required a priori knowledge about the number of textures in the image nor their behavior. The segmentation was performed using the *K-means* algorithm with a newly defined distance measure. The algorithm was compared to algorithms using a model-based approach. The GMRF was used in the comparison. The results indicate that the proposed algorithm is much simpler and faster while using less storage and achieving better segmentation. The new algorithm also gives good results even with small block sizes (giving less coarse segmentation).The main problem that faces the proposed algorithm is the choice of the blocks sizes. If these blocks are chosen too large, regions of differing textures may be lost. If the blocks are chosen too small, discriminating among similar textures may be difficult. Also, if the textures in the image are of significantly.

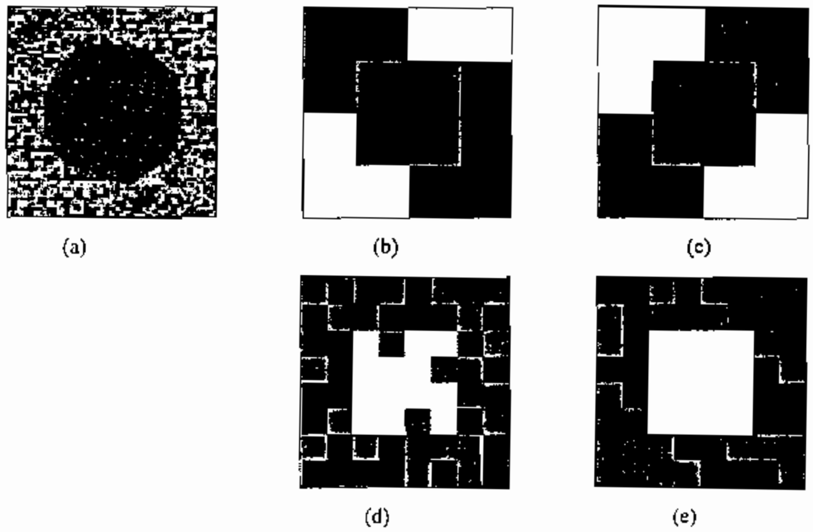

Fig. 2. (a) Original image composed of "Flowers, Water, and Sand" (b) Coarse seg. using GMRF, Euclidean distance, and K-means, 16×16 window (c) Coarse seg. using new algorithm, 16×16 window (d) Coarse seg. using GMRF, Euclidean distance, and K-means, 8×8 window

(e) Coarse seg, using new algorithm, 8×8 window

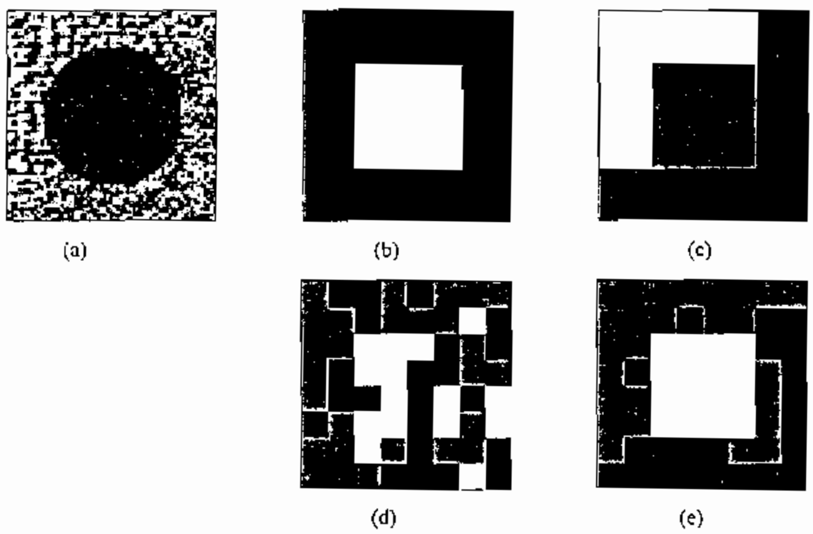

Fig. 3. (a) Original image composed of "Raffia, Grass, and Rocks" (b) Coarse seg. using GMRF, Euclidean distance, and K-means, 16×16 window (c) Coarse seg, using new algorithm, 16×16 window (d) Coarse seg. using GMRF, Euclidean distance, and K-means, 8×8 window

(e) Coarse seg, using new algorithm, 8×8 window

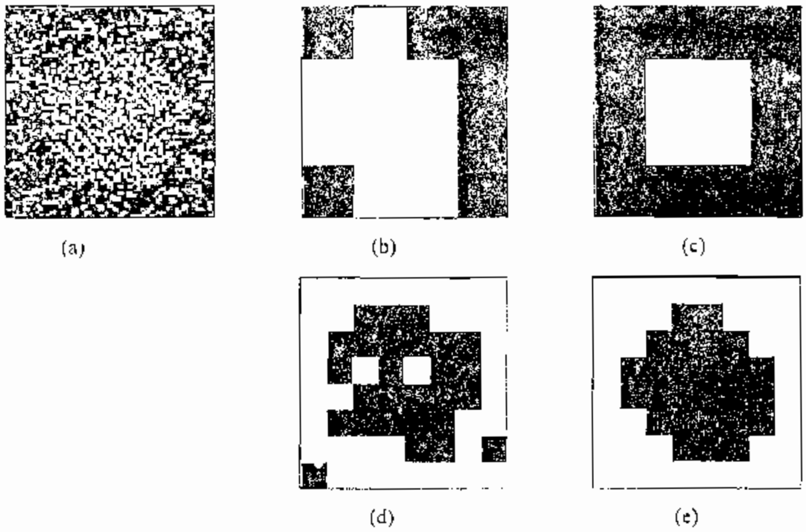

Fig 4. (a) Original image composed of "Rocks and Stucco" (b) Coarse seg. using GMRF, Euclidean distance and K-means, 16×16 window (c) Coarse seg, using new algorithm, 16×16 window (d) Coarse seg using GMRF, Euclidean distance, and K-means, 8×8 window

(e) Coarse seg, using new algorithm, 8×8 window

different degrees of smoothness or coarseness, discrimination between them is a difficult task. Therefore, some method should be applied to take into account the grain size of the different textures

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