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CATHODE REGION OF HIGH-CURRENT ARC-DISCHARGE

BY

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ABSTRACT

By analysing the processes in the cathode region of high current arcs, a closed set of equations describing this region is proposed. The Steenbeck condition of minimum cathode potential usually used in literature is found unnecessary and is replaced by the condition of balance between the flux of ions striking the cathode and the flux of atoms in the opposite direction. Effects of the presence of plasma and the random character of the field near the cathode are taken into account when calculating the electron emission from the cathode. It is found that these effects lead to a considerable increase of the emission current density.

Numerical results are compared with those of previous authors and with experimental data.

I- INTRODUCTION

Study of the cathode region in d-c gas discharge is a problem of great interest from both the theoretical and the practical points of view. Its theoretical importance is due to the many physical processes that may take place in the cathode region. Many of these processes have been the subject of intensive study during the last decades. A survey of earlier literature is given in books [1], [2]. Review of recent contributions has been presented by Eckert in [3] and [4].

The practical importance of studying the cathode region appears when constructing high-current arc-type plasma generators (plasmatrons). The current density at the cathode of these generators may reach 10^4 A/cm2 and the heat flux $10^4 - 10^5$ W/cm^2. It is not effective to reduce the current density and the heat flux to the cathode by increasing the cathode surface since a cathode spot is formed, i.e. a small part of the cathode surface which receives the main current and heating. It is desirable to be able to calculate the cathode operating conditions, or at least to have some quantitative criteria to indicate optimum conditions. The present work services this purpose.

In this paper, a mathematical model is presented which describes the cathode region of high current arcs at high pressures.

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The model can be used to compute the main parameters of the cathode: the cathode temperature "T_c", the cathode drop "V_c", the net current density on the cathode "j" and the heat flux to the cathode "q" in terms of cathode operating conditions, properties of the discharge gas, and pressure inside the discharge chamber. It has been found experimentally that these are the main factors that determines the discharge characteristics, and in the long run, the electrode life.

Similar models have been presented in some recent works [5 - 7]. In our view, the main deficiency in these models is that they make use of an additional hypothesis, namely, the Steenbeck principal of minimum cathode voltage, to determine the radius of the cathode spot. The present model does not rely on such additional hypothesis. Instead, an obvious and natural boundary condition is involved. This is the condition of balance between the flux of ions striking the cathode and the flux of neutral atoms leaving the cathode surface. A similar condition has been applied in [8] for the development of the theory of the electric probe in a dense low-temperature plasma. It will be shown that the use of this condition makes it possible to construct a model for the cathode region, comprising a closed set of equations, in the sense that the number of unknowns is equal to the number of equations, and it is therefore unnecessary to appeal to experimental data or additional hypothesis as in [5 - 7]. Also, as distinct from these works, in the present work an accurate formula of the emission current density is used which takes into account the effect introduced by the plasma on the work function of the cathode. As shown in [9], this effect may cause a considerable increase of the emission current density.

II- Mathematical Model of the Cathode Region:

Diagnosis of high current arc discharge [10] has shown that it is possible to isolate from the plasma a narrow cathode layer, where ions are generated which subsequently reach the cathode. Outside this layer, the plasma can be regarded as in local thermal equilibrium (LTE). This latter portion of the discharge will be called, conventionally, the column.

Inside the column the LTE plasma is characterized by a single parameter; the plasma temperature T_p. This is the temperature of electrons as well as heavy particles (ions and neutral atoms). Particle concentration is determined from Saha equation [1]:

$$\frac{n_e\ n_i}{N_a} = \frac{n^2}{N_a} = \frac{g_e\ g_i}{g_A}\ \frac{(2\pi mk\ T_p)^{3/2}}{h^3}\ \exp(-\ eV_i/kT_p) \quad ....(1)$$

where:

N_a = neutral particles (atom) concentration,

n_e = electron concentration,

n_i = ion concentration,

g_a, g_e, g_i are statistical weights

At equilibrium $n_e = n_i = n$.

Portion of the ions in the cathode region will be attracted towards the cathode and form the contribution "j_i" of the ions to the total current density "j" on the cathode surface. It has been shown that [6,11], when the cathode layer is thin enough, then

$$j_i \simeq \frac{1}{4}\ nq\ \bar{v}_i \quad ....(2)$$

where $\bar{v}_i$ is the mean ion velocity in the column.

Also, a portion of the electrons in the column will have sufficient energy to penetrate the potential barrier at the cathode and fall on the cathode surface. These electrons constitute the so called "reverse current" to the cathode. At low plasma temperatures and high potential barrier the reverse current is negligibly small. In the opposite case, the density of the reverse electron current is given by [6]

$$j_r = \frac{1}{4}\ qn\ \bar{v}_e\ \exp(-V_c/T_p) \quad ....(3)$$

The third contribution to the current density at the cathode is due to thermoionic emission from the cathode surface. For a hot cathode emitting in vacuum, the thermoelectron current density j_e is given by Richardson Law with correction due to Schottky effect. This is the way in which j_e is calculated in [5 - 7]. However, in high current arcs operating at high temperatures and pressures, the presence of plasma at the cathode boundary is shown to increase j_e to a large extent.

In [9] it is found that j_e is given by:

$$j_e = A\ T^2\ \exp\left(\frac{-e\phi}{k\ T} + \frac{e\Delta\phi}{k\ T} + \frac{\gamma}{2}\ \frac{T_p}{T_k}\right) \quad ....(4)$$

Here $\Delta\phi$ is the apparent reduction in the work function of the cathode due to electric field E at the cathode surface, $\Delta\phi = \sqrt{eE}$ (Schottky effect). The factor $\exp(\frac{\gamma}{2} \frac{T_p}{T_k})$ gives the increase in j_e due to presence of the plasma, which is characterized by the parameter $\gamma = \frac{e^2}{kT_p D}$ (D is the Debye length).

γ is a measure of the ratio of the mean interparticle potential energy to the mean plasma kinetic energy. For nearly ideal plasma, $\gamma \ll 1$, and the equation of state is given by:

$$p = R\, N_a\, T_p \qquad \text{.........(5)}$$

where P is the pressure inside the discharge chamber.

The emission of electrons from the hot cathode is also affected by the random and discrecte character of the distribution of charged particles near the cathode. This phenomenon has been studied by Astrotsov and Others [12 - 13] and it has been found that the average current density of the emission current can be obtained by putting in equation (4):

$$A = A_o \exp\left(\frac{.61e}{kT} \frac{e^{3/2} n^{2/5}}{\sqrt{E}}\right) \qquad \text{.........(5)}$$

where

$$A_o = \text{Sommerfeld constant} = \frac{4\pi e\, mk^2}{(2\pi k)^3}$$

and

E = average value of the electric field near

$$\text{the cathode} = \sqrt{4\left(\frac{\pi}{2}\, j_i \left(\frac{2M}{e}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} V_c^{\frac{1}{2}}}$$

For a cathode emitting in vacuum $A = A_o$, so that the exponential in (5) represents the increase in emission current due to the field near the cathode.

The net current density at the cathode is obtained from (2), (3), and (4) as:

$$j = j_e + j_i - j_r \qquad \text{.........(6)}$$

The plasma temperature T_p in the vicinity of the cathode (the generation region) is determined by setting the equation of energy balance in this region. Assuming that LTE is established close enough to the cathode, which is the case of high pressure arcs [10] , we get (Fig. 1-c)

$$j_e(V_c + 2T_c) = j_i(V_i + 2T_p) + j_r(V_c + 2T_p) + \beta j\, T_p \qquad \text{.......(7)}$$

The expression to the right is the energy carried by the emitted electrons into the generation region. The quantity $j_i V_i$ is the energy necessary for the production of ions (V_i is the ionization potential of the gas). The quantity $j_i T_p$ is the thermal energy carried by the ions. The quantity $j_r (V_c + 2T_p)$ is the energy carried by the electron capable of penetrating the cathode barrier. Finally, $\beta j T_p$ is the energy carried out of the generation region up-stream. β is a factor of order unity. For a plasma with predominant Coulomb electron scattering, $\beta = 3.2$ [14] . This is the value used in our calculations.

On formulating equation (7) we have neglected energy loss due to thermal conduction or radiation. The latter is negligibly small since the generation region is very thin. Following [6] , we can estimate thermal conduction as follows. Heat transfer due to thermal conduction is of the order of

$$R^2 x_p \frac{\partial T_p}{\partial r} \simeq x_p R T_p$$

where x_p is the coefficient of thermal conduction of the plasma. Let us compare this quantity with the energy dissipated in the cathode region-IU. For $I \sim 100$ A, $U \sim 10$ V, $T_p \sim 10^4$ K°, $x_p \sim 10^{-2}$ W/cm-deg., $R \sim 0.1$cm, we have $x_p R T_p / IU$ 10^{-2}, which justifies our assumption.

We now consider the conditions at the cathode. In order to determine the cathode potential drop V_c, we have to solve Poisson's equation subject to the boundary conditions. However, a simple expression for V_c can be obtained proceeding from the schematic picture of the equipotential surfaces and lines of force shown in Fig.(1-a). It is assumed that the cathode is of infinite extent (or at least much larger than the spot size). Under these conditions, we may assume that at distance from the cathode equal to few times the spot radius, the potential settles at a constant level U-which we shall take as reference (Fig. 1-b). This picture of the potential distribution is justified by recent results of probe diagnosis inside the discharge [16] . As follows from Fig. (1-a) the electric lines of force terminate on a surface charge density j/σ at the cathode. Hence, the cathode potential drop is given by [6] ,[15]:

$$V_c = U - \frac{1}{2\pi} \oint \frac{j}{\sigma(\bar{r}-\bar{r}')} ds \qquad \text{.........(8-a)}$$

where integration is over the spot area. The plasma conductivity is, in general, not constant. However, it is not a strong function of temperature (for a plasma in which Coulomb electron scattering predominates, $\sigma \sim T_p^{3/2}$), so that in the range of temperatures encountered in the present problem ($\sim 10^4$ °K), σ can be treated as constant, For argon, $\sigma \simeq 120$ MM/cM. Assuming that the current density over the spot is uniform, eq.(8-a) reduces to

$$V_c = U - \frac{I}{4 R\sigma} \qquad \text{.........(8)}$$

To determine the cathode temperature T_c and the heat flux to the cathode we have to solve the equations of heat conduction and energy balance at the cathode. The latter is given by

$$q = j_i(V_c + E + 2T_p - \phi) + j_r(2T_p + \phi) - j_e(2T_c + \phi) \quad /....(9)$$

The first term to the right is the energy carried by ions striking the cathode surface, the second is the energy of the "reverse electrons" and the last term is the energy of the electrons emitted from the cathode.

From the analogy between Poisson's equation and the equation of heat conduction, we see that the cathode temperature is given by an expression similar to (8);

$$T_c = T_{co} + \frac{Q}{4R\ x} \quad ....(10)$$

where T_{co} is the temperature of the far (cold) side of the cathode and $Q = \pi R^2\ q$ is the total heat to the cathode.

The last equation in the present model expresses conservation of mass at the cathode, or in other words, the balance of the flux of ions falling in the cathode layer and the flux of atoms leaving it: This balance implies that

$$N_a\ V_a = j_i/q \quad ....(11)$$

where V_a is the average thermal velocity of neutrals (atoms) on leaving the cathode surface,

$$V_a = (2\ k\ T_c/\pi M)^{1/2}$$

A condition similar to (11) has been used in [8] for the developement of the theory of electric probe in a dense low-temperature plasma. The physical basis of this condition consists in the following. Ions reaching the cathode, are neutralized by the emitted electrons. The resulting atoms are subsequently reflected backward into the generation region where thermal ionization takes place.

We now have a system of eleven equations (1) - (11) in eleven unknowns: j_e, j_r, j_i, n, N_a, T_p, V_k, U, T_c, q and R.

II- Results and Discussion:

Equations (1) - (11) constitute a closed set of equations, whose solution determines the main cathode parameters corresponding to given operating conditions. These conditions are specified by: total discharge current I, type and pressure of discharge gas and cathode cooling rate (expressed by the temperature T_{co}). The system is similar to the one proposed in [5,6], except for the equation of thermoionic emission (4) and the inclusion of equation (11) in the present system. Due to the nonlinearity of the system, exact solution is too difficult. We have applied a conventional iterative technique similar to that described in detail in [6] . However, since the present system does not contain integral or differential equations, its solution is much easier and requires less time than the system in [6] .

Table 1 gives the main parameters of the cathode region calculated for a tungsten cathode (ϕ = 4.5 eV, x = 0.7 W/cm deg). There is also given some experimental results of [15], [18] and results obtained by using Steenbeck principle [5] . The discharge gas is argon and the pressure is atmospheric. The results of [5] do not agree with experiment in some aspects: the existence of max current density and cathode temperature at a discharge current of about 100 Amp., and the fall of the plasma temperature with increasing discharge current. Such disagreement is not observed in the results of the present work. Also, as compared with our computations the Steenbeck method predicts higher cathode drop V_c, lower plasma temperature, lower current density and, consequently, larger spot radius. In our opinion, this disagreement is due to accurate calculation of electron and ion current densities in the present method.

The present computations show that with increasing total discharge current "I", increases the temperature of the hot spot "T_c", the electron-emission current density and the total current density "j" at the cathode surface (Table 1). The ratio s = j_e/j representing the contribution of the emission current to the total current is also a monotonically increasing function of I (Fig. 2). This is in agreement with the experimental results of [17] . However, the values of S computed here is larger than those measured in [17] . The latter are in the range 0.7 - 0.8.

The present results show, at least qualitatively the stability of the cathode processes with respect to small external disturabnces. For example, a sudden increase in the cathode barrier potential V_c causes an increase of the energy of the ions striking the cathode. The heat flux to the cathode "q" increases (equation (9)), and the spot temperature rises. More electrons are emitted from the cathode and, consequentlg, the total discharge current increases. However, since $\partial V_c/\partial I < 0$ (Fig. 2), the increase in I leads to a decrease in V_c so that the action of the external disturbance is compensated.

It is worth notice that aur computed results are not in complete agreement with experiment (Table 1 and Fig. 2). In our opinion, this discrepancy is in part due to the simplifications we have made on constructing the mathematical model. A more rigorous analysis should take into account the distribution of potential, temperature and current densities over the cathode surface as well as the actual geometry of the cathode. Radiation from the cathode region must be considered at very high discharge currents and large spot size.

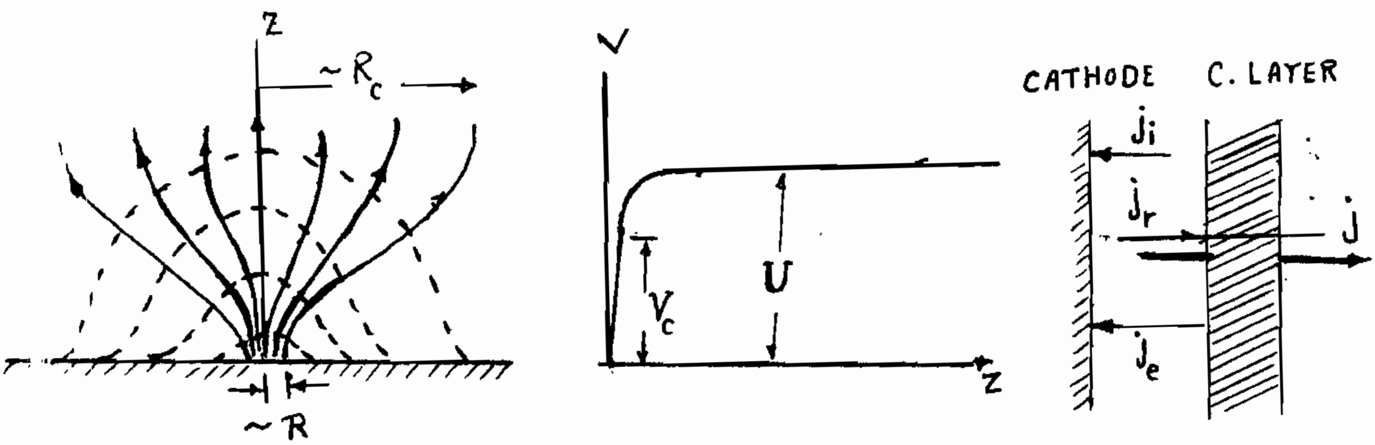
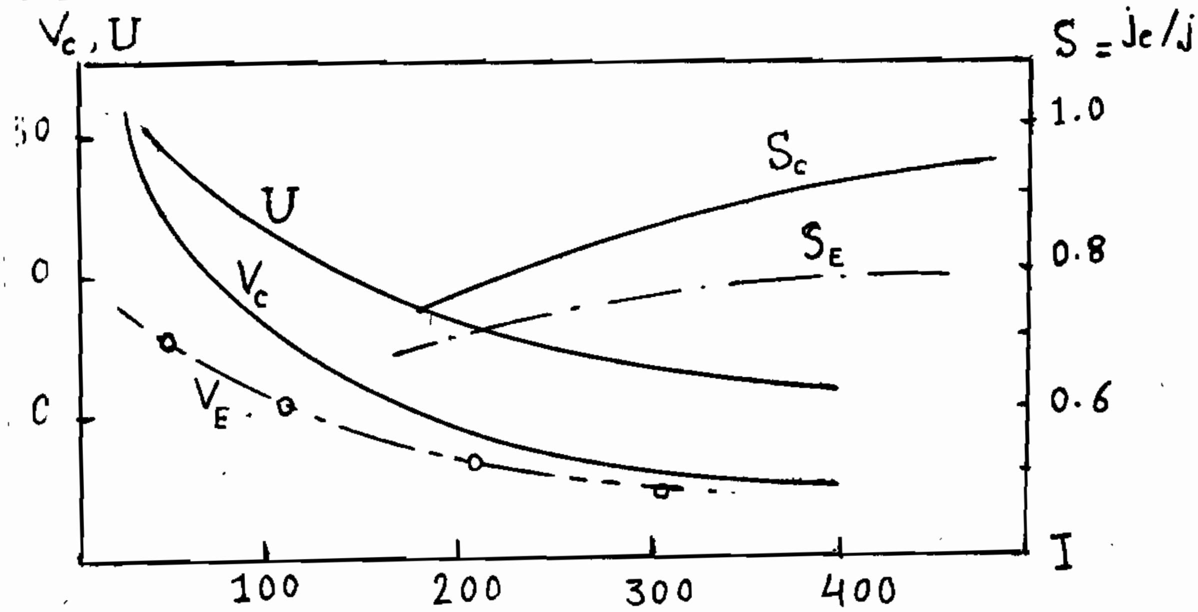


Fig. 1(a,b): Schematic of Potential and Lines of Force Near the Cathode.

1(c): Current Components.



S_E, V_E Experimental Results of 17 , 18

S_c, V_c Computed Results

Fig.(2): Variation of Cathode Drop and Ratio of Emission to Total Current Densities with Discharge Current.
(Argon at Atmospheric Pressure)

I(Amp)	V_c, V (Comp.)	V_E (Exp.)	V_S (St.)	T_c °K (Comp.)	T_c °K (St.)	J_{A/Cm^2} (Comp.)	J_{A/Cm^2} (Exp.)	J_{A/Cm^2} (St.)	T_p °K (Comp.)	T_p °K (St.)
10	18.5	15.6	28.1	3700	3670	4200	-	3190	19300	17500
100	12.3	9.2	14.2	3850	3930	6600	6400	5090	21400	18200
150	11.7	-	-	3900	-	7500	8000	4090	22500	18500
250	11.2	8.1	-	3980	-	8400	9300	-	24500	17200
300	10.3	7.5	13.2	4000	3700	8800	-	1400	25700	16100
500	9.5	7.3	13.2	4100	3560	9500	-	670	28500	16100

Exp. Experimental Results of 16

St. Results of Steenbeck method 5

Comp. Results of Pres ent Computations.

Table (1) : Cathode Region Parameters by Different Methods.

IV- CONCLUSION

A mathematical model of the cathode region of high current d.c. arcs has been developed. The model can be used to compute the main parameters of the discharge in the cathode region and to estimate the performance of the cathode under specified operating conditions.

Computations show that the results of the present model are, generaly, in agreement with available experimental results.

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