CRITERIA FOR FILLING INCLINED PIPES CARRYING NEWTONIAN OR NON-NEWTONIAN LIQUIDS

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Abstract

A simple analysis has been carried out for determining criteria for filling inclined pipes carrying liquids. The analysis resulted in criteria which are weakly dependent on Reynolds number and hence liquid properties. Experimental study on upward facing inclined pipes, carrying Newtonian and non-Newtonian liquids, has been done. Aqueous solutions of carboxy methyl cellulose in water were used as the non-Newtonian liquids. A plexi-glass open end test section was used for visualization of the liquid free surface at exit. Comparison between theoretical and both present and early experimental results was made. The comparison shows that the theoretical criteria predict safely the critical Froude number for filling inclined pipes. It has been concluded that for both horizontal and upward facing pipes the design value of Froude number is safely taken as Fr=0.725. On the other hand, for downward facing inclined pipes, the safe design value is Fr=0.92.

Introduction

Calculations of liquid flows through pipes have been derived based on single phase (i.e liquid) flow only. When the pipe runs partially filled, two phase (liquid and gas) flow may occur and the single phase-conventional-calculations can be significantly in error. As the flow rate - of a liquid - through an open ended pipe is gradually reduced, at some point, the pipe ceases to remain filled and a free surface is formed within the pipe. This surface moves closer to the pipe entrance, as the flow rate is reduced, until finally the pipe runs partially filled. This problem is met in many engineering applications such as freely discharging pipes and interconnected tanks.

Review of previous work has shown that most of the early studies have been concerned with either two phase flow [e.g. 1-8] or full-pipe Newtonian and non-Newtonian flow systems [e.g. 9]. Thus, the counter current air-water flow was studied by many authors [e.g. 1,2] while others [e.g. 3,4] were concerned with the flooding phenomenon. Also, many publications [e.g. 5,6] were concerned with flows including air-water interface. Other studies [e.g. 7,8] dealt with the characteristics of mixing layers. Studies on full pipe Newtonian and non-Newtonian fluid flow are neomerous [e.g. 9] while the characteristics of open channel flow are widely presented [e.g.10].

Studies on criteria for filling of pipes seem to be little. However, the available previous work-which was concerned with determining criteria for filling

MANUSCRIPT RECEVED FROM DR:M.H. EMPAPY AT: 8/7/1996,
ACCEPTED AT 6/8/1996, PP 35-49
ENGINEERING RESEARCH BULLETIN, VOL, 19, NO. 3, 1996
MENOUFIYA UNIVERSITY, FACULTY OF ENGINEERING,
SHEBINE EL-KOM, EGYPT. ISSN. 1110-1180

of Newtonian liquid carrying pipes - was reported by Krishnakumar et al. [11]. They carried out an experimental work and developed empirical criteria in terms of the flow Reynolds and Froude numbers for a horizontal and a vertical submerged end pipes. Similar studies for non-Newtonian liquids seem not to be available. Therefore, the present work aims at determining criteria for establishing full pipe flow of Newtonian and non-Newtonian liquids in inclined pipes.

Analysis

Figure (1.a) shows a schematic sketch of an upward facing inclined pipe at critical conditions. When the pipe begins to run partially full, the liquid free surface begins moving in the upstream direction. According to the prevailing conditions, it begins at a distance Xc upstream the open end. The corresponding critical velocity is Vc and the free surface falls a distance ϵin the normal symmetry plane at exit section. Figure (1.b) shows an experimental photograph for a near exit partial filling condition. Neglecting the surface tension effects, assuming steady flow and applying the integral continuity and energy equations between sections 1 and 2 gives [10]:

continuity eq. : $$A1 .\, Vc = A2 .\, V2 \qquad (1)$$

energy eq. :

$$D \cos \theta + \frac{V_c^2}{2g} = (D-\epsilon)\cos \theta + Xc.\sin \theta + \frac{V_2^2}{2g} + \lambda \frac{X_c}{D}\frac{V_c^2}{2g} \qquad (2)$$

Using eq. (1) to substitute for V_2 into eq. (2), dividing by D, eq. (2) gives:

$$Fr_c = \sqrt{\frac{2\bar{\epsilon} \cos \theta - 2\bar{X}_c \sin \theta}{AR^2 - 1 + \lambda \bar{X}_c}} \qquad (3)$$

where Fr_c is the critical Froude number, $Fr_c = V_c / \sqrt{gD}$ corresponding to an unwetted distance $\bar{X}_c = X_c / D$, λ is Darcy's friction coefficient and $\bar{\epsilon} = \epsilon / D$. AR is the wetted area ratio between the full and partially filled sections 1 and 2 respectively. From the geometry of exit section, see figure (1.a), AR is given by:

$$AR = \pi / (\pi - \tan^{-1}(\alpha/\beta) + \alpha\beta) \ ; \qquad \bar{\epsilon} < 0.5 \qquad (4)$$

$$, \quad AR = \pi / (\tan^{-1}\alpha/\beta + \alpha\beta) \ ; \qquad \bar{\epsilon} > 0.5 \qquad (5)$$

and $AR = 2$ for $\bar{\epsilon} = 0.5$

where $$\alpha = \sqrt{\bar{\epsilon} - \bar{\epsilon}^2} \qquad (6.a)$$

$$, \beta = 1 - 2\bar{\epsilon} \qquad \text{for } \bar{\epsilon} < 0.5 \qquad (6.b)$$

and $$\beta = 2\bar{\epsilon} - 1 \qquad \text{for } \bar{\epsilon} > 0.5 \qquad (6.c)$$

The free surface drop $\bar{\epsilon}$ is assumed to increase exponentially with $\bar{X}_c$. Also, two conditions are to be satisfied: the first is that $\bar{\epsilon} = 0$ as $\bar{X}_c = 0$ and the second is that $\bar{\epsilon} = 1$ as $\bar{X}_c = \infty$. These requirements are fulfilled by assuming :

$$\bar{\epsilon} = (1 - e^{-\bar{X}_c}) \qquad (7)$$

This function, also, gives a convex free surface which is similar to that shown in Fig.(1.b).
Similar steps with facing downward pipes will lead to similar equations with θ having a negative sign.

Theoretical results and discussion

The above simple analysis resulted in a general equation for determining criteria for partial filling of inclined pipes carrying Newtonian or non-Newtonian liquids. Inspection of eqs.(2) and (3) shows that the critical Froude number depends on the energy changes between the full and partially filled sections. Thus, the numerator of eq. (3) represents the change in potential energy while the denominator includes the increment in kinetic energy plus friction energy loss. In other words, eq.(3) shows that the critical Froude number Fr_c depends on the pipe inclination "θ", the friction coefficient "λ" and the unwetted distance $\overline{X}_c$. The effects of these parameters are discussed below.

(i) Effect of friction coefficient "λ"

In order to investigate the effects of friction coefficient, the conditions corresponding to $\overline{X}_c = 0.5$ will be considered as critical conditions. Thus, Fr_c is obtained from eqs. (3) through (7) as:

$$Fr_c = \sqrt{\frac{0.787 \cos\theta - \sin\theta}{1.49 + 0.5\,\lambda}} \tag{8}$$

It is known that λ depends on Reynolds number "Re" for laminar flow while it depends on both "Re" and pipe roughness for the turbulent regime.

The effects of "Re" may be examined by considering the smooth pipe case. In this case, the following full pipe λ-Re relationships are considered [9] :

$$\lambda = 64 / Re \quad ; \quad Re < 2000 \tag{9.a}$$

and

$$\lambda = 0.312\,(Re)^{-0.25} \quad ; \quad Re \geq 2000 \tag{9.b}$$

"Re" is a general Reynolds number , for both Newtonian and non-Newtonian power law fluids given by:

$$Re = \frac{\rho D^n V_c^{2-n}}{8^{n-1}\left(\frac{3n+1}{4n}\right)^n K} \tag{10}$$

where K is the consistency index while n is the flow behavior index for a power law fluid.

Equations (8) through (10) are used for predicting Fr_c and typical results are shown in Fig. (2). The figure shows the results for horizontal ($\theta = 0^o$), upward inclined ($\theta = 20^o$) and downward inclined ($\theta = -20^o$) pipes. It is seen, generally, that Fr_c is weakly dependent on Re for Re > 1000. Thus, for laminar flow Fr_c decreases as Re decreases while it approaches a constant value for turbulent flow.

The effect of pipe roughness is to increase the friction coefficient as it increases. This means that Fr decreases as roughness increases as can be depicted from eq.(8).

The above discussion suggests that the value of Fr_c, obtained by setting $\lambda = 0$, may be taken as a safe design criterion. This is because higher values of

Fr_c - and hence Vc-will be obtained.

(ii) Effect of inclination angle θ

As depicted from eq.(3) the potential energy change is affected by the inclination angle θ . Equations (8) through (10) are used for predictions at Re = 20000 and the results are shown in Fig.(3). The effect of inclination is discussed as follows:

(A) Upward facing inclined pipes:

Equation (8) and Fig.(3) show that for upward facing inclined pipes, Fr_c for any inclination angle is always lower than that corresponding to horizontal pipe ($\theta = 0$). Also, eq.(8) shows that there is a limiting value of the upward inclination angle, namely $\theta \simeq 38^o$, at which $Fr_c = 0$. This condition means that the pipe runs full irrespective of the flow velocity. On the other hand, for higher inclinations - i.e. $\theta > 38^o$ - eq.(8) is no more valid. This, in turn, means that eq.(2) - for free surface flow - is no more applicable and only full pipe energy balance that should be applied. Therefore, for higher inclinations - $\theta > 38^o$ - the pipe runs full at any flow velocity. Thus, the critical value $Fr_c = 0.725$ - corresponding to $\bar{X}_c = 0.5$ for the horizontal pipe with $\lambda = 0$ may be safely taken as a design value for the upward facing inclined pipes.

(B) Downward Facing inclined pipes

Equation (8) and Fig.(3) show that for downward facing inclined pipes, Fr_c at any angle is always higher than that at $\theta = 0^o$ (horizontal pipe).

Also, Fr_c has a maximum value at $\theta \simeq -60^o$. Therefore the value of Fr corresponding to $\bar{X}_c = 0.5$, $\theta = -60^o$ and $\lambda = 0$ - namely $Fr_c = 0.92$ - may be taken as a design value for these pipes.

(iii) Effect of upstream distance $\bar{X}_c$

Equation (3) shows that the changes in both the potential energy - i.e. the numerator in eq.(3) - and the kinetic energy , i.e. the denominator in eq.(3), are affected by $\bar{X}_c$. Figure (4) shows that , generally, as $\bar{X}_c$ increases Fr_c decreases. This is physically expected since lower velocities result in longer unwetted lengths. Also, the figure shows a steep decrease in Fr_c at $\bar{X}_c > 0.5$. This is mainly due to the steep increase in the kinetic energy term - i.e. the denominator - in eq.(3). It is also seen that for upward facing pipes Fr_c decreases faster than in the case of horizontal or downward facing pipes. This is attributed to the additional decrease in the potential energy term - i.e. the numerator in eq.(3) - as $\bar{X}_c$ exceeds the value of 0.5.

Experimental Work

A constant overhead circuit, shown in Fig.(5), was used in the experimental tests. It consists of a main tank, an over flow tank, the main flow pipe and a circulating pump. Control valves were used to control the flow rate and hence the flow velocity. Flow rates were measured using a collecting tank. The test section, shown in Fig.(5), was established at the end of the main pipe. It is a 5 cm dia. plexi-glass pipe of 15 cm length. The upstream end of the main pipe was connected to a rubber tube so that it can be tilted up with respect to the horizontal plane at the required inclination.

Water was used as the working Newtonian fluid while carboxymethyl cellulose (CMC) solutions in water, at different concentrations, were used as the non-Newtonian fluids. A capillary tube viscometer was used for determining the rheological parameters of the fluids. The shear stress shear rate data could be fitted to a power law formula. Under one dimensional laminar flow conditions, this formula reduces to:

$$\tau = K(\frac{du}{dy})^n \qquad (11)$$

where "τ"is the shear stress and $\frac{du}{dy}$ is the shear rate.

The parameters K and n are presented in table (1) below for the tested fluids.

TABLE (1) Power-law parameters for the tested CMC solutions

CMC concentration (p.p.m)	0(water)	5000	10000	2000 0
Flow behavior index "n"	1	0.94	0.85	0.75
Consistency index "K", Pa.s^n	0.001	0.005	0.01	0.1

The experiments were aimed at determining the critical velocity "V_c" which keeps the liquid free surface at a distance X_c upstream the test pipe end when the pipe inclination is θ [see Fig. (1)].

Under the test conditions, the control valve is opened gradually to increase the flow rate until the free surface achieves a distance X_c upstream from the open end under steady state condition. Then the flow rate is measured. This procedure is repeated for different pipe inclinations and different CMC concentrations. Measurements were restricted to upward facing inclinations because of the limited constant level deriving head. This head did not allow the high velocities required for downward facing inclinations.

Figures (6) through (9) show typical variations of the critical velocity V_c, and hence Fr_c, under the effects of varying θ, X_c and CMC concentration "C".

Figures (6) through (8) show that, as physically expected, V_c decreases with increase of both θ and X_c. These trends were theoretically predicted as previously shown in Figs.(3) and (4). On the otherhand, Fig.(9) shows that V_c decreases as C increases. This may be discussed as follows. It is known that [9] as the flow behavior index n decreases, the velocity profile-normalized by average velocity - becomes more flat. This means that, for the same average velocity, the near wall layer velocity increases as n decreases. Therefore, the near wall layer separates at a lower average velocity for lower values of "n", i.e., for higher CMC concentrations.

Comparison Between Theoretical and Experimental Results

The present experimental results for facing upward inclined pipes carrying Newtonian and non-Newtonian liquid in addition to early results [11], for horizontal and vertical submerged end pipes carrying Newtonia liquid, are compared with theoretical results. The non-Newtonian parameters, presented in table (1), were used for calculating the generalized Reynolds number - Eq. (10) - for these fluids. The dimensionless experimental results, for $\bar{x}_c$ = 0.5 are compared with the theoretical ones and presented in Figs.(10) through (12). The comparison shows that the derived simple-criteria determining-equation (8) safely predicts the critical Froud numbers, and hence the critical velocities, which are seen to be always higher than the experimental values. This may be attributed to the neglection of surface tension and roughness effects in the analysis. These effects, as physically expected, tend to delay free surface formation and hence decrease the critical velocities. It can be seen, from Fig. (10), that the disadvantage of discontinuity for the early obtained empirical criteria [11] does not exist for the present derived criteria. It is worthnoting that, as Fig.(12) shows, only two points among thirteen data points were under estimated by eq. (8). However, it was mentioned by ref. [11] that these two points were obtained for high viscosity liquids. The deviation between theoretical and experimental results for these points may be attributed to the approximations used in the theoretical analysis. The experimental measuring errors may also add to the apparent remarkable deviation. Also, the error in measuring the unwetted length $\bar{x}_c$ may result in a significant error in the value of Fr_c. For example, Fig. (12) shows predictions using eq. (3) for $\bar{x}_c$ = 0.2 and 0.08 which compare well with the two prementioned points.

Conclusions

I. A simple analysis, for determining criteria for filling of open end pipes carrying liquids has been presented. The analysis resulted in a simple general criteria determining equation. Comparison with experimental results, for half pipe diameter unwetted length criterion, showed that the theoretically predicted Froude numbers are safely higher than those obtained experimentally. The comparison included both Newtonian and non-Newtonian liquids.

II. The derived simple equation may be used for any liquid of known friction factor correlations.

III. The results, for half pipe diameter unwetted length criterion showed that:
   (1) for both horizontal and upward inclined pipes the safe design value of Froude number is about 0.725.
   (2) For downward facing inclined pipes the safe design value is about 0.92.

Nomenclature

C CMC polymer concentration, p.p.m.;
D Pipe diameter, m;
F_{rc} Froude number at critical conditions, $V_c/\sqrt{gD}$;
g Gravitational acceleration, m/s^2.
K Consistency index, $P_a.\ s^n$;

n Flow behavior index;

Re generalized Reynolds number, $\dfrac{\rho V_c^{2-n} D^n}{8^{n-1}(\frac{1+3n}{4n})^n K}$;

X_c, unwetted upstream pipe length, m;

$\bar{X}_c$ dimensionless value of $x_c(\bar{x}_c = x_c / D)$,

V_c critical flow velocity, m/s,

θ pipe inclination angle, degrees;

ρ fluid density, kg/m^3;

λ Darcy's friction coefficient = $8\,\tau_w/\rho V_c^2$.

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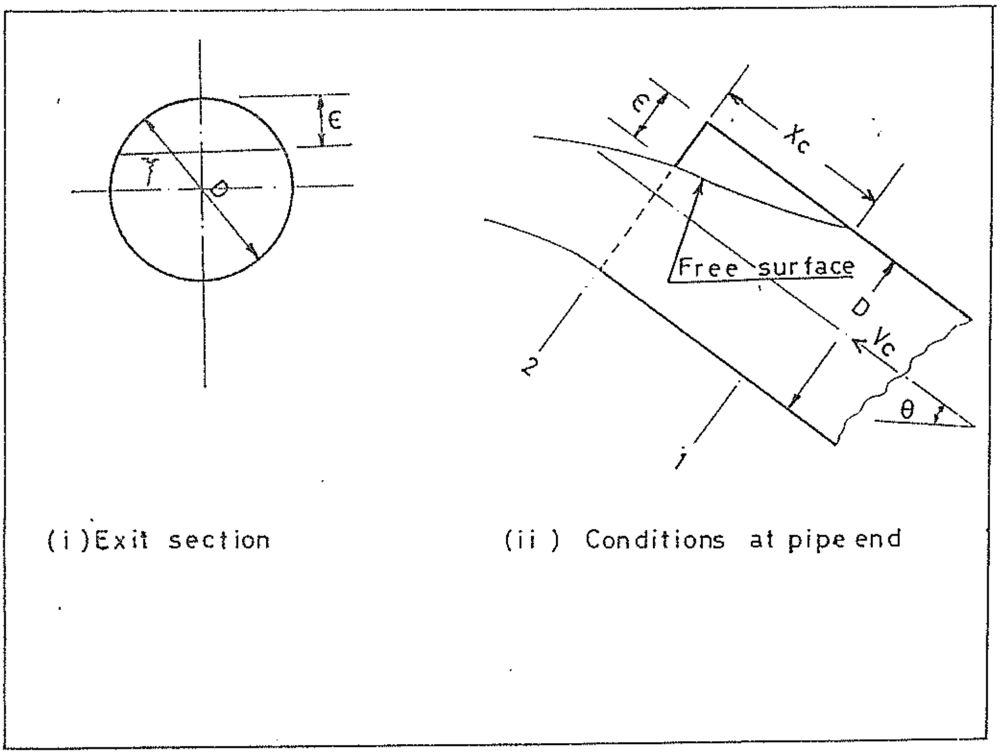

Fig. (1.a) Parameters at pipe end under critical conditions.

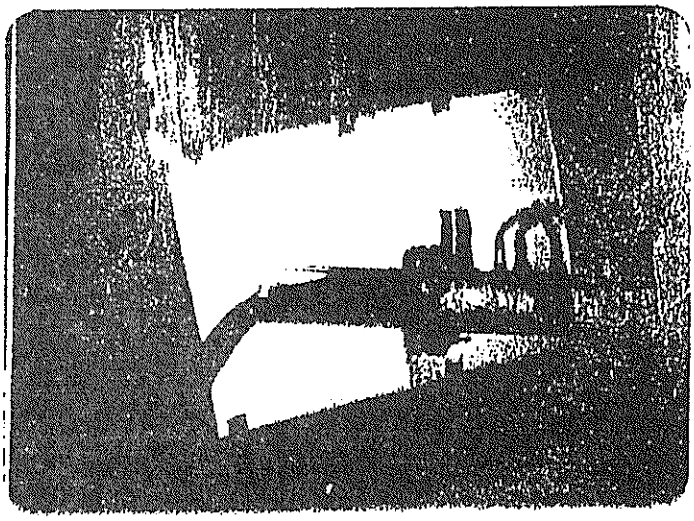

Fig. (1.b) The near exit partial Filling

($\bar{x}c = 1.5$)

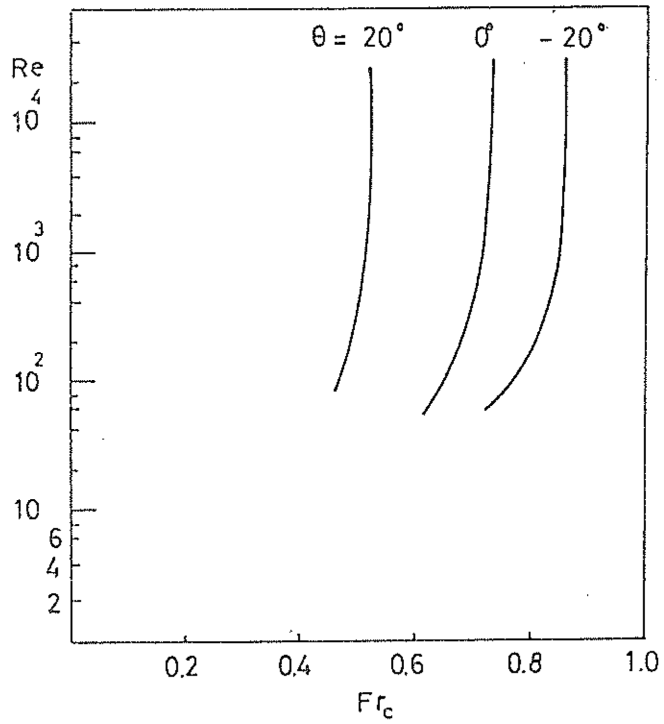

Fig. (2) Typical Fr_c - Re relationship.

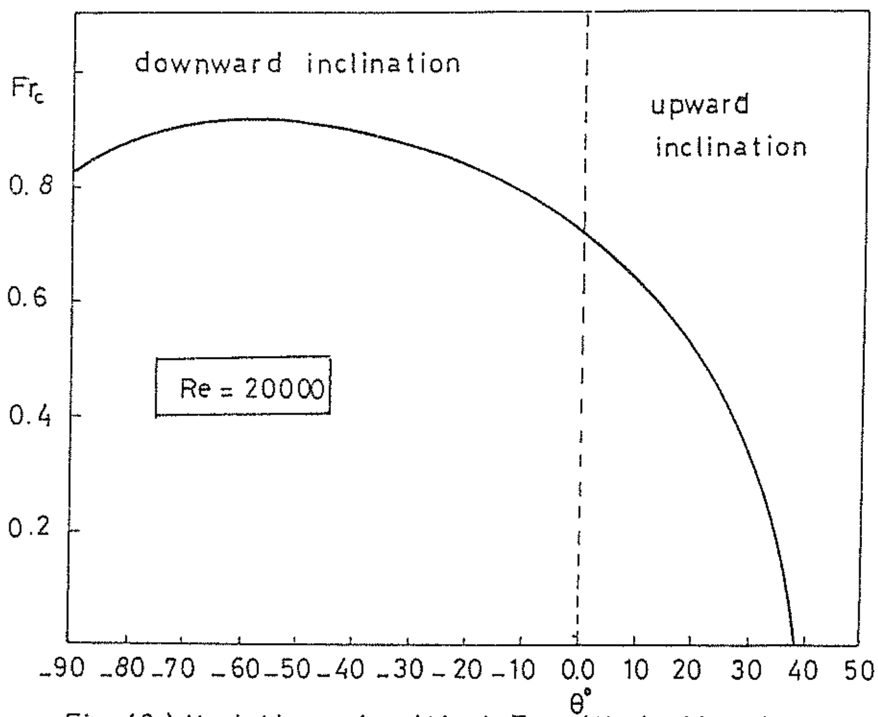

Fig. (3) Variation of critical Fr with inclination angle θ for turbulent flow.

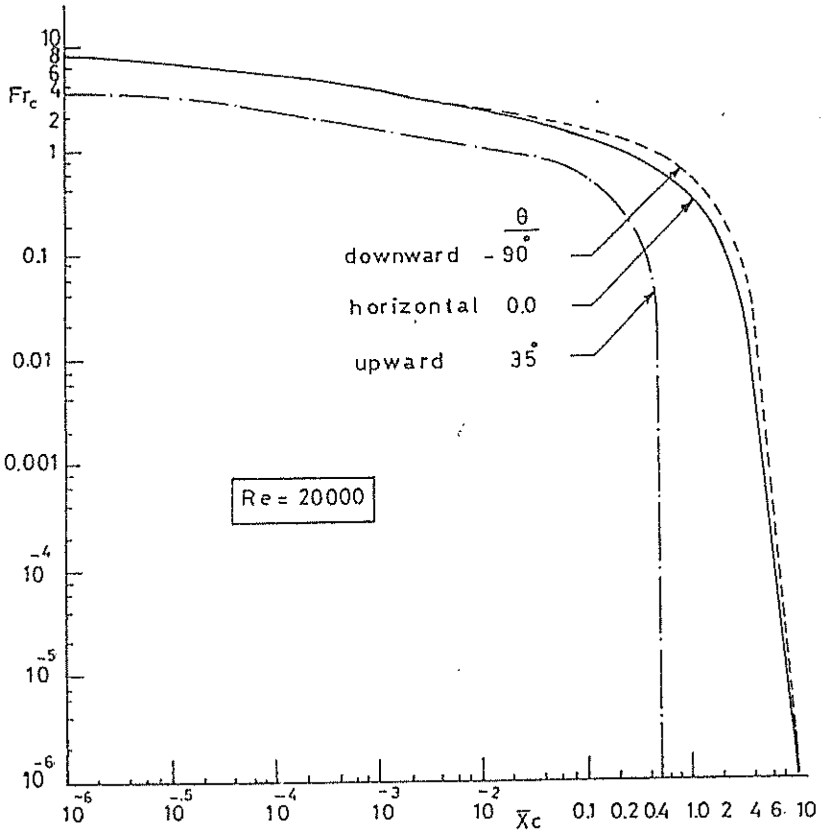

Fig.(4) Variation of Fr_c with $\bar{X}c$ at turbulent flow.

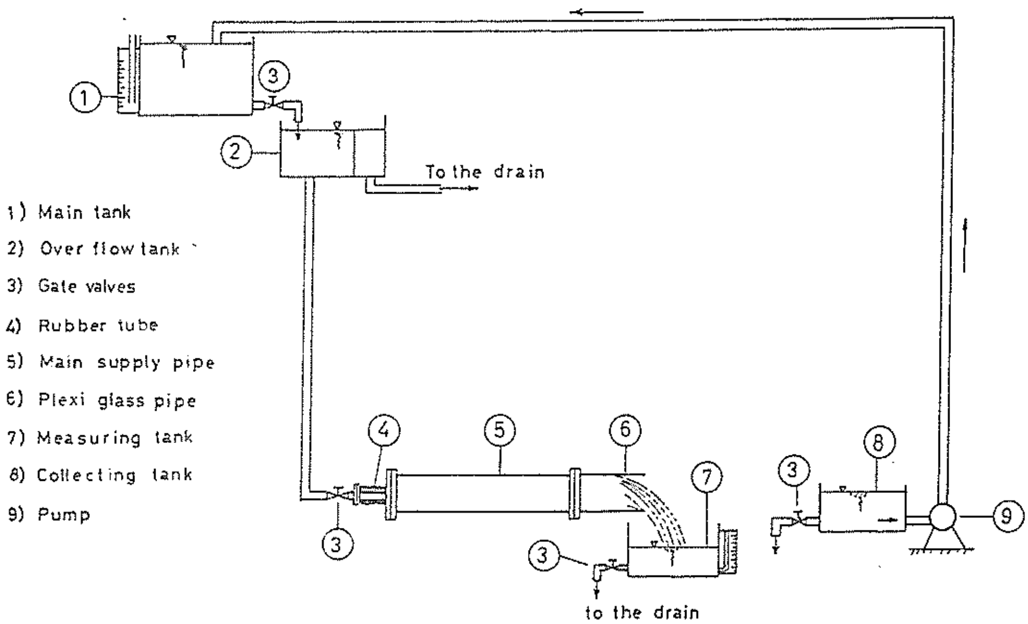

Fig (5) Experimental appratus.

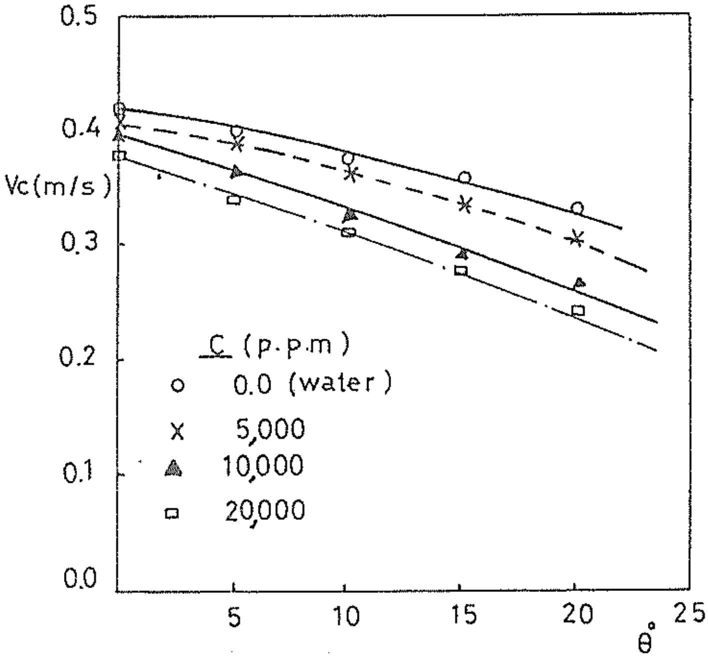

Fig. (6) Effect of upward inclination angle on critical velocity ($\overline{X}c = 0.5$) .

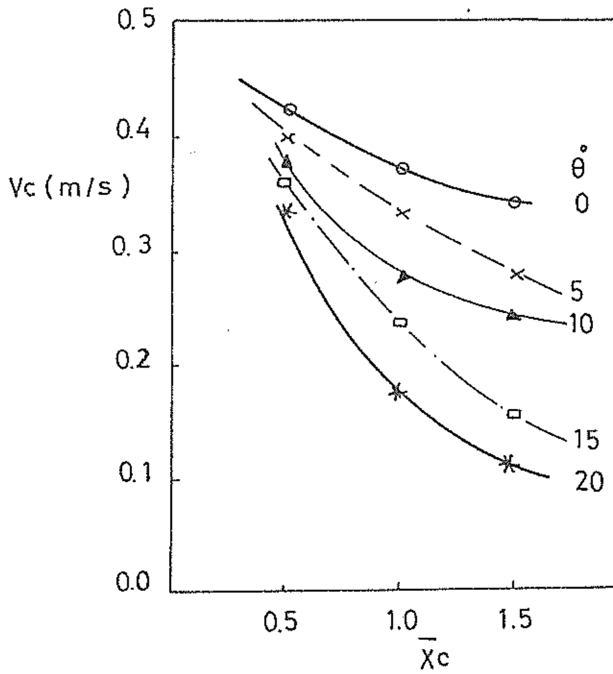

Fig.(7) Effect of $\overline{X}c$ on critical velocity, C=0 p.p.m. (water).

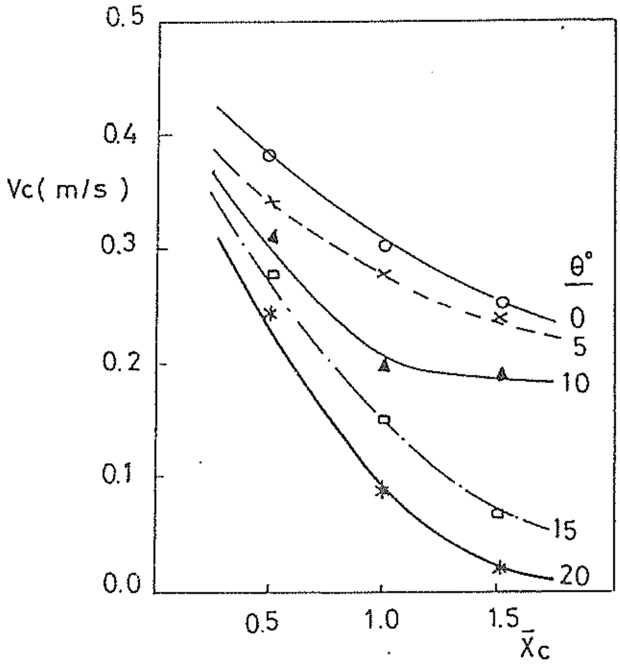

Fig. (8) Contd. (C = 20,000 p.p.m.)

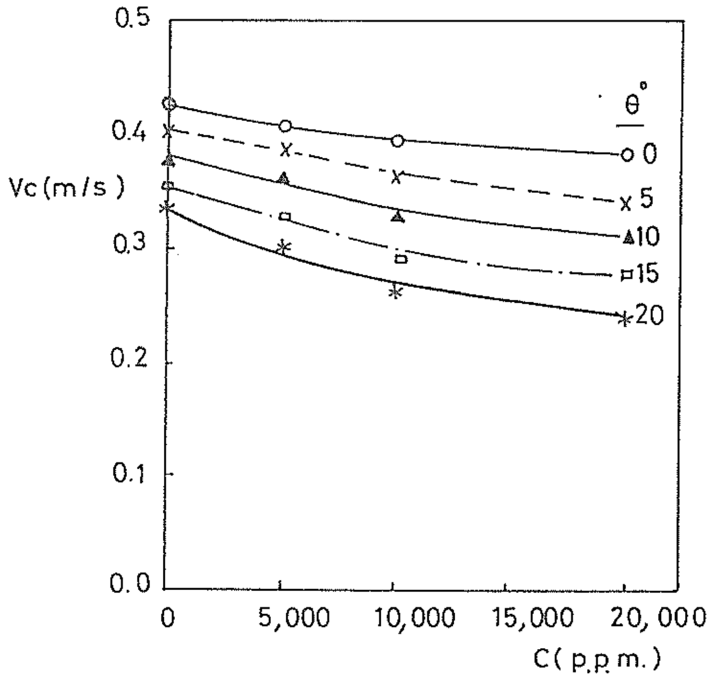

Fig.(9) Effect of concentration "C" on critical velocity, ($\bar{X}c = 0.5$)

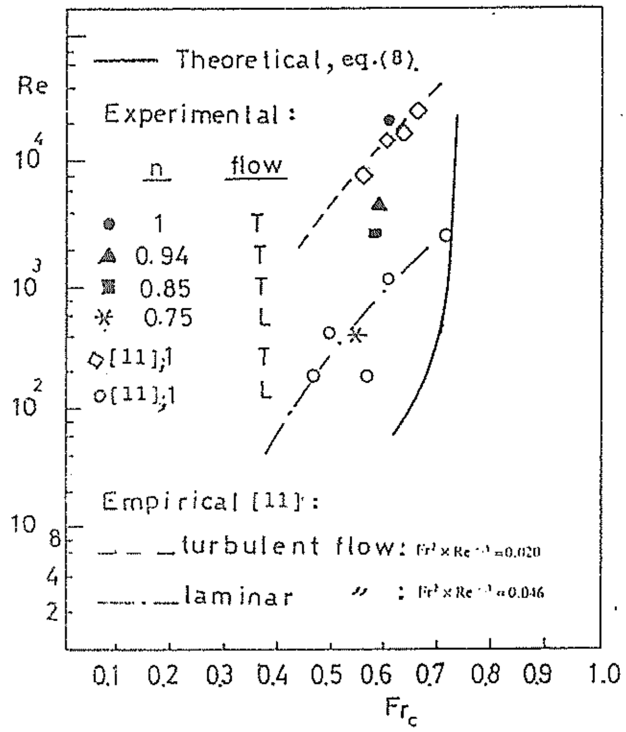

Fig. (10) Comparison between theoretical and experimental results. Horizontal pipe ($\theta = 0$); L=laminar, T=turbulent flow.

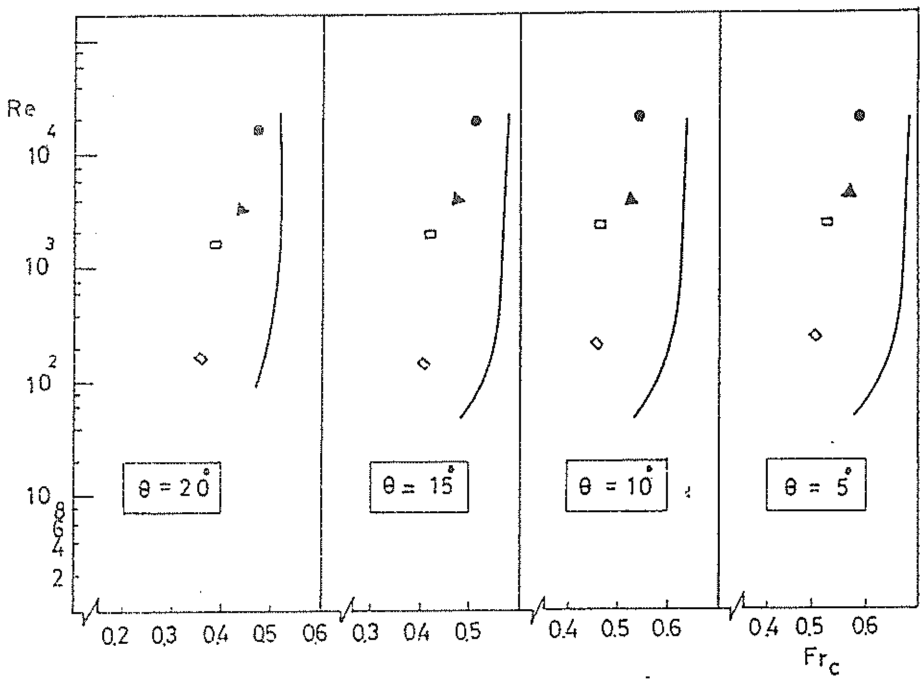

Fig. (11) Comparison between theoretical and experimental results, for upward facing pipes. (—) theoretical; (●) water n = 1 ; (▲) n = 0.94; (□) n = 0.85 and (◇) n = 0.75 .

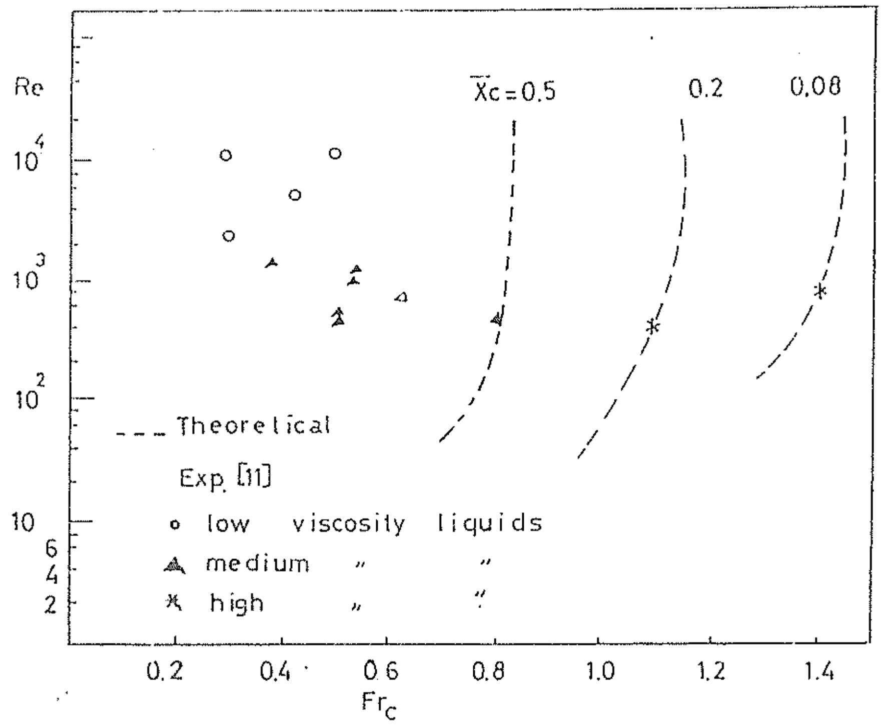

Fig.(12) Comparison between theoretical and experimental results, for vertical submerged end pipes ($\theta = -90^{\circ}$).

ملخص البحث

CRITERIA FOR FILLING INCLINED PIPES CARRYING NEWTONIAN OR NON-NEWTONIAN LIQUIDS

(دالة تحديد إمتلاء الأنابيب المائلة التى تحمل سوائل نيوتونية أو غير نيوتونية)

أجريت دراسة نظرية وعملية على خاصية امتلاء الأنابيب المائلة التى تسرى يها سوائل . الدراسة النظرية أدت الى أستنباط دالة إمتلاء لا تعتمد كثيرا على رقم رينولدز وبالتالى على خصائص السائل. أجريت دراسة عملية على الانابيب المائلة لأعلى واستخدمت محاليل مادة CMC فى الماء بتركيزات مختلفة.كسوائل غير نيوتونية بينما إستخدم الماء كسائل نيوتونى. وقد استخدم مقطع إختبار شفاف لمشاهدة سطح الماء عند مخرج أنبوبة الاختبار وتحديد إمتلاء الأنبوبة . وقد أجريت مقارنة بين النتائج النظرية والعملية بينت أن دالة الإمتلاء المستنبطة نظريا تعطى سرعات إمتلاء أعلى دائما من السرعات العملية مما يعنى إمكانية إعتماد عليها فى تصميم خطوط الأنابيب . وقد تم إستنتاج أن للأنابيب المائلة لأعلى فإن رقم فرويد التصميمى يكون فى حدود ٧٢٥, ٠ بينما للأنليبب المائلة لأسفل يكون هذا الرقم أعلى وفى حدود ٩٢, ٠