Sci. J. Fac. Sci. Menoufia Univ. Vol. VIII (1994).

ORDER MANIFOLDS

Shoukry I. Nada
Department of Mathematics, Faculty of Science,
Menoufia Univerisity,

ABSTRACT

The idea of order manifolds is introduced in such a way if the order is equality, then we get the orginal definitions and properties of manifolds. Some theorems and properties with examples are shown.

1. INTRODUCTION

The connection of topology and ordering was introduced by L. Nachbin[2]. He gave most basic definirtions and proved many generalization of well-knwon results concerning general topology and functional analysis. Since then , many authors have investigated the subject systematically.

In this paper we shall introduce the idea of order manifolds which is a generalization of differentiable manifolds. We give some properties with examples .

2. Order differentiable manifolds

2.1 Definitions

An order n-dimensional topological manifold M is an ordered space with a countable base for the topology which is locally order homeomorphic to R^n .

The last condition means that, for each point $x \in M$, there exists an open neighbourhood U of x and an order homeomorphism.

$h : U \to U'$ onto an open set $U' \subseteq R^n$

2.2. Examples

(i) Figure (1) shows two examples of order 1-manifolds. It should be noticed that each order 1-manifold has the totolly order.

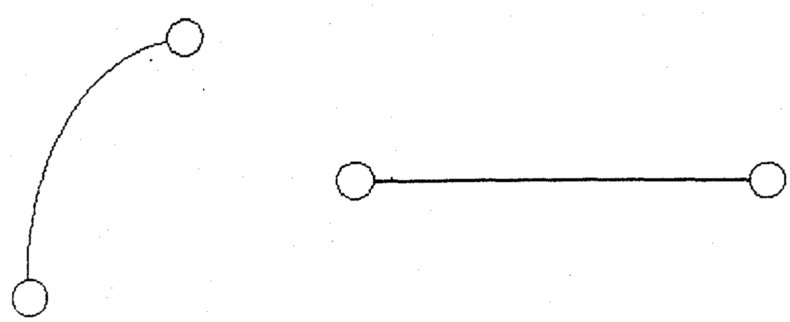

Fig. 1

(ii) The circle is not an order 1-manifold although it is a 1-manifold.

(iii) R^n with the usual topology and the vector order is an order n-manifold.

(iv) Any open subset of an order n-manifold is again an order n-manifold.

If M is an order n-manifold and $h : U \to U'$ is an order homeomorphism of an open subset $U \subseteq R^n$ onto an open subset $U' \subseteq R^n$, then h is called an order chart of M and U is the associated chart domain. The collection of order charts $\{h_\alpha : U_\alpha \to U'_\alpha \ , \alpha \in \Omega\}$ is called an order atlas for M if

- $\cup_{\alpha\in\Omega} U_\alpha = M$

The first question to be answered is that : Given two order charts, both are order-homeomorphisms h_α , h_β are defined on the intersection of their domain $U_{\alpha\beta} := U_\alpha \cap U_\beta$. Is $h_{\alpha\beta}$ an order homeomerphism on which

$$h_\beta o h_\alpha^{-1} = h_{\alpha\beta}\ ?$$

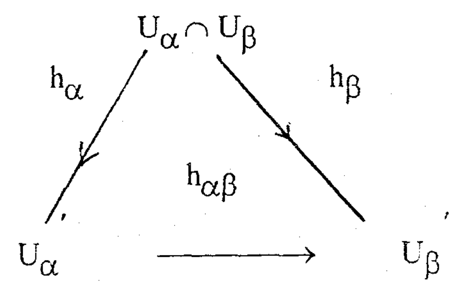

Fig. 2

Since the composition of two order homeomerphisms is again an order homeomorphism[3], it follows that the answer of the given question is yes.

2.3. Theorem : If M is an order m-manifold and N is an order n-manifold, then the product space M X N is an order (m+n) -manifold .
Proof : The product of two ordered space is again an ordered space[3], and the dimension of the resulted space is the sum of

dimensions of the two spaces. Also the $U^n \times U^m$ is an order homeomorphism to U^{n+m}. Hence the rest of the proof is similar to the proof of the ordinary case.

As one expected an order atlas of an order manifold is order differentiable if all its order chart transformations are order differentiable. Also a continuous order mapping between two order differentiable manifolds defined similarily.

2.4. Theorem : Every order differentiable manifold, M, possesses a countable order atlas.

Proof: Since the manifold M has a countable base, each open set $U \in M$ contains an open set $V \in B$ where B is a countable base for the order manifold. Thus one can select these order homeomerphisms from the differentable atlas. This completes the proof.

The difinition of order embedding was introduced in [2]. If R^k and R^n are given the usual topology and the vector order and $k \leq n$, then we conclude.

2.5. Lemma : R^k is an order embedded in R^n

Proof: $(x_1,\ldots,x_k) \xrightarrow{f} (x_1,\ldots,x_k,0,\ldots,0)$ is a continuous mapping from R^k to R^n and preseving order as well. Obviosly f is one to one, hence the order embedding follows.

Using the previous lemma, one can easly proof the following

2.6. Theorem: If $M \neq \phi$ is an order n-manifold and $k \leq n$, then there exists an order embedding $R^k \longrightarrow M$.

REFRENCES

[1] ***BROCKER, TH & Janich, k*** Introduction to differential topology , cambridge University press (1982).

[2] *Nachbin, L.* Topology and order, Van Nostrand, Inc., New York (1965).

[3] *Nada, S. I.* studies on topological order spaces ph.D. Thesis, Southamphton University, U. K. (1986).

فراغ عديد الطيات المرتب

شكرى ابراهيم ندا

قسم الرياضيات البحته –كلية العلوم –جامعة المنوفية

الملخص العربى

فى هذا البحث قمنا بتعريف فراغ عديد الطيات المرتب و هذه الفكره هى امتداد لافكار (Nachbin) التى ظهرت سابقا فى فروع التوبولوجى والتحليل الدالى .ذكرنا بعض الامثلة وقمنا ببرهان بعض النظريات مثل حاصل ضرب فراغين عديدا الطيات المرتبه هو فراغ عديد الطيات المرتب. ونلاحظ انه اذا افترضنا ان الترتيب المقترح هو التساوى فاننا نحصل على النتائج المعروفه فى فراغ عديد الطيات العادى.