



Answer all the following questions.

**1-a)** The open tank in Fig. 1-a contains water at 20°C and is being filled through section 1. Assume incompressible flow. First derive an analytic expression for the water-level change  $dh/dt$  in terms of arbitrary volume flows ( $Q_1, Q_2, Q_3$ ) and tank diameter  $d$ . Then, if the water level  $h$  is constant, determine the exit velocity  $V_2$  for the given data  $V_1 = 3$  m/s and  $Q_3 = 0.01$  m<sup>3</sup>/s. [12 Marks]

**1-b)** The horizontal lawn sprinkler in Fig. 1-b has a water flow rate of 4.0 lit/sec introduced vertically through the center. Estimate (a) the retarding torque required to keep the arms from rotating and (b) the rotation rate ( $r/min$ ) if there is no retarding torque. [13 Marks]

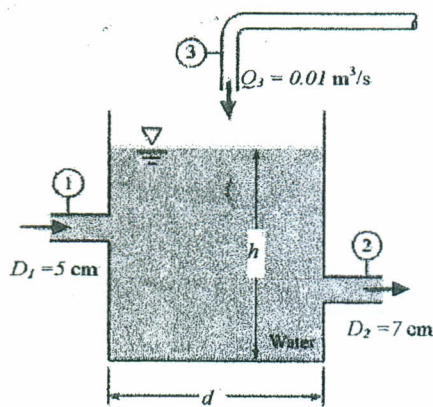


Fig.1-a

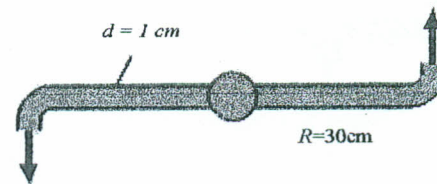


Fig. 1-b

**2-a)** Given the steady, incompressible velocity distribution:

$$V = 3x \mathbf{i} + Cy \mathbf{j} + 0 \mathbf{k}, \text{ where } C \text{ is a constant.}$$

If conservation of mass is satisfied, the value of  $C$  should be:

- (i) 3, (ii) 3/2, (iii) 0, (iv) -3/2, (v) -3 [5 Marks]

**2-b)** A viscous liquid of constant  $\rho$  and  $\mu$  falls due to gravity between two plates a distance  $2h$  apart, as in Fig. 2-b. The flow is laminar and fully developed, with a single velocity component  $w = w(x)$ . There are no applied pressure gradients, only gravity. Solve the Navier-Stokes equation for the velocity profile between the plates then calculate the volume flow rate per unit depth of walls. [20 Marks]

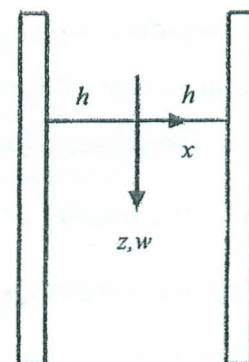
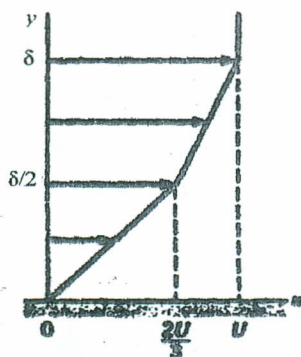


Fig 2-b

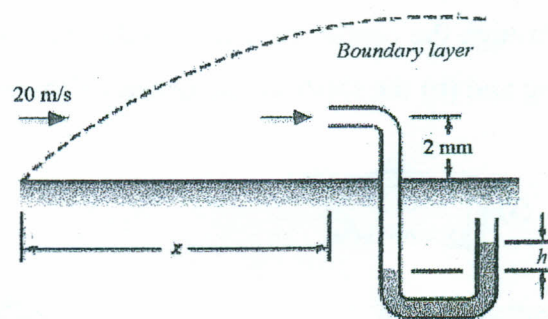
**3-a)** Explain the effect of pressure gradient on the boundary layer velocity profile. [5 Marks]

**3-b)** A laminar boundary layer velocity is approximated by the two straight-line segments indicated in the figure 3-a. Use momentum integral equation to determine the boundary layer thickness  $\delta = \delta(x)$ , and wall shear stress,  $\tau_o = \tau_o(x)$ . [12 Marks]

**3-c)** Air at 20°C and 1 atm flows at 20 m/s past the flat plate in Fig. 3-c. A pitot stagnation tube, placed 2 mm from the wall, develops a manometer head  $h = 16$  mm of Meriam red oil,  $SG = 0.827$ . Use this information to estimate the downstream position  $x$  of the pitot tube. Assume laminar flow. [8 Marks]



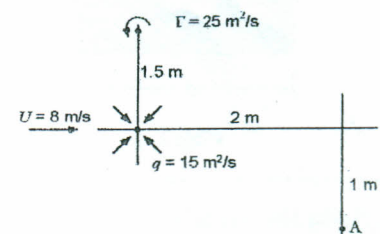
**Fig.3-b**



**Fig 3-c**

**4-a)** Define: i) ideal flow ii) stream function  
iii) velocity potential. [5 Marks]

**4-b)** Find the resultant velocity vector induced at point A in Fig 4.b by the uniform stream, vortex and line sink. [8 Marks]



**Fig 4-b**

**4-c)** The two-dimensional steady flow past a circular cylinder is formed by combining a uniform stream of speed  $U$  in the positive  $x$ -direction and doublet of strength  $\mu$  at the origin. The pressure for upstream of the origin is  $P_\infty$ . [12 Marks]

- Drive the velocity potential ( $\Phi$ ) and the stream function ( $\Psi$ ) for this flow field.
- Velocity components in cylindrical coordinate ( $u_r$  and  $u_\theta$ ).
- Determine the pressure in this flow field on the surface of the cylinder.

GOOD LUCK

Prof. Dr. Lotfy Rabie  
Dr. Hossam AbdelMeguid

**Note:**

**The continuity equation for incompressible fluid:**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

**The Navier-Stokes equations for a newtonian fluid with constant density and viscosity are :**

$$\text{x-momentum: } \rho g_x - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = \rho \frac{du}{dt}$$

$$\text{y-momentum: } \rho g_y - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) = \rho \frac{dv}{dt}$$

$$\text{z-momentum: } \rho g_z - \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) = \rho \frac{dw}{dt}$$

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**The Blasius Velocity Profile.**

$y[U/(vx)]^{1/2}$	$u/U$	$y[U/(vx)]^{1/2}$	$u/U$
0.0	0.0	2.8	0.81152
0.2	0.06641	3.0	0.84605
0.4	0.13277	3.2	0.87609
0.6	0.19894	3.4	0.90177
0.8	0.26471	3.6	0.92333
1.0	0.32979	3.8	0.94112
1.2	0.39378	4.0	0.95552
1.4	0.45627	4.2	0.96696
1.6	0.51676	4.4	0.97587
1.8	0.57477	4.6	0.98269
2.0	0.62977	4.8	0.98779
2.2	0.68132	5.0	0.99155
2.4	0.72899	$\infty$	1.00000
2.6	0.77246		