OIL FILM THICKNESS FOR THE GEARS OF CIRCULAR-ARC TOOTH-PROFILE

Dr. Ahmed M.M. EL-BAHLOUL
Lecturer, Production Engineering Dept.,
Faculty of Engineering, Mansoura University
Mansoura, EGYPT

" سمك طبقة الزيت بين أسنان المسننات ذات الأسنان الدائرية الجانبية "

الخلاصة هذا العمل يبين كيفية استنتاج معادلة سمك طبقة الزيت بين أسنان المسننات ذات
الأسنان الدائرية الجانبية وذلك فى حالة التزليق المرن وهذا يتطلب حل معادلة رينولد
ومعادلة المرونه بواسطة استخدام طريقة الفروق المحدده . هذه المعادلة المستنتجة تمثل
أداه بسيطة وسهلة للمصمم حيث يمكنه تحديد سمك طبقة الزيت بين أسنان المسننات وبالتالى
تحديد سعة تحميل المسننات ذات الأسنان الدائرية الجانبية وذلك تحت ظروف التشغيل المختلفة
من السرعة وزاوية اللولبة الحلزونية وأنصاف أقطار التقوس للأسنان ونوعية الزيوت المستخدمة
وكذلك المعدن المصنع منه المسنن . وكذلك تمت مقارنة سمك طبقة الزيت المحسوب من هذه
المعادلة المستنتجة بثلاث قيم محسوبة من معادلات التزليق المرن لكل من همروك ودوسون
شنج وكذلك آرترد وكوى وأظهرت هذه المقارنة أن سمك طبقة الزيت المحسوب من المعادلة
المستنتجة أكبر من كل من الثلاث قيم المحسوبة من المعادلات الأخرى تحت نفس ظروف التشغيل

ABSTRACT

A formula for the oil film thickness in elasto-hydrodynamic lubrication for the gears of circular-arc tooth-profile is derived in terms of all parameters. This required the simultaneous solutions of the Reynolds equation with the elasticity equation by using the finite difference method. This formula represents a simple tool for the designers, where the oil film thickness can be calculated, and the corresponding load capacity of the gears of circular-arc tooth-profile are determined, for any given speed, helix angle, radii of curvature, lubricant properties and material of the gears. The calculated oil film thickness obtained from the presented formula is compared with the existing theories of elastohydrodynamic lubrication equations developed by Hamrock and Dowson, Cheng and Archard and Cowking and shows that the calculated values of the oil film thickness obtained from the presented formula are greater

INTRODUCTION

Helical gears of circular-arc tooth-profiles are conformal gears with point contact between teeth changing to an elliptical area under load [1-3]. Contact between these gears is along the face and no progressive contact occurs on the profile.

The first step towards a theoretical solution of elastohydrodynamic lubrication EHL of point contact conditions was presented by Archard and Cowking [4]. They adopted an approach similar to that used by Grubin for line contact conditions. The Hertzian contact zone is assumed to form a parallel film region and the generation of high pressure in the approaches to the Hertzian zone is considered. Cameron and Gohar [5] derived an approximate equation for the EHL of point contact , using a number of assumptions similar to those used successfolly with the line contact . They also used an optical interferometry technique in getting the shape of the oil film . Cheng [6] presented a numerical solution of the elastohydrodynamic film thickness in an elliptical contact , also used an approach similar to that used by Grubin in determining a minimum film thickness for point contact . Hamrock and Dowson [7 , 8 , 9 and 10] presented the theoretical solution of the isothermal EHL of point contact . [7] presented the elasticity model in which the conjunction is divided into equal rectangular areas with a uniform pressure applied over each area . [8] presented a complete approach for solving the EHL problem for point contact. [9] presented the influence of the ellipticity parameter upon solutions to the point contact problem, the ellipticity parameter was varied from one (a ball on a plate) to eight (a configuration approaching line contact) . [10] presented the influence of the ellipticity parameter, the dimensionless speed, load, and material parameters on the minimum oil film thickness.

This paper presents a numerical solution for the EHL of the gears of circular-arc tooth-profiles to obtain the oil film thickness and load capacity for a given oil viscosity, surface velocities, pressure viscosity exponent, maximum Hertzian pressure, size of the Hertzian ellipse, helix angle of the gear and the radii of curvature along the helix angle and radii of curvature in the normal plane to the tooth. This is achieved by solving the Reynolds' equation numerically with the elasticity equation using the finite difference method. The calculated oil film thickness obtained from the presented formula is compared with the existing theories of EHL equations.

NOMENCLATURE

a = semiaxis of the ellipse in plane normal to the tooth, mm "in x-direction"
$\bar{A}$ = parameter, $\bar{P}H^3 (\partial f/\partial \bar{p})_o / (\partial f/\partial \beta)$
A_o = dimensionless parameter = $3W / 8bh_o E'$
AA_1, AA_2, ... = constants in a computer programme and equations.
b = semiaxis of the ellipse along the helix angle, mm "in y-direction"
$\bar{B}$ = parameter, $\bar{P}H^3 \cdot (\partial f/\partial \theta)/(\partial f/\partial \bar{p})$
B1, B2,..... = constants in a computer programme and equations
$\bar{C}$ = dimensionless parameter $(12\, \eta_o a V)/(h_o^2\, P_{HZ})$
C_{AP} and C_{BP} = the pressure density coefficients
$\bar{D}$ = dimensionless parameter, U/V
E_1, E_2 = modulus of elasticity of the pinion and wheel respectively, Kp/mm^2
$E' = [(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2]^{-1}$
$\bar{F}$ = dimensionless parameter, V_2/V
f = viscosity exponent function
h = oil film thickness, mm
ho = minimum oil film thickness, mm
H = dimensionless oil film thickness = h/ho
HI = dimensionless oil film thickness ho/R , $\bar{H}=H^3$
L_1, L_2, ... = constants in equations and computer programme
m = constant in equations
n = constant in equations
p = pressure in the oil film, kp/mm^2.
P_{HZ} = Hertzian pressure, kp/mm^2
P_o = central pressure = $1.5\, W/\pi ab$ kp/mm^2
$\bar{P}$ = dimensionless pressure = P/P_{HZ}
q = reduced pressure
$\bar{q} = q/\bar{C}$
r = radius of polar coordinate
R_{1x}, R_{1y} = radii of tooth curvature of the pinion in the plane normal to the tooth and along the helix angle, mm
R_{2x}, R_{2y} = radii of tooth curvature of the wheel in the plane normal to the tooth and along the helix angle, mm
R'_x = effective radius of curvature in the plane normal to the teeth = $[1/R_{1x} - 1/R_{2x}]^{-1}$ mm
R'_y and R = effective radius of curvature along the helix angle = $[1/R_{1y} + 1/R_{2y}]^{-1}$ mm
u_1, u_2 and U = surface velocities in the plane normal to the teeth m/sec
v_1, v_2 and V = surface velocities along the helix angle m/sec
w_1, w_2 = surface velocities in z-direction "plane normal to the oil film" m/sec
$\bar{w}_1$, $\bar{w}_2$ = elastic deformations of the tooth for the pinion and wheel in z direction mm,
W = tooth load
x, $\bar{X}$ = coordinate and dimensionless coordinate x/a in the plane normal to the tooth
y, $\bar{Y}$ = coordinate and dimensionless coordinate y/b along the helix angle
z = coordinate across the oil film.

α = pressure viscosity component, mm²/kp.
$\bar{\alpha}$ = αPHz
β = ellipticity parameter = b/a
η = local viscosity of the lubricant, kp. sec/mm²
η_o = inlet viscosity of the lubricant, kp. sec/mm²
$\bar{\eta}$ = dimensionless viscosity of the oil film = η/η_o
θ = ratio of the absolute temperature of the oil film to the ambient temperature
ν_1, ν_2, = Possion's ratio of the pinion and wheel
ξ = transformed variable for r
ρ = density of the lubricant, kp/mm³
ρ_o = ambient density, kp/mm³
$\bar{\rho}$ = dimensionless density = ρ/ρ_o
ϕ = angle of polar coordinate
ψ = helix angle.

THEORETICAL ANALYSIS

A- Reynolds' Equation :

The equation which governs the generation of pressure in lubricating films is known as the Reynolds' equation. This equation is derived by applying the basic equations of motion and the continuity of the lubricant. This equation is written as follows, [11] see Fig. (1)

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial y}\right)=\frac{\partial}{\partial x}\left(\rho U h\right)+\frac{\partial}{\partial y}\left(\rho V h\right)$$
$$-\rho u_2\frac{\partial h}{\partial x}-\rho v_2\frac{\partial h}{\partial y}+\rho\left(w_2-w_1\right) \qquad (1)$$

The relation between the viscosity, pressure and temperature are in the from, [6]

$$\eta=\eta_o e^{f(\bar{P},\theta)} \quad \text{or} \quad \bar{\eta}=e^{f(\bar{P},\theta)} \quad \therefore\ 1/\bar{\eta}=e^{-f(\bar{P},\theta)} \qquad (2)$$

The reduced pressure can be written as follow;

$$q=\frac{1}{\left(\frac{\partial f}{\partial \bar{P}}\right)_o}\left[1-e^{-f(\bar{P},\theta)}\right] \qquad (3)$$

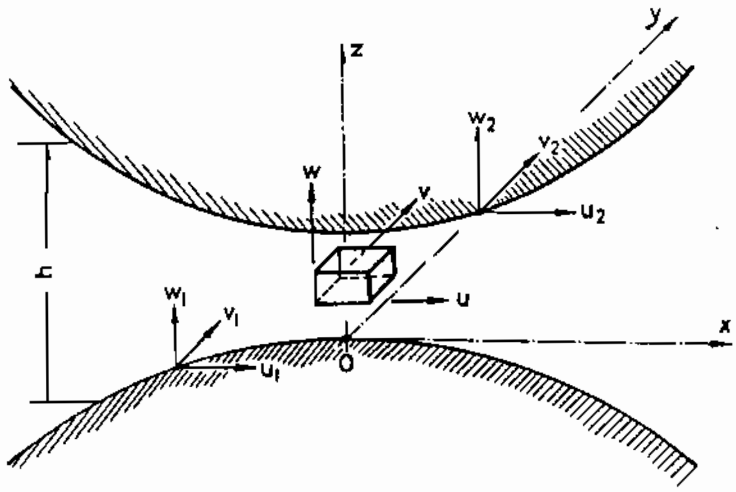
The density of the lubricant, according to [12], is

$$\rho=\rho_o\left(1+\frac{C_A P}{1+C_B P}\right) \qquad (4)$$

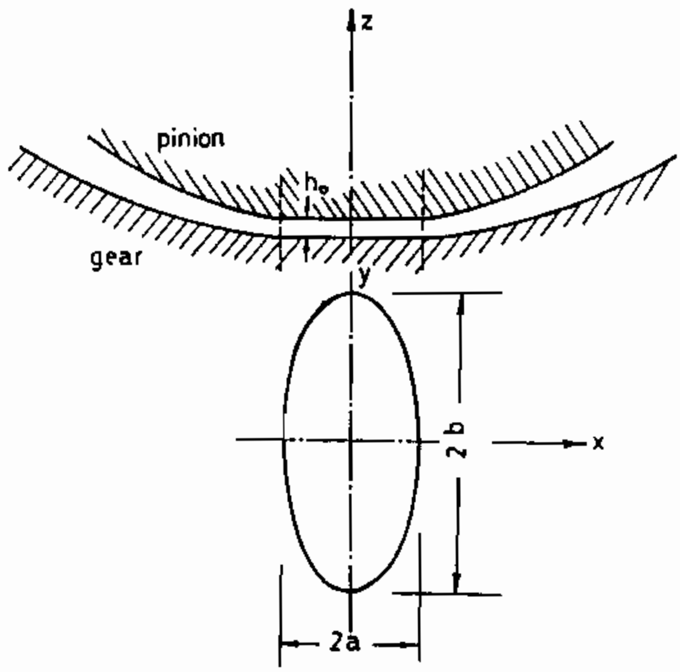
To convert Reynolds' equation to dimensionless form see Fig. (2), put $\bar{X}=\frac{x}{a}$, $\bar{Y}=\frac{y}{b}$, $\bar{\rho}=\frac{\rho}{\rho_o}$, $H=\frac{h}{h_o}$, $\bar{P}=\frac{p}{P_{Hz}}$, $\bar{\eta}=\frac{\eta}{\eta_o}$, $w_1=0$, $\beta=\frac{b}{a}$,

and $w_2=u_2\frac{\partial h}{\partial x}=u_2\frac{h_o}{a}\frac{\partial H}{\partial \bar{X}}$

$$\therefore\frac{\partial}{\partial \bar{X}}\left(\frac{\bar{\rho}H^3}{\bar{\eta}}\frac{\partial \bar{P}}{\partial \bar{X}}\right)+\frac{1}{\beta^2}\frac{\partial}{\partial \bar{Y}}\left(\frac{\bar{\rho}H^3}{\bar{\eta}}\frac{\partial \bar{P}}{\partial \bar{Y}}\right)=\frac{12\eta_o a V}{h_o^2 P_{Hz}}\left[\frac{U}{V}\frac{\partial}{\partial \bar{X}}(\bar{\rho}H)+\frac{1}{\beta}\frac{\partial}{\partial \bar{Y}}(\bar{\rho}H)\right.$$
$$\left.-\frac{v_2}{V}\frac{1}{\beta}\bar{\rho}\frac{\partial H}{\partial \bar{Y}}\right] \qquad (5)$$



Fig(1)



Fig(2)Geometry of the Hertzian area of contact

Differentiating equation (3) with respect to $\bar{X}$

$$\frac{\partial q}{\partial \bar{X}} = \frac{1}{\left(\frac{\partial f}{\partial \bar{P}}\right)_o}\left\{\left[-e^{-f(\bar{P},\theta)}\left(-\frac{\partial f}{\partial \bar{P}}\frac{\partial \bar{P}}{\partial \bar{X}}\right)\right]+\left[-e^{-f(\bar{P},\theta)}\left(-\frac{\partial f}{\partial \theta}\frac{\partial \theta}{\partial \bar{X}}\right)\right]\right\}$$

From equation (2)

$$\frac{\partial q}{\partial \bar{X}} = \frac{1}{(\partial f/\partial \bar{P})_o}\left\{\left(\frac{1}{\bar{\eta}}\frac{\partial f}{\partial \bar{P}}\frac{\partial \bar{P}}{\partial \bar{X}}\right)+\left(\frac{1}{\bar{\eta}}\frac{\partial f}{\partial \theta}\frac{\partial \theta}{\partial \bar{X}}\right)\right\}$$

$$\therefore \frac{\partial \bar{P}}{\partial \bar{X}} = \frac{(\partial f/\partial \bar{P})_o}{(\partial f/\partial \bar{P})}\bar{\eta}\frac{\partial q}{\partial \bar{X}} - \frac{(\partial f/\partial \theta)}{(\partial f/\partial \bar{P})}\frac{\partial \theta}{\partial \bar{X}} \qquad (6)$$

Also, differentiating with respect to $\bar{Y}$

$$\therefore \frac{\partial \bar{P}}{\partial \bar{Y}} = \frac{(\partial f/\partial \bar{P})_o}{(\partial f/\partial \bar{P})}\bar{\eta}\frac{\partial q}{\partial \bar{Y}} - \frac{(\partial f/\partial \theta)}{(\partial f/\partial \bar{P})}\frac{\partial \theta}{\partial \bar{Y}} \qquad (7)$$

Substituting equations (6) and (7) into equation (5), then

$$\frac{\partial}{\partial \bar{X}}\left[\frac{\bar{\rho}H^3}{\bar{\eta}}\left\{\frac{(\partial f/\partial \bar{P})_o}{(\partial f/\partial \bar{P})}\bar{\eta}\frac{\partial q}{\partial \bar{X}} - \frac{(\partial f/\partial \theta)}{(\partial f/\partial \bar{P})}\frac{\partial \theta}{\partial \bar{X}}\right\}\right]+\frac{1}{\beta^2}\frac{\partial}{\partial \bar{Y}}\left[\frac{\bar{\rho}H^3}{\bar{\eta}}\left\{\frac{(\partial f/\partial \bar{P})_o}{(\partial f/\partial \bar{P})}\bar{\eta}\frac{\partial q}{\partial \bar{Y}}\right.\right.$$

$$\left.\left. - \frac{\partial f/\partial \theta}{\partial f/\partial \bar{P}}\frac{\partial \theta}{\partial \bar{Y}}\right\}\right] = \frac{12\eta_o a V}{h_o^2 P_{Hz}}\cdot\left\{\frac{U}{V}\frac{\partial}{\partial \bar{X}}(\bar{\rho}H)+\frac{1}{\beta}\frac{\partial}{\partial \bar{Y}}(\bar{\rho}H) - \frac{V_2}{V}\frac{1}{\beta}\bar{\rho}\frac{\partial H}{\partial \bar{Y}}\right\}$$

$$\therefore \frac{\partial}{\partial \bar{X}}\left[\bar{A}\frac{\partial q}{\partial \bar{X}} - \frac{\bar{B}}{\bar{\eta}}\frac{\partial \theta}{\partial \bar{X}}\right]+\frac{1}{\beta^2}\frac{\partial}{\partial \bar{Y}}\left[\bar{A}\frac{\partial q}{\partial \bar{Y}} - \frac{\bar{B}}{\bar{\eta}}\frac{\partial \theta}{\partial \bar{Y}}\right] = \bar{C}\left[\bar{D}\frac{\partial}{\partial \bar{X}}(\bar{\rho}H) + \right.$$

$$\left.\frac{1}{\beta}\frac{\partial}{\partial \bar{Y}}(\bar{\rho}H) - \frac{\bar{F}}{\beta}\bar{\rho}\frac{\partial H}{\partial \bar{Y}}\right] \qquad (8)$$

This is a general form of the dimensionless Reynolds' equation

For the simplification of the equation (8), the oil is considered incompressible and isothermal, i.e. ρ and θ are constants.

$$f = \bar{\alpha}\bar{P} \qquad \therefore \eta = \eta_o e^{\alpha\bar{P}} \qquad \text{or} \qquad \bar{\eta} = e^{\bar{\alpha}\bar{P}} \qquad (9)$$

The reduced pressure can be written as

$$q = \frac{1}{\bar{\alpha}}(1 - e^{-\bar{\alpha}\bar{P}}) \qquad \text{and} \qquad \frac{\partial q}{\partial \bar{X}} = e^{-\bar{\alpha}\bar{P}}\frac{\partial \bar{P}}{\partial \bar{X}} \qquad (9\text{-a})$$

Substituting equation (9-a) into the equation (8), the dimensionless Reynolds' equation can be written as follow ;

$$\frac{\partial}{\partial \bar{X}}\left(H^3\frac{\partial q}{\partial \bar{X}}\right)+\frac{1}{\beta^2}\frac{\partial}{\partial \bar{Y}}\left(H^3\frac{\partial q}{\partial \bar{Y}}\right) = \bar{C}\left[\bar{D}\frac{\partial H}{\partial \bar{X}}+\frac{1}{\beta}\frac{\partial H}{\partial \bar{Y}} - \frac{\bar{F}}{\beta}\frac{\partial H}{\partial \bar{Y}}\right]$$

put $\bar{q} = \frac{q}{\bar{C}}$ and $\bar{H} = H^3$

$$\therefore \frac{\partial}{\partial \bar{X}}\left(\bar{H}\frac{\partial \bar{q}}{\partial \bar{X}}\right) + \frac{1}{\beta^2}\frac{\partial}{\partial \bar{Y}}\left(\bar{H}\frac{\partial \bar{q}}{\partial \bar{Y}}\right) = \bar{D}\frac{\partial H}{\partial \bar{X}} + \frac{1}{\beta}\frac{\partial H}{\partial \bar{Y}} - \frac{\bar{F}}{\beta}\frac{\partial H}{\partial \bar{Y}} \qquad (10)$$

To convert equation (10) from cartezian to polar coordinates, Fig. (3) one uses

$$\bar{X} = r\cos\phi\ , \quad \bar{Y} = r\sin\phi\ , \quad \bar{X}^2 + \bar{Y}^2 = r^2 \quad \text{and} \quad \phi = \tan^{-1}\left(\frac{\bar{Y}}{\bar{X}}\right)$$

Hence

$$\frac{\partial r}{\partial \bar{X}} = \frac{2\bar{X}}{2\sqrt{\bar{X}^2+\bar{Y}^2}} = \cos\phi\ , \quad \frac{\partial r}{\partial \bar{Y}} = \frac{2\bar{Y}}{2\sqrt{\bar{X}^2+\bar{Y}^2}} = \sin\phi\ ,$$

$$\frac{\partial \phi}{\partial \bar{X}} = \frac{1}{1+(\bar{Y}/\bar{X})^2}(-\bar{Y}/\bar{X}^2) = -\frac{1}{r}\sin\phi \quad \text{and} \quad \frac{\partial \phi}{\partial \bar{Y}} = \frac{1/\bar{X}}{1+(\bar{Y}/\bar{X})^2} = \frac{1}{r}\cos\phi$$

$$\text{but} \quad \frac{\partial}{\partial \bar{X}} = \frac{\partial}{\partial r}\frac{\partial r}{\partial \bar{X}} + \frac{\partial}{\partial \phi}\frac{\partial \phi}{\partial \bar{X}}, \ \text{Thus} \ \frac{\partial}{\partial \bar{X}} = \cos\phi\frac{\partial}{\partial r} - \frac{1}{r}\sin\phi\frac{\partial}{\partial \phi}$$

$$\text{and} \quad \frac{\partial}{\partial \bar{Y}} = \frac{\partial}{\partial r}\frac{\partial r}{\partial \bar{Y}} + \frac{\partial}{\partial \phi}\frac{\partial \phi}{\partial \bar{Y}}, \ \text{Thus} \ \frac{\partial}{\partial \bar{Y}} = \sin\phi\frac{\partial}{\partial r} + \frac{1}{r}\cos\phi\frac{\partial}{\partial \phi}$$

Substituting these values into equation (10), then

$$L_1\frac{\partial}{\partial r}\left(\bar{H}\frac{\partial \bar{q}}{\partial r}\right) + L_2\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(\bar{H}\frac{\partial \bar{q}}{\partial \phi}\right) - L_{13}\left[\bar{H}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \bar{q}}{\partial \phi}\right) + \bar{H}\frac{\partial}{\partial \phi}\left(\frac{1}{r}\frac{\partial \bar{q}}{\partial r}\right) + \frac{1}{r}\frac{\partial \bar{H}}{\partial r}\frac{\partial \bar{q}}{\partial \phi} + \frac{1}{r}\frac{\partial \bar{H}}{\partial \phi}\frac{\partial \bar{q}}{\partial r}\right] + L_1\frac{\partial \bar{H}}{\partial r}\frac{\partial \bar{q}}{\partial r} + L_2\frac{1}{r^2}\frac{\partial \bar{H}}{\partial \phi}\frac{\partial \bar{q}}{\partial \phi} = \bar{D}\left[\cos\phi\frac{\partial H}{\partial r} - \frac{1}{r}\sin\phi\frac{\partial H}{\partial \phi}\right]$$

$$+\frac{1}{\beta}\left[\sin\phi\frac{\partial H}{\partial r} + \frac{1}{r}\cos\phi\frac{\partial H}{\partial \phi}\right] - \frac{\bar{F}}{\beta}\left[\sin\phi\frac{\partial H}{\partial r} + \frac{1}{r}\cos\phi\frac{\partial H}{\partial \phi}\right] \qquad (11)$$

Where : $L_1 = \cos^2\phi + \frac{1}{\beta^2}\sin^2\phi$, $L_2 = \sin^2\phi + \frac{1}{\beta^2}\cos^2\phi$ and $L_{13} = \sin\phi\cos\phi\left(1 - \frac{1}{\beta^2}\right)$

This equation is solved numerically by using the finite difference method. The variation of $\bar{q}$ is expected to be more drastic near r = 1 and very gradual as $r = r_{final}$, it is beneficial to transform the variable r into a new variable ξ by the following relation :

$$r = e^{\xi} - 1$$

This transformation enables one to use even grids in the radial direction

$$\partial r = e^{\xi}\,\partial \xi$$

substituting these values into equation (11) gives

$$\therefore L_1\frac{\partial}{\partial \xi}\left(\frac{\bar{H}}{e^{\xi}}\frac{\partial \bar{q}}{\partial \xi}\right) + \frac{L_2 e^{\xi}}{(e^{\xi}_{1})^2}\frac{\partial}{\partial \phi}\left(\bar{H}\frac{\partial \bar{q}}{\partial \phi}\right) - L_{13}\left[\bar{H}\frac{\partial}{\partial \xi}\left(\frac{1}{e^{\xi}_{1}}\frac{\partial \bar{q}}{\partial \phi}\right) + \right.$$

$$\bar{H}\frac{\partial}{\partial \phi}\left(\frac{1}{e^{\xi}_{1}}\frac{\partial \bar{q}}{\partial \xi}\right) + \frac{1}{e^{\xi}_{1}}\frac{\partial \bar{H}}{\partial \xi}\frac{\partial \bar{q}}{\partial \phi} + \left.\frac{1}{e^{\xi}_{1}}\frac{\partial \bar{H}}{\partial \phi}\frac{\partial \bar{q}}{\partial \xi}\right] + \frac{L_1}{e^{\xi}}\frac{\partial \bar{H}}{\partial \xi}\frac{\partial \bar{q}}{\partial \xi} +$$

$$\frac{L_2 e^{\xi}}{(e^{\xi}_{1})^2}\frac{\partial \bar{H}}{\partial \phi}\frac{\partial \bar{q}}{\partial \phi} = \bar{D}\left[\cos\phi\frac{\partial H}{\partial \xi} - \frac{e^{\xi}}{e^{\xi}_{1}}\sin\phi\frac{\partial H}{\partial \phi}\right] + \frac{1}{\beta}\left[\sin\phi\frac{\partial H}{\partial \xi} + \right.$$

$$\left.\frac{e^{\xi}}{e^{\xi}_{1}}\cos\phi\frac{\partial H}{\partial \phi}\right] - \frac{\bar{F}}{\beta}\left[\sin\phi\frac{\partial H}{\partial \xi} + \frac{e^{\xi}}{e^{\xi}_{1}}\cos\phi\frac{\partial H}{\partial \phi}\right] \qquad (12)$$

B- Application of the Finite Difference Method on Reynolds Equation :

Refering to Fig (4) the difference form of equation (12)

$$L_1\frac{1}{(\Delta\xi)^2}\Big[\left(\frac{\bar H}{e^{\xi}}\right)_{(i+1,j)}\times\bar q_{(i+1,j)}-\left(\left(\frac{\bar H}{e^{\xi}}\right)_{(i+1,j)}+\left(\frac{\bar H}{e^{\xi}}\right)_{(i,j)}\right)\bar q_{(i,j)}+\left(\frac{\bar H}{e^{\xi}}\right)_{(i,j)}\bar q_{(i-1,j)}\Big]+L_2\left(\frac{e^{\xi}}{(e^{\xi_1})^2}\right)_{(i,j)}\frac{1}{(\Delta\phi)^2}$$

$$\Big[\bar H_{(i,j+1)}\bar q_{(i,j+1)}-\left(\bar H_{(i,j+1)}+\bar H_{(i,j)}\right)\bar q_{(i,j)}+\bar H_{(i,j)}\bar q_{(i,j-1)}\Big]-L_{13}\Big[\frac{\bar H_{(i,j)}}{\Delta\phi\Delta\xi}\Big(\frac{1}{(e^{\xi_1})_{(i,j)}}\left(\bar q_{(i+1,j+1)}-\bar q_{(i+1,j)}-\bar q_{(i,j+1)}+\bar q_{(i,j)}\right)$$

$$+\left\{\frac{1}{(e^{\xi_{-1}})_{(i+1,j)}}-\frac{1}{(e^{\xi_1})_{(i,j)}}\right\}\left(\bar q_{(i,j+1)}-\bar q_{(i,j)}\right)\Big)+\frac{1}{\Delta\xi\Delta\phi}\left(\frac{\bar H}{e^{\xi_1}}\right)_{(i,j)}\times\left(\bar q_{(i+1,j+1)}-\bar q_{(i+1,j)}-\bar q_{(i,j+1)}+\bar q_{(i,j)}\right)+$$

$$\frac{1}{\Delta\xi\Delta\phi}\frac{1}{(e^{\xi_1})_{(i,j)}}\left(\bar H_{(i+1,j)}\bar q_{(i,j+1)}-\bar H_{(i+1,j)}\bar q_{(i,j)}-\bar H_{(i,j)}\bar q_{(i,j+1)}+\bar H_{(i,j)}\bar q_{(i,j)}\right)+\frac{1}{\Delta\xi\Delta\phi}\frac{1}{(e^{\xi_1})_{(i,j)}}\Big($$

$$\bar H_{(i,j+1)}\bar q_{(i+1,j)}-\bar H_{(i,j+1)}\bar q_{(i,j)}-\bar H_{(i,j)}\bar q_{(i+1,j)}+\bar H_{(i,j)}\bar q_{(i,j)}\Big)\Big]+\frac{L_1}{(\Delta\xi)^2}\frac{1}{[e^{\xi}]_{(i,j)}}\Big[\bar H_{(i+1,j)}\bar q_{(i+1,j)}-\bar H_{(i+1,j)}\bar q_{(i,j)}-\bar H_{(i,j)}\bar q_{(i+1,j)}$$

$$+\bar H_{(i,j)}\bar q_{(i,j)}\Big]+\frac{L_2}{(\Delta\phi)^2}\left(\frac{e^{\xi}}{(e^{\xi_1})^2}\right)_{(i,j)}\Big[\bar H_{(i,j+1)}\bar q_{(i,j+1)}-\bar H_{(i,j+1)}\bar q_{(i,j)}-\bar H_{(i,j)}\bar q_{(i,j+1)}+\bar H_{(i,j)}\bar q_{(i,j)}\Big]=$$

$$\bar D\Big[\frac{1}{\Delta\xi}\cos\phi\left(H_{(i+1,j)}-H_{(i,j)}\right)-\frac{1}{\Delta\phi}\sin\phi\left(\frac{e^{\xi}}{e^{\xi_1}}\right)_{(i,j)}\times\left(H_{(i,j+1)}-H_{(i,j)}\right)\Big]+\frac{1}{\beta}\Big[\frac{1}{\Delta\xi}\sin\phi\Big($$

$$H_{(i+1,j)}-H_{(i,j)}\Big)+\frac{1}{\Delta\phi}\cos\phi\left(\frac{e^{\xi}}{e^{\xi_1}}\right)_{(i,j)}\times\left(H_{(i,j+1)}-H_{(i,j)}\right)\Big]-\frac{\bar F}{\beta}\Big[\frac{1}{\Delta\xi}\sin\phi\left(H_{(i+1,j)}-H_{(i,j)}\right)+\frac{1}{\Delta\phi}\cos\phi$$

$$\times\left(\frac{e^{\xi}}{e^{\xi_1}}\right)_{(i,j)}\times\left(H_{(i,j+1)}-H_{(i,j)}\right)\Big] \qquad (13)$$

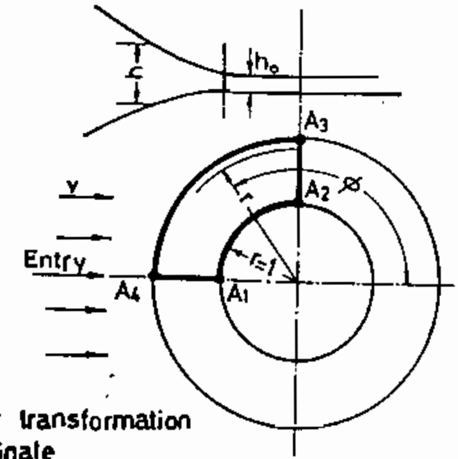
For simplicity of manipulations, equation (13) is written in the form [13, 14 and 15]

$$\bar q_{(i,j)}=\Big[1/\left(B_{363}\bar H_{(i+1,j)}+B_{13}\bar H_{(i,j)}+B_{39}\bar H_{(i,j+1)}\right)\Big]\times\Big[AA_3H_{(i+1,j)}+$$

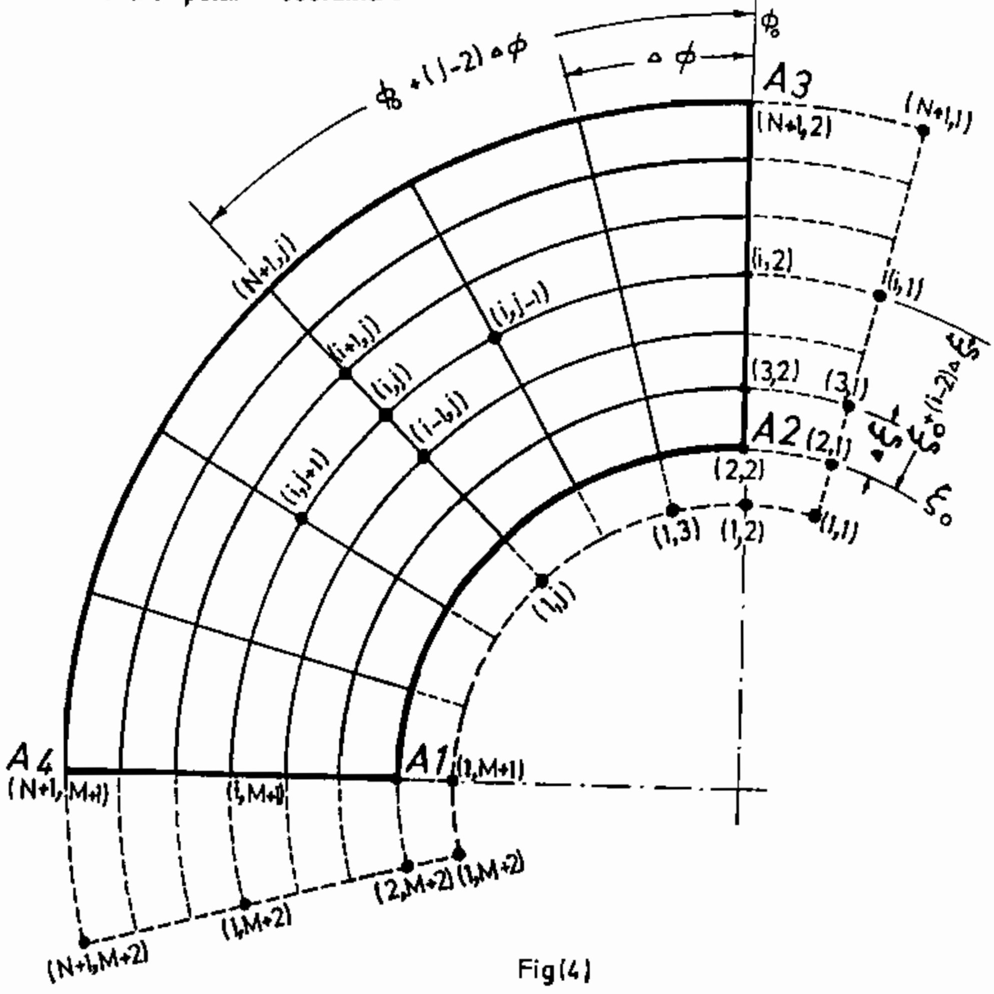
$$AA_4H_{(i,j)}-AA_5H_{(i,j+1)}\Big)-\left(B_{63}\bar H_{(i+1,j)}+B_{33}\bar H_{(i,j)}-L_3\bar H_{(i,j+1)}\right)\bar q_{(i+1,j)}$$

$$-\left(B_{73}\bar H_{(i,j)}\right)\bar q_{(i-1,j)}-\left(-L_3\bar H_{(\ldots,j)}+B_{319}\bar H_{(i,j)}+B_{190}\bar H_{(i,j+1)}\right)\bar q_{(i,j+1)}$$

$$-\left(B_{19}\bar H_{(i,j)}\right)\bar q_{(i,j-1)}-\left(B_{300}\bar F_{(i,j)}\right)\bar q_{(i+1,j+1)}\Big] \qquad (14)$$



Fig(3)Geometry of contact after transformation into the polar coordinate



Fig(4)

C - Boundary Condition :

According to Figs(3 and 4) there is an axis of symmetry across A1A4. The pressure inside the contact area $r \leq 1$ is constant and maximum. This pressure then falls to zero at $r = \infty$, but by analogy with disk it must be practically at $r = 2$ as that of [5 and 6]. Due to the symmetry of the entry we use a quadrant A1 A2 A3 A4 A1 in the analysis. The boundary conditions are:

a- Along A1 A2 the pressure is maximum

b- Along A1 A4, $\phi = \pi, \frac{d\bar{q}}{d\phi} = 0$

c- Along A3 A4 the pressure equal zero

d- Along A2 A3, $\phi = \pi/2 \; \bar{q} = \frac{\Phi(\eta)}{\Phi(o)}\bar{q}^*$, $\Phi(\eta) = \int_{\eta}^{\pi/2} \frac{\tan\eta \, d\eta}{H'}$, $\eta = \mathrm{Sec}^{-1} r$. Where: $\bar{q}^* = \bar{q}$ at A1

D- Oil Film Thickness Equation :

The oil film thickness between the two mating gear teeth is given by as follow, see Fig. (5)

$$h = h_o + \frac{x^2}{2}\left(\frac{1}{R_{1x}} - \frac{1}{R_{2x}}\right) + \frac{y^2}{2}\left(\frac{1}{R_{1y}} + \frac{1}{R_{2y}}\right) - (\bar{w}_1 + \bar{w}_2) \qquad (15)$$

The elastic deformation of the two mating teeth in the z-direction ($\bar{w}_1$ and $\bar{w}_2$) is written in general form as [16] :

$$\bar{w} = \frac{1-\nu^2}{\pi E}\iint \frac{q.dA}{r} \qquad (16)$$

qdA, is the pressure acting on an infinitely small element of the surface of contact, and r is the distance of this element from the point under consideration.

This form is written in cartezian coordinates as follows, similar to [17].

$$\bar{w}(x,y) = \frac{1}{\pi E'}\int_{-b}^{b}\int_{-a}^{a} \frac{p(x,y)\,dx.dy}{\sqrt{(x-x_1)^2 + (y-y_1)^2}}$$

The elastic deformation at the centre $x_1 = y_1 = 0$

$$\therefore \bar{w}(0,0) = \frac{1}{\pi E'}\int_{-b}^{b}\int_{-a}^{a} \frac{p(x,y)\,dx\,dy}{\sqrt{x^2 + y^2}} \qquad (17)$$

According to Hertz the intensity of pressure p over the surface of contact is represented by the ordinates of a semi-ellipsoid constructed on the surface of contact, thus according to [18] ,

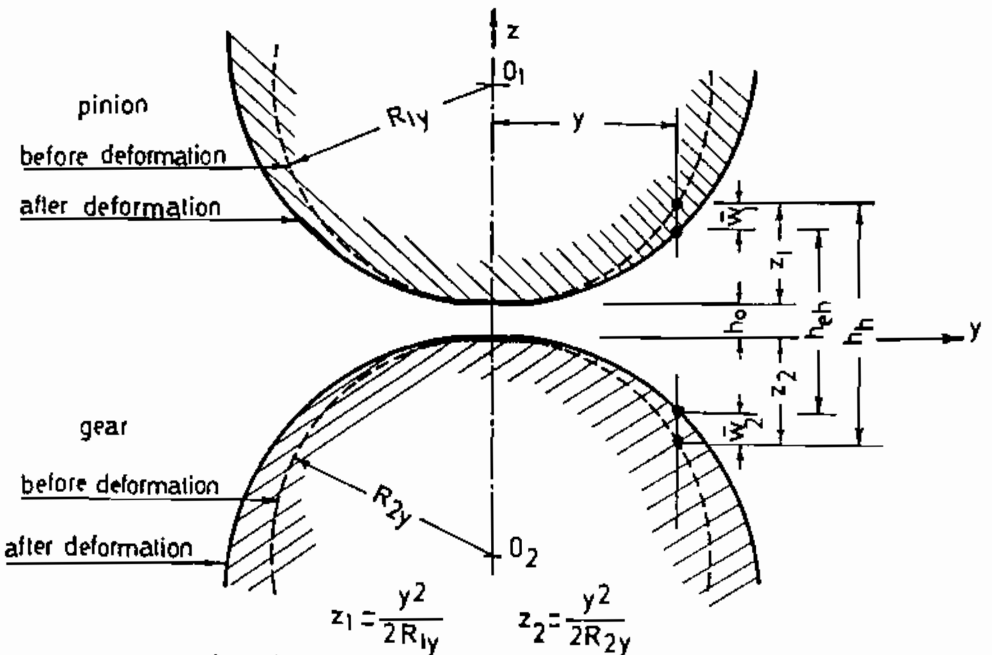
$$p(x,y) = P_o\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{\frac{1}{2}},$$

Substituting the values of P_o , $p(x,y)$ in equation (17) gives :

$$\bar{w} = \frac{1}{\pi E'}\,\frac{3}{2}\,\frac{W}{\pi ab}\int_{-b}^{b}\int_{-a}^{a} \frac{\left[1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right]^{\frac{1}{2}}}{\left[x^2 + y^2\right]^{\frac{1}{2}}}\,dx\,dy$$

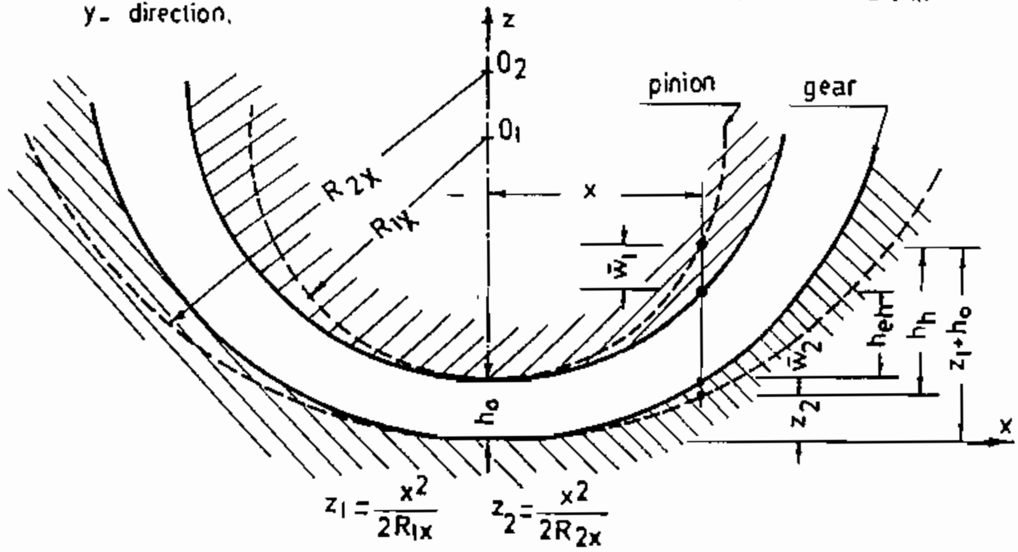
Substituting this equation into the oil film thickness equation (15)

$$h = h_o + \frac{x^2}{2}\left(\frac{1}{R_{1x}} - \frac{1}{R_{2x}}\right) + \frac{y^2}{2}\left(\frac{1}{R_{1y}} + \frac{1}{R_{2y}}\right) - \frac{3}{2}\,\frac{1}{\pi^2 E'}\,\frac{W}{ab}\int_{-b}^{b}\int_{-a}^{a} \frac{\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{\frac{1}{2}}}{\left(x^2 + y^2\right)^{\frac{1}{2}}}\,dx\,dy$$



$$z_1 = \frac{y^2}{2R_{1y}} \qquad z_2 = \frac{y^2}{2R_{2y}}$$

a_ representation of the oil film between convex_convex radii of curvature in y_ direction.



$$z_1 = \frac{x^2}{2R_{1x}} \qquad z_2 = \frac{x^2}{2R_{2x}}$$

b_representation of the oil film between convex_concave radii of curvature in x_ direction.

h_h= hydrodynamic oil film thickness.

h_{eh}= elastohydrodynamic oil film thickness

Fig(5)

$$\text{or } \frac{h}{h_o} = 1 + \frac{x^2}{2h_o}\left(\frac{1}{R_{1x}} - \frac{1}{R_{2x}}\right) + \frac{y^2}{2h_o}\left(\frac{1}{R_{1y}} + \frac{1}{R_{2y}}\right) - \frac{3}{2}\frac{1}{\pi^2 E'}\frac{W}{a\,b}\int_{-b}^{b}\int_{-a}^{a}\frac{\left(1-\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)^{\frac{1}{2}}}{\left(x^2+y^2\right)^{\frac{1}{2}}}dx\,dy \quad (18)$$

At no load the contact between the two mating gears teeth is a point. In a normal plane to the tooth the contact is considered as that of sphere and spherical socket "convex-concave", the semi-ellipsoid a-is written as follows [18]

$$a = \sqrt[3]{\frac{3}{4}\frac{W\frac{1}{E'}}{\left(\frac{1}{R_{1x}}-\frac{1}{R_{2x}}\right)}} \qquad \therefore \left(\frac{1}{R_{1x}}-\frac{1}{R_{2x}}\right) = \frac{3}{4}\frac{W}{a^3E'} \quad (a)$$

In the plane along helix angle the contact between the two mating gears teeth is similar to two spheres "convex-convex", the semi-ellipsoid b is written as follows [18]

$$b = \sqrt{\frac{3}{4}\frac{W\frac{1}{E'}}{\left(\frac{1}{R_{1y}}+\frac{1}{R_{2y}}\right)}} \qquad \therefore \left(\frac{1}{R_{1y}}+\frac{1}{R_{2y}}\right) = \frac{3}{4}\frac{W}{b^3E'} \quad (b)$$

Substituting the equations (a) and (b) into the dimensionless oil film thickness equation (18) and putting $\bar{X} = x/a$, $\bar{Y} = y/b$ and $\beta = b/a$, we have

$$\frac{h}{h_o} = 1 + \frac{\bar{X}^2}{2ah_o}\frac{3}{4}\frac{W}{E'} + \frac{\bar{Y}^2}{2bh_o}\frac{3}{4}\frac{W}{E'} - \frac{3}{2}\frac{1}{\pi^2E'}\frac{W}{bh_o}\int_{-1}^{1}\int_{-1}^{1}\frac{\left(1-\bar{X}^2-\bar{Y}^2\right)^{\frac{1}{2}}}{\left((\bar{X}/\beta)^2+\bar{Y}^2\right)^{\frac{1}{2}}}d\bar{X}\,d\bar{Y}$$

$$= 1 + \frac{3W}{8bh_oE'}\left[\frac{b}{a}\bar{X}^2 + \bar{Y}^2 - \frac{4}{\pi^2}\int_{-1}^{1}\int_{-1}^{1}\frac{\left(1-\bar{X}^2-\bar{Y}^2\right)^{\frac{1}{2}}}{\left((\bar{X}/\beta)^2+\bar{Y}^2\right)^{\frac{1}{2}}}d\bar{X}\,d\bar{Y}\right]$$

$$\therefore H = 1 + A_o\left[\beta\bar{X}^2 + \bar{Y}^2 - \frac{4}{\pi^2}\int_{-1}^{1}\int_{-1}^{1}\frac{\left(1-\bar{X}^2-\bar{Y}^2\right)^{\frac{1}{2}}}{\left((\bar{X}/\beta)^2+\bar{Y}^2\right)^{\frac{1}{2}}}d\bar{X}\,d\bar{Y}\right] \quad (19)$$

This equation is written in polar coordinates as follows

$$H = 1 + A_o\left[r^2(\beta\cos^2\Phi + \sin^2\Phi) - \frac{4}{\pi^2}\int_0^1\int_0^{2\pi}\frac{(1-r^2)^{\frac{1}{2}}\,d\Phi\,dr}{(\cos^2\Phi/\beta^2+\sin^2\Phi)^{\frac{1}{2}}}\right] \quad (20)$$

$$\text{or } H = 1 + A_o\left[r^2(\beta\cos^2\Phi + \sin^2\Phi) - \frac{4}{\pi^2}\,WW\right]$$

$$\text{Where: } WW = \int_0^1\int_0^{2\pi}\frac{(1-r^2)^{\frac{1}{2}}\,d\Phi\,dr}{(\cos^2\Phi/\beta^2+\sin^2\Phi)^{\frac{1}{2}}}$$

The dimensionless oil film thickness equation is written at a point (i,j) as follow ; see Fig. (4)

$$H_{(i,j)} = 1 + A_o\left[(e^{\xi}-1)^2 \cdot (\beta \cos^2\phi + \sin^2\phi) - \frac{4}{\pi^2} WW\right] \tag{21}$$

where $\xi = \xi_o + (i-2)\Delta\xi$ and $\phi = \phi_o + (j-2)\Delta\phi$

The integration WW is solved numerically as follow and according to [19] see Fig (6) ;

$$\frac{1}{2\pi h^2}\iint f(X,Y)\,dx\,dy = \sum_{i=1}^{n} W_i f(X_i, Y_i) \text{ ; Where}$$

(x_i, y_i)	W_i
$(0,0)$	$1/9$
$\left(\sqrt{\frac{6-\sqrt{6}}{10}}\,h\cos\frac{2\pi K}{10},\ \sqrt{\frac{6-\sqrt{6}}{10}}\,h\sin\frac{2\pi K}{10}\right)$	$\frac{16+\sqrt{6}}{360}$
$\left(\sqrt{\frac{6+\sqrt{6}}{10}}\,h\cos\frac{2\pi K}{10},\ \sqrt{\frac{6+\sqrt{6}}{10}}\,h\sin\frac{2\pi K}{10}\right)$	$\frac{16-\sqrt{6}}{360}$

$(K = 1, 2, \ldots\ldots, 10)$

This form is written in polar coordinate as follow

$$\int_0^1\int_0^{2\pi} \frac{(1-r^2)^{\frac{1}{2}}\,d\phi\,dr}{(\cos^2\phi/\beta^2 + \sin^2\phi)^{1/2}} = \pi h^2 \sum_{i=1}^{n} W_i f(r_i, \phi_i)$$

THEORETICAL RESULTS :

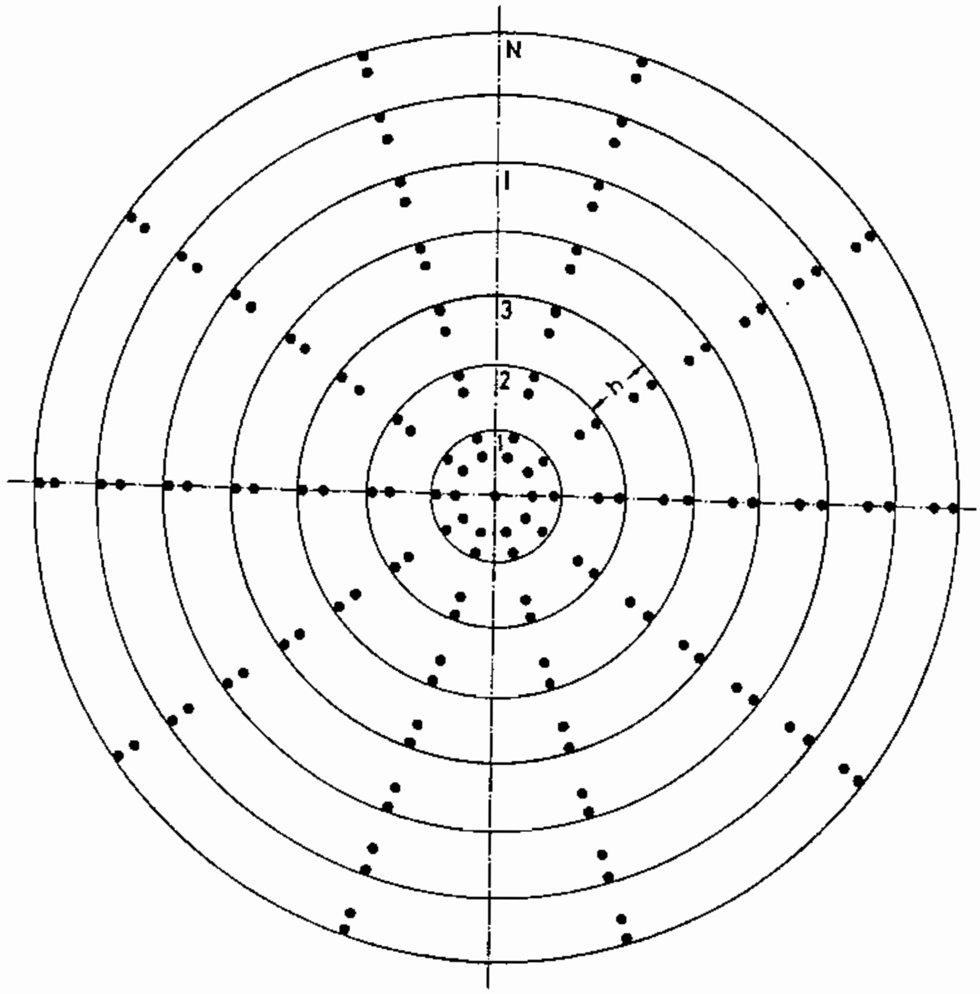
The numerical solutions of the Reynolds' equation (14) with the oil film thickness equation (21) gives the distribution of the dimensionless reduced pressure $\bar{q}$ as a function of Ao, β and ψ . Fig (7) shows the main flow chart, and the computer programme is made on the IBM personal computer AT with a core capacity of 640 k. Fig (8) shows the change of the reduced pressure $\bar{q}$ with the change of the dimensionless parameter Ao at different values of the elapticity parameters β and different helix angles. It indicates that the reduced pressure $\bar{q}$ decreases linearly with increasing the parameter Ao for different values of β and ψ. The linear relationship between $\bar{q}$ and Ao is written in the form;

$$\bar{q} = m A_o^{-n} \tag{22}$$

but $\bar{q} = \dfrac{1/\bar{\alpha}\,(1 - e^{-\bar{\alpha}\bar{P}})}{12\,\eta_o a V / h_o^2\, P_{Hz}}$ and $\bar{\alpha} = \alpha P_{Hz}$

or $\bar{q} = \dfrac{1}{12\alpha}\,\dfrac{(1 - e^{-\bar{\alpha}\bar{P}})h_o^2}{\eta_o a V}$

The pressure is maximum at r = 1 and the value of $e^{-\bar{\alpha}\bar{P}}$ can be neglected,



Fig(6)

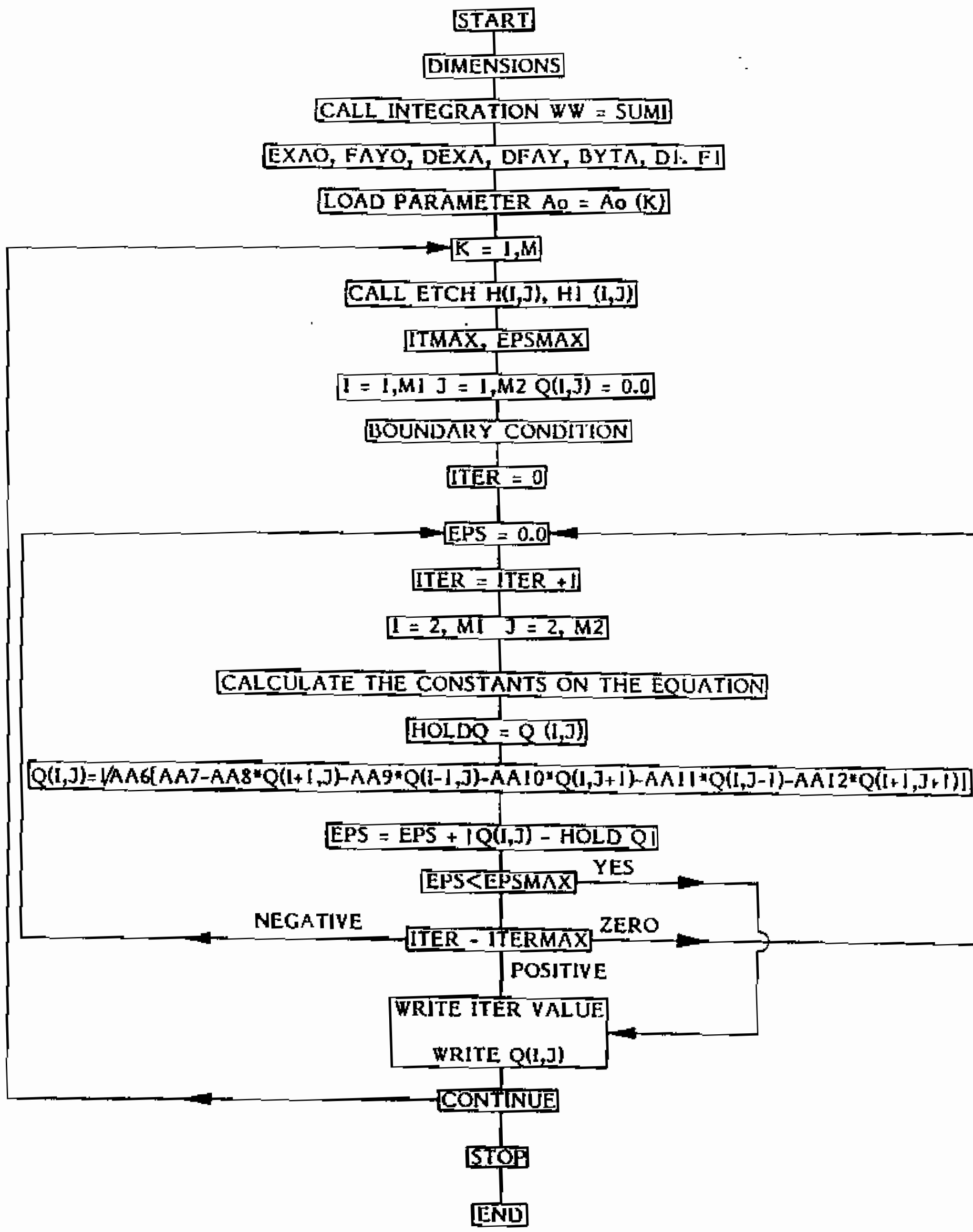


Fig. (7) Main flow chart.

$$\therefore \bar{q} = \frac{1}{12\alpha} \frac{h_o^2}{\eta_o a V}$$

Substituting this value and the value of A_o in the equation (22) and rewrite this equation as follow :

$$\frac{1}{12\alpha} \frac{h_o^2}{\eta_o a V} = m\left(\frac{3W}{8bh_o E'}\right)^{-n}$$

$$h_o^2 = 12m(\alpha\eta_o V)*(a)*\left(\frac{3W}{8bE'}\right)^{-n}*\left(\frac{1}{h_o}\right)^{-n}$$

By dividing this equation on R^2 to get a dimensionless form

$$\left(\frac{h_o}{R}\right)^2 = 12m\left(\frac{\alpha\eta_o V}{R}\right)*\left(\frac{a}{R}\right)*\left(\frac{3W}{8bE'}\right)^{-n}*\left(\frac{1}{h_o}\right)^{-n}$$

$$\frac{h_o^{2-n} R^{-n}}{R^2 R^{-n}} = 12m\left(\frac{\alpha\eta_o V}{R}\right)*\left(\frac{a}{R}\right)*\left(\frac{3W}{8bE'}\right)^{-n}$$

$$\left(\frac{h_o}{R}\right)^{2-n} = 12m\left(\frac{\alpha\eta_o V}{R}\right)*\left(\frac{a}{R}\right)*\left(\frac{3W}{8bRE'}\right)^{-n}$$

$$\therefore \frac{h_o}{R} = (12m)^{\frac{1}{2-n}}*\left(\frac{\alpha\eta_o V}{R}\right)^{\frac{1}{2-n}}*\left(\frac{a}{R}\right)^{\frac{1}{2-n}}*\left(\frac{3W}{8bRE'}\right)^{\frac{-n}{2-n}} \longrightarrow ** \quad (23)$$

From Fig(8), for using a pair of gears of 22° helix angle m=0.15855 n=0.702603

$$\frac{h_o}{R} = 1.642\left(\frac{\alpha\eta_o V}{R}\right)^{0.771}*\left(\frac{a}{R}\right)^{0.771}*\left(\frac{3W}{8bRE'}\right)^{0.542}$$

for pair of gears of 34° helix angle m=0.12425 , n=0.75968

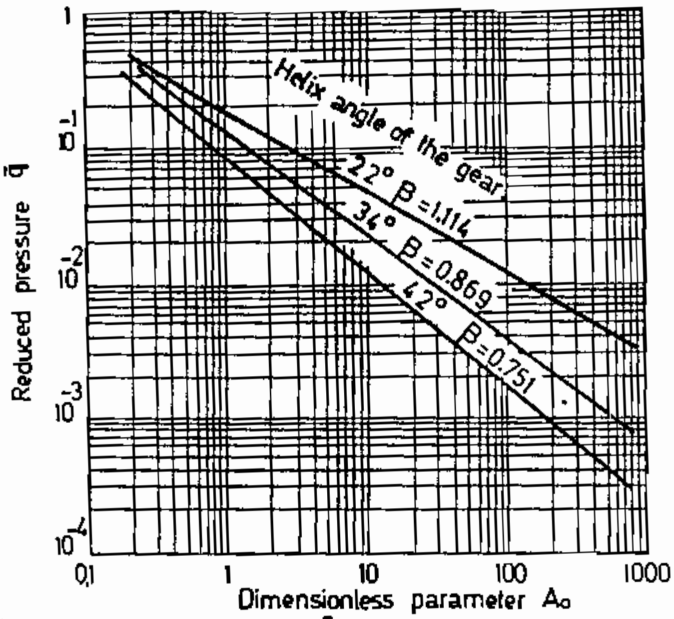
$$\frac{h_o}{R} = 1.38\left(\frac{\alpha\eta_o V}{R}\right)^{0.806}*\left(\frac{a}{R}\right)^{0.806}*\left(\frac{3W}{8bRE'}\right)^{-0.612}$$

for pair of gears of 42° helix angle m=0.0861 , n=0.8426

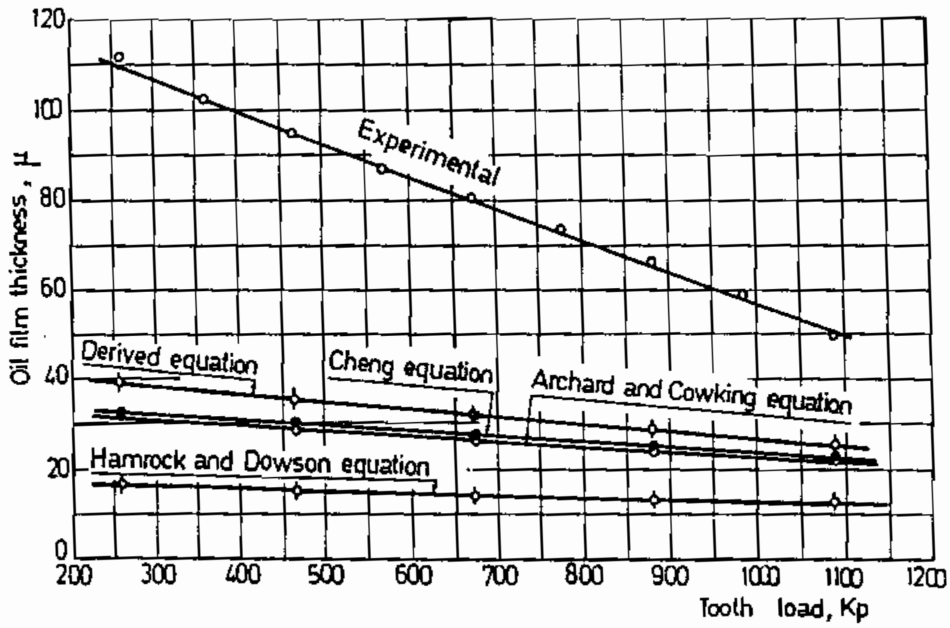
$$\frac{h_o}{R} = 1.029\left(\frac{\alpha\eta_o V}{R}\right)^{0.864}*\left(\frac{a}{R}\right)^{0.864}*\left(\frac{3W}{8bRE'}\right)^{-0.728}$$

COMPARISON OF THE DERIVED THEORETICAL FORMULA WITH THE EXPERIMENTAL RESULTS AND THE EXISTING THEORETICAL FORMULAE :

Values of the oil film thickness obtained from the derived theoretical formula are compared with the experimental values of the oil film thickness [20] and the theoretical values obtained from the existing theoretical formulae carried out by Archard & Cowking, Cheng and Hamrock & Dowson. These formulae are presented in appendix (I). Fig (9) shows the calculated



Fig(8) Change of reduced pressure $\bar{q}$ with the change of the dimensionless parameter A_o at different helix angles of the gears



Fig(9) Change of the calculated values of the oil film thickness obtained from the derived equation with the tooth load as compared with the experimental values and with those obtained from Archard & Cowking, Cheng and Hamrock & Dowson equations at speed 3000 r.p.m using oil of kinematic viscosity 462 cSt at 40°C for a pair of gears of 22° helix angle values of the change of the oil film thickness obtained from the derived formula with the applied tooth load as compared with the experimental value and with those obtained from Archard & Cowking, Cheng and Hamrock & Dowson at speed 3000 r.p.m, oil of kinematic viscosity 462 cSt at 40 °C using pair of gears of 22° helix angle. From this figure it is noticed that ; the experimental value of the oil film thickness is higher than that of the theoretical one. The existing theoretical formulae are based on a disc machine which give point or elliptical area of contact, the radii of curvature are convex for the two planes which differed than the experimental conditions. For the experimental conditions the concave convex radii of curvature for the plane normal to the tooth make a reservoir for the oil supplied to the track of contact and make a pumping effect, in addition to this the effect of helix angle make the wedge angle effect. These conditions make a better effect on the formation of the oil film. The rate of decrease of the experimental oil film thickness with applied tooth load is greater than the theoretical one due to increasing the effect of dynamic load, decreasing the effect of wedge action and friction effect on the experimental values. Furthermore, load parameter on the theoretical equations has a slight effect upon the oil film. Oil film thickness obtained from the derived theoretical formula is greater than any value obtained from the existing theoretical formulae and is in more agrrement with the experimental values. This is due to the fact that all parameters of the Reynolds' equation were taken into consideration ; effect of velocities along the helix angle and in normal plane of the gear tooth, effect of motions due to variations of the oil film thickness in normal plane of the tooth and along the helix angle, and the effect of motion in the plane normal to the oil film thickness. Also the effect of convex and concave radii of curvature in normal plane and the convex convex radii of curvature along the helix angle. The obtained oil film thickness by using Cheng equation are greater than the other values obtained from the other existing theoretical formulae due to the assumptions mentioned in appendix (1). While Archared & Cowking equation give oil film slight differed and smaller than the oil film by Cheng. Oil film thickness obtained by Hamrock & Dowson equation is minimum. This is due to using the model of elasticity mentioned in appendix (1).

CONCLUSION

A procedure for the numerical solution of the elastohydrodynamic lubrication for the gears of circular-arc tooth-profile is presented. This calls for the simultaneous solution of the elasticity and Reynolds equations using the finite difference technique. The derived theoretical formula represents a simple tool for the designers, where the oil film thickness can be calculated, and the corresponding load capacity of the gears of circular-arc tooth-profile are determined for any given speed, helix angle, radii of curvature, lubricant properties and kind of material of the gears. The calculated oil film thickness obtained from the presented formula is compared with the existing theories of elastohydrodynamic lubrication equations developed by Archard & Cowking, Cheng and Hamrock & Dowson and shows that the calculated values of the oil film thickness obtained from the presented formula are greater. Also the oil film thickness obtained from the derived theortical formula is compared with the experimental results [20] and shows the oil film thickness obtained from the derived formula is smaller.

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APPENDIX (I)

EXISTING THEORETICAL FORMULAE

Archard and Cowking equation:

$$\frac{h}{R} = 1.37\, \bar{G}^{0.74} \cdot \bar{U}^{0.74} \cdot \bar{P}_{Hz}^{-0.22} \quad , \quad \bar{G} = \alpha \cdot E_d \quad , \bar{U} = \frac{\mu_o (u_1 + u_2)}{2 E_d R} \, , \, \bar{P}_{Hz} = \frac{P_{Hz}}{E_c} \, ,$$

$$P_{Hz} = \frac{3}{2} \frac{W}{\pi a^2} \, , \, \frac{1}{E_d} = \frac{1}{2} \left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right) \quad \text{and} \quad \frac{1}{R} = \frac{1}{R_{1y}} + \frac{1}{R_{2y}}$$

Cheng equation:

$$\frac{h}{\bar{R}_y}=\left(12C\,\frac{\mu_o \alpha U}{\bar{R}_y}\right)^{\frac{1}{2-n}} * \left(\frac{\pi}{2}\right)^{-\frac{n}{2-n}} * \left(\frac{P_{Hz}}{\bar{E}}\right)^{-\frac{1-2n}{2-n}} * \left(\frac{m^3\pi^2}{\beta^2(1+\bar{R}_y/\bar{R}_x)}\right)^{\frac{1-n}{2-n}},$$

$$m=\frac{b}{\sqrt[3]{\frac{3}{2}\pi\,\frac{W}{(1/2\bar{R}_x+1/2\bar{R}_y)\bar{E}}}}, \quad P_{Hz}=\frac{3}{2}\,\frac{W}{\pi a b}, \quad \frac{1}{\bar{E}}=\frac{1-\nu_1^2}{\pi E_1}+\frac{1-\nu_2^2}{\pi E_2},$$

$$\beta=\frac{b}{a}, \quad \bar{R}_x=\frac{R_{1x}\cdot R_{2x}}{R_{2x}-R_{1x}} \quad \text{and} \quad \bar{R}_y=\frac{R_{1y}\cdot R_{2y}}{R_{1y}+R_{2y}}$$

Values of C and n for each β are given:

β	C	n
0.5	0.065	0.548
1	0.088	0.620
2	0.095	0.642

Hamrock and Dowson equation:

$$\frac{h}{\bar{R}_y}=3.63\,\bar{U}^{0.68} * \bar{G}^{0.49} * \bar{W}_o^{-0.073} * (1-e^{-0.68K}),$$

$$\bar{W}_o=\frac{W}{E_d\bar{R}_y^2}, \quad \bar{U}=\frac{(u_1+u_2)\mu_o}{2E_d\bar{R}_y} \quad \text{and} \quad K=1.03\left(\frac{\bar{R}_y}{\bar{R}_x}\right)^{0.64}$$

General Assumptions For All Equations :

1- Oil film is incompressible and isothermal.

2- These equations based on a disc machine which give point or elliptical area of contant. the radii of curvature in the two planes are convex. This is differed than the experimental condition.

3- Neglecting the effect of the dynamic load and friction.

Cheng solved the elastohydrodynamic problem for elliptical area of contact. The deformation contour in the inlet region was calculated according to Hertz theory for elliptical contact. The Hertzian contact zone is assumed to form a parallel film region and the generation of high pressure in the approaches to the Hertzian zone is considered.

Archard & Cowking treated the point contact as an assembly of elemental line contact. They assumed a Hertzian deformation for the case of a sphere on a plate. The Hertzian contact zone is assumed to form a parallel film region and the generation of high pressure in the approaches to the Hertzian zone is considered.

Hamrock & Dowson solve the elastohydrodynamic problem for point contact this required the solution of the elasticity and Reynolds equations. They presented an elasticity model in which the conjunction was divided into equal rectangular areas with a uniform pressure applied over each area.