جامعة المنوفية
كلية هندسة شبين الكوم
قسم العلوم الأساسية الهندسية
الفصل الدراسي الثاني ٢٠١٤م

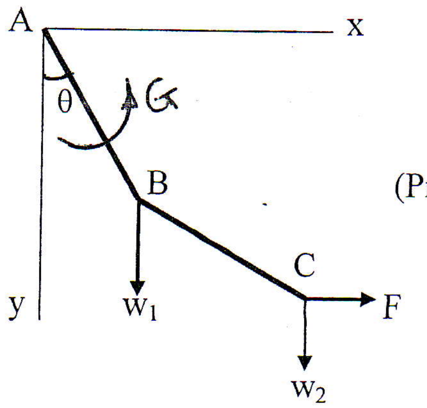
إختبار دراسات عليا مستوى ٥٠٠
أسس الميكانيكا التحليلية (BES513)
التاريخ: ١١/٦/٢٠١٤م
الزمن: ثلاث ساعات

Solve all the following questions:

1) The parametric equations of a moving particle are: $x=s^2-4s+3$, $y=s^2$, $z=2s$, where s is the traveling distance measured on the path of the particle from a known point. The distance s is expressed in terms of time t as: $s=t^2+2t$. Determine the physical coordinates, generalized coordinates, and generalized velocities and their conditions. Deduce the velocity equation and prove that it represents a holonomic constraint which satisfies the path equation of the particle.

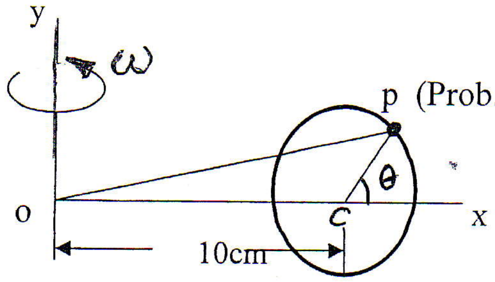
2) If the origin of a moving two-dimensional rectangular coordinate system (X_2,Y_2) has an initial velocity (20,10) m/s and constant acceleration (32,16)m/s^2 while the axes rotate with constant angular velocity ω. Determine the transformation equations of the moving coordinates relative to the fixed reference frame (X_1,Y_1) at any time t. If the lengths (x_1,y_1) m locate the point p relative to the fixed frame and (-2,4) m locate the point p relative the moving frame. Find the lengths (x_1,y_1) when $\omega=\pi$ rad/sec and $t=1/2$ sec.

3) In Fig (1) two light rods AB and BC of lengths a and b are smoothly joined at B. The end A is smoothly hinged at a vertical wall, a weight w_1 is attached at B and a weight w_2 and a horizontal force F act at point C. The rod AB is acted upon by couple G. Find the number of degrees of freedom of the system. What are the generalized coordinates of the system? For equilibrium apply the principle of virtual work to find the relation between w_1, w_2 ,G and F. Find the horizontal force F and the condition of equilibrium when $\theta=\phi=\pi/4$.



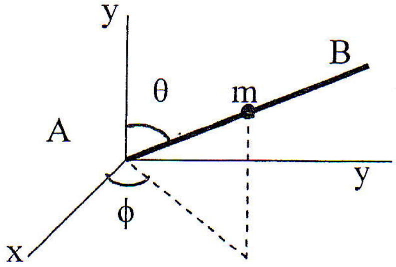
(Prob. 3)

4) A bead moves on a vertical smooth circular wire of radius 5cm which is made to rotate about y axis with constant angular speed $\omega=3$ rad/s. The center of the wire is at distance 10 cm from the axis y. Find the velocity and acceleration of the bead at $t=\pi/10$sec and $\dot{\theta}=5$ rad/sec.



(Prob.4)

5) A ring of mass m=2kg is forced to start to move from an end A along a uniform heavy rod of mass M=3kg and length L=2m with constant relative velocity v=1m/s. If the rod is free to rotate about end A which is fixed. Derive and discuss Lagrange's equations of motion of the system.



(Prob.5)

6) Using Lagrange's equations show that the equations of motion of a right circular cone rolls and slides on a smooth horizontal flat surface are given by the system of equations $M\underline{q}=\underline{f}$, where M is the mass matrix, $\underline{q}$ is the velocity vector, and $\underline{f}$ is the force vector.

7) A particle of mass m moves in x-y plane under the influence of a central force $\underline{\mathbf{F}}=F_r\,\underline{\mathbf{u}}_r+F_\theta\,\underline{\mathbf{u}}_\theta$. If the potential of this force is $V=cr^2$ and the velocity vector in polar coordinates is $\underline{\mathbf{v}}=r\,\mathbf{u}_r+r\theta\,\mathbf{u}_\theta$. Deduce Lagrange's function and equations of motion of the particle. Also, deduce Hamilton's function and Hamilton's equations.