Menoufia University
Faculty of Engineering, Shebin El-Kom,
Basic Engineering sciences Department
Second Semester Examination, 2013-2014
Date of Exam: 9/6/2014

Subject: Introduction to ordinary differential equations.
Code : BES 506
Year : postgraduate students
Time Allowed : 3 hours
Total Marks: 100 marks

Answer the following questions

Question 1 (20 marks)

a) Solve the following D.E by using the Variation of parameters:

$$y''' - 3y'' + 2y = \frac{e^x}{e^{-x} + 1}$$

b) Find the general solution of the differential equation:

$$x^2 y'' + 2xy' + (x^2 - 1)y = 0$$

Question 2 (20 marks)

a) Solve the following D.E using the method of undetermined coefficients:

$$y'' - 3y' - 4y = 3e^{2t} + 2\sin t - 8e^{2t}\cos 2t$$

b) Show that $y_1(t) = t^{\frac{1}{2}}$, and $y_2(t) = t^{-1}$ form a fundamental set of solutions for the equation using Wronskian determinant:

$$2t^2 y'' + 3ty' - y = 0 \qquad t > 0$$

Question3 (20 marks)

a) Write briefly the steps of Euler algorithm to solve the differential equations.

b) Solve the initial value problem using Euler's method with step size h=0.1, then compare the approximate solution with the actual solution :

$$y' = 3 + e^{-t} - \frac{1}{2}y, \qquad y(0) = 1$$

Question 4

(20 marks)

a) Solve the following Riccati Equation:

$$y' = \frac{2\cos^2 x - \sin^2 x + y^2}{2\cos x} \qquad y(0) = -1$$

(Knowing that $y = \sin x$ is a particular solution of the given differential equation)

b) For Legendre D.E $(1-x^2)y'' - 2xy' + n(n+1)y = 0$. Obtain the Legendre $P_4(x)$ directly from Legendre's equation of order 4 by assuming a polynomial of degree 4, i.e. $y = ax^4 + bx^3 + cx^2 + dx + e$

Question 5

(20 marks)

a) Show that $y = xJ_1(x)$ is a solution of $x^2y'' - y' - x^2J_0'(x) = 0$

b) A mass weighing 4 lb stretches a spring 2 in suppose that the mass is displaced an additional 6 in the positive direction and then released as in Figure. The mass is in a medium that exerts a viscous resistance of 6 lb when the mass has velocity of 3ft/s. Formulate the initial value problem that governs the motion of the mass.

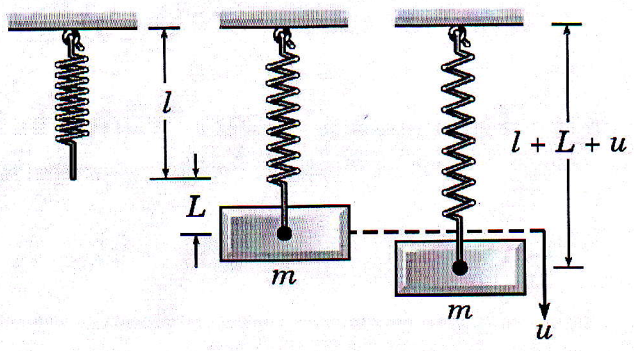

Fig. Spring mass system

With my best wishes

Dr. Eng. Rizk Masoud