Sci. J. Fac. Sci. Minufia Univ., 2007, Vol. XXI, 45-55

ON FUZZY δ-CONTINUOUS MULTIFUNCTIONS

R.A. MAHMOUD (*) KHALED M. A. AL-HAMADI (**) S. B. NIMSE(**)
(*) Department of Mathematics, Faculty of Science, Menoufia University, Shbeen El-Koom, EGYPT.
(**)Department of Mathematics, University of Pune, INDIA

ABSTRACT

Multifunctions on topological spaces are one of the important tools for constructing a new spaces from previously known ones and establishing several new properties of these spaces. By the concept of α-compact fuzzy topological spaces which known before, we devoted this paper to characterize the concept of fuzzy δ-continuous multifunctions [13] utilizing each of fuzzy δ^*-operator closed sets, fuzzy δ^*-open sets and fuzzy δ-closure operator. Also we presented the relationships between fuzzy δ-continuous multifunctions and some other corresponding forms of fuzzy continuity. Moreover, we gave counter examples which showed that the inverse of these relationships doesn't hold in general. Finally, we investigated the effect of these new classes of fuzzy multifunctions on some types of fuzzy topological spaces.

1. INTRODUCTION AND PREREQUISITES.

Throughout the last half century, some types of weaker forms of continuity on fuzzy topological spaces have been considered by several researchers (e.g.cf [2, 4, 7, 12, and 13]), by corresponding concepts of fuzzy openness such as: fuzzy regular-open [2], fuzzy semi-open [2] and fuzzy preopen [3]. On the other hand, α-compact fuzzy topological spaces were first introduced by Ganter et al [8], in 1986. Weaker forms of α-compactness on fuzzy topological spaces were also constricted throughout the last quarter century by other staff of researchers (e.g.cf [1, 6, 9, 11, 16]).

Throughout this paper, (X,τ) and (Y,σ) mean fuzzy topological spaces (sometimes will be denoted by X and Y, for short, respectively) on which no separation axioms are assumed unless explicitly stated. For any fuzzy point x and any fuzzy set λ, we write $x_\varepsilon \in \lambda$ iff $\varepsilon \leq \lambda(x)$, where ε is a membership of a fuzzy point x. A fuzzy set λ is called quasi-coincident with a fuzzy set μ, denoted by

($\lambda\ q\ \mu$), iff there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. Otherwise, λ and μ are quasi-noncoincident which denoted by ($\lambda\ \not{q}\ \mu$) Therefore, throughout this paper the imitated notions related with the above mentioned concepts are given. New characterizations of fuzzy δ-continuous multifunction are established and other new basic and useful results which have been discussed.

Definition 1.1:

[13] A fuzzy multifunction F: X → Y is called fuzzy lower (resp. upper) δ-continuous at any point $x_o \in X$, iff for every fuzzy open set μ in (Y,σ) with $x_o \in F^-(\mu)$ (resp. $x_o \in F^+(\mu)$), there exists $\lambda \in \tau(x_o)$ such that $int(cl(\lambda)) \subseteq F^-(int(cl(\mu)))$ (resp. $int(cl(\lambda)) \subseteq F^+(int(cl(\mu)))$). While, F is fuzzy lower and fuzzy upper δ-continuous on (X,τ) iff it is, respectively, so at each point of X.

Definition 1.2:

[16] A fuzzy subset μ of a fuzzy topological space X is said to be RQ-neighbourhood (RQ-nbd, for short) of a fuzzy point xε if and only if there is a fuzzy regular open set λ in X such that $(x\varepsilon\ q\ \lambda) \leq \mu$.

Definition 1.3:

[16] A fuzzy point xε is said to be fuzzy δ*-cluster point of a fuzzy subset μ in a fuzzy topological space X if and only if each RQ-nbd of xε is quasi-coincident with μ. The union of all fuzzy δ*-cluster points of μ is defined to be a δ*-closure of μ and denoted by δ*-cl(μ). If μ = δ*-cl(μ), then μ is called fuzzy δ*-closed. The complement of a fuzzy δ*-closed set is called fuzzy δ*-open.

Theorem 1.4:

[16] A fuzzy subset λ is δ*-open if and only if each fuzzy point xε with fuzzy point (xε q λ), there is a fuzzy regular open set μ in X such that $(x\varepsilon\ q\ \mu) \leq \lambda$.

Definition 1.5:

[16] A fuzzy subset λ of a fuzzy topological space X is said to be fuzzy α-nearly compact relative to X, if for each fuzzy open α-shading of S(λ) has a finite subfamily such that the fuzzy interiors of whose fuzzy closures of its members are an α-shading of S(λ).

Theorem 1.6:

[16] A fuzzy subset λ is α-nearly compact relative to a fuzzy topological space X, if and only if each α-shading of S(λ) by fuzzy δ*-open sets has a finite α-subshading.

Corollary 1.7:

[16] A fuzzy topological space X is an α-nearly compact if and only if each fuzzy δ*-open α-shading of X has a finite α-subshading.

Theorem 1.8:

[16] If X is a fuzzy Hausdorff space, then for any pair distinct points x, y $\in$ X, there are a fuzzy regular open sets μ, ρ of X such that $\mu(x) = \rho(y) = 1$, $\rho(x) = \mu(y) = 0$, $\delta^*\text{-cl}(\mu) \leq 1 - \rho$ and $\delta^*\text{-cl}(\rho) \leq 1 - \mu$.

Theorem 1.9:

[16] If λ is a fuzzy δ^*-closed set of a fuzzy α-nearly compact space X, then λ is fuzzy α-nearly compact relative to X.

Definition 1.10:

[9] A fuzzy multifunction F: X $\to$ Y is said to be fuzzy almost open S, if F(μ) is fuzzy open of Y for each fuzzy regular open set μ of X.

Lemma 1.11:

[10] Let (X,τ) be a fuzzy topological space $\lambda \in \tau$ and $\mu \subseteq X$ be a fuzzy regular open. Then $(\lambda \wedge \mu)$ is a fuzzy regular open set of $(\lambda, \tau\lambda)$.

2. New charactizations of fuzzy δ continuous multifunctions.

Theorem 2.1:

Let F: X $\to$ Y, then the following statements are equivalents:

(1) F is fuzzy lower δ-continuous.

(2) For each $x_\varepsilon \in X$ and for each fuzzy open set μ with $(F(x_\varepsilon)\ q\ \mu)$, there is a fuzzy open set λ with $(x_\varepsilon\ q\ \lambda)$ such that $\text{int}(\text{cl}(\lambda)) \leq F^-(\text{int}(\text{cl}(\mu)))$.

(3) For each fuzzy regular open set μ in Y, $F^-(\mu)$ is a fuzzy δ^*-open in X.

(4) For each fuzzy regular closed set μ in Y, $F^+(\mu)$ is a fuzzy δ^*-closed in X.

(5) For each $x_\varepsilon \in X$ and for each fuzzy regular open set μ in Y with $(F(x_\varepsilon)\ q\ (\mu))$, there is a fuzzy regular open set β with $(x_\varepsilon\ q\ \lambda)$ such that $\lambda \leq F^-(\mu)$.

(6) For each fuzzy δ^*-open set μ of Y, $F^-(\mu)$ is a fuzzy δ^*-open in X.

(7) For each fuzzy δ^*-closed set λ of Y, $F^-(\lambda)$ is a fuzzy δ^*-closed in X.

(8) $F(\delta^*\text{-cl}(\lambda)) \leq \delta^*\text{-cl}(F(\lambda))$ for each fuzzy subset λ of X.

(9) $\delta^*\text{-cl}(F^+(\mu)) \leq F^+(\delta^*\text{-cl}(\mu))$ for each fuzzy subset μ of Y.

Proof:

(1) $\Rightarrow$ (2) By the fact that: if x_ε is a fuzzy point in X and μ is a fuzzy set of X, then $(x_\varepsilon\ q\ \mu)$ if and only if $x_{1-\varepsilon} \in \mu$. So, let x_ε be a fuzzy point in X and μ be a fuzzy open set of Y with $(F(x_\varepsilon)\ q\ \mu)$. First take $\varepsilon < 1$, then $F(x_{1-\varepsilon}) \in \mu$. Thus there is a fuzzy open set λ of X with $x_{1-\varepsilon} \in \lambda$ such that $\text{int}(\text{cl}(\lambda)) \leq F^-(\text{int}(\text{cl}(\mu)))$. Hence there is a fuzzy open set λ of X with $(x_\varepsilon\ q\ \lambda)$ such that $\text{int}(\text{cl}(\lambda)) \leq F^-(\text{int}(\text{cl}(\mu)))$. When $x_\varepsilon = 1$, $S(F(x)) = t$ (say) > 0. We may consider the fuzzy point P with support F(x)) and $P(F(x)) = 1 - \rho$, $0 < \rho < t$. By [15], P q μ, and hence

$F(x_\rho) \in \mu$, so it follows that, there is a fuzzy open set λ of X with $x_\rho \in \lambda$ such that $int(cl(\lambda)) \le F^-(int(cl(\mu))$. Hence $x_{1-\rho} \le \lambda$ and $int(cl(\lambda)) \le F^-(int(cl(\mu))$. Therefore $x_\varepsilon \le \lambda$ and $int(cl(\lambda)) \le F^-(int(cl(\mu))$.

(2) $\Rightarrow$ (1) Let x_ε be a fuzzy point in X and μ be a fuzzy open set of Y with $F(x_\varepsilon) \in \mu$. Then $(F(x_{1-\varepsilon}\ q\ \mu)$. By (2), there is a fuzzy open set λ of X with $(x_{1-\varepsilon}\ q\ \lambda)$ such that $int(cl(\lambda)) \le F^-(int(cl(\mu))$. Hence there is a fuzzy open set λ of X with $x_\varepsilon \in \mu$ and $int(cl(\lambda)) \le F^-(int(cl(\mu))$. Therefore F is fuzzy lower δ-continuous.

(1) $\Rightarrow$ (3) Let μ be a fuzzy regular open set in Y and x_ε be a fuzzy point in X with $(x_\varepsilon\ q\ F^-(\mu))$. Then $(F(x_\varepsilon)\ q\ \mu)$. By (2), there is a fuzzy open set λ of X with $(x_\varepsilon\ q\ \lambda)$ such that $int(cl(\lambda)) \le F^-(int(cl(\mu)) = \mu$. Putting $int(cl(\lambda)) = \eta$ hence η is fuzzy regular open set of X such that $(x_\varepsilon\ q\ \eta) \le F^-(\mu)$. By Theorem (1.4), $F^-(\mu)$ is a fuzzy δ^*-open.

(3) $\Rightarrow$ (4) Obvious.

(3) $\Rightarrow$ (5) Let x_ε be a fuzzy point in X and μ be a fuzzy regular open set in Y with $(F(x_\varepsilon)\ q\ \mu)$. By (3), $F^-(\mu)$ is a fuzzy δ^*-open with $(x_\varepsilon\ q\ F^-(\mu))$. By Theorem (1.4) above, there is a fuzzy regular open set λ of X such that $(x_\varepsilon\ q\ \lambda) \le F^-(\mu)$. Therefore, there is a fuzzy regular open set λ of X with $(x_\varepsilon\ q\ \lambda)$ such that $\lambda \le F^-(\mu)$.

(5) $\Rightarrow$ (6) Let μ be a fuzzy above δ^*-open set of Y and x_ε be a fuzzy point in X with $(x_\varepsilon\ q\ F^-(\mu))$. Then $(F(x_\varepsilon)\ q\ \mu)$. By Theorem (1.4) above, there is a fuzzy regular open set η of Y such that $(F(x_\varepsilon)\ q\ \eta) \le \mu$. By (5), there is a fuzzy regular open set λ of X with $(x_\varepsilon\ q\ \lambda)$ such that $\lambda \le F^-(\eta)$. Thus we obtain $(x_\varepsilon\ q\ \lambda) \le F^-(\eta) \le F^-(\mu)$. By Theorem (1.2) above, $F^-(\mu)$ is a fuzzy δ^*-open.

(6) $\Rightarrow$ (7) Obvious.

(7) $\Rightarrow$ (8) Let λ be a fuzzy set in X, then $\delta^*\text{-}cl(F(\lambda))$ is fuzzy δ^*-closed set of Y. By (7), $F^-(\delta^*\text{-}cl(F(\lambda)))$ is a fuzzy δ^*-closed in X. Let x_ε be a fuzzy point in X and $x_\varepsilon \notin F^-(\delta^*\text{-}cl(F(\lambda)))$, then for some RQ-nbd μ of x_ε, $(\mu\ \not q\ F^-(\delta^*\text{-}cl(F(\lambda))))$. By proposition [16], $\mu \le [F^-(\delta^*\text{-}cl(F(\lambda)))]' \le \lambda'$ and hence $(\mu \not q\ \lambda)$. Then $x_\varepsilon \notin \delta^*\text{-}cl(F(\lambda))$. Therefore $F(\delta^*\text{-}cl(\lambda)) \le \delta^*\text{-}cl(F(\lambda))$.

(8) $\Rightarrow$ (9) Let μ be a fuzzy subset of Y. By (8), we have $F(\delta^*\text{-}cl(F^+(\mu)) \le \delta^*\text{-}cl(F(F^+(\mu))) \le \delta^*\text{-}cl(\mu)$ and hence $\delta^*\text{-}cl(F^+(\mu)) \le F^+(\delta^*\text{-}cl(\mu))$.

(9) $\Rightarrow$ (2) Let x_ε be a fuzzy point in X and μ be a fuzzy subset of Y with $(F(x_\varepsilon)\ q\ \mu)$. Put $\gamma = 1 - int(cl(\mu))$. Then γ is fuzzy δ^*-closed set of Y. By (9), $\delta^*\text{-}cl(F^+(\gamma)) \le F^+(\delta^*\text{-}cl(\gamma)) = F^+(\gamma)$ and hence $F^+(\gamma)$ is fuzzy δ^*-closed set in X. Then $F^-(int(cl(\mu))$ is a fuzzy δ^*-open in X with $x_\varepsilon\ q\ F^-$

(int(cl(μ))). By Theorem (1.4), there is a fuzzy regular open set λ of X such that (x_ε q λ) ≤ F^-(int(cl(μ))). Thus we obtain int(cl(λ)) = F(λ) ≤ F^-(int(cl(μ))).

Theorem 2.2:

Let F: X → Y, then the following statements are equivalents:

(1) F is fuzzy upper δ-continuous.

(2) For each x_ε ∈ X and for each fuzzy open set μ with (F(x_ε) q μ), there is a fuzzy open set λ with (x_ε q λ) such that int(cl(λ)) ≤ F^+(int(cl(μ)).

(3) For each fuzzy regular open set μ in Y, F^+(μ) is a fuzzy δ^*-open in X.

(4) For each fuzzy regular closed set μ in Y, F^-(μ) is a fuzzy δ^*-closed in X.

(5) For each x_ε ∈ X and for each fuzzy regular open set μ in Y with (F(x_ε) q μ), there is a fuzzy regular open set λ with (x_ε q λ) such that λ ≤ F^+(μ).

(6) For each fuzzy δ^*-open set μ of Y, F^+(μ) is a fuzzy δ^*-open in X.

(7) For each fuzzy δ^*-closed set μ of Y, F^-(μ) is a fuzzy δ^*-closed in X.

(8) F(δ^*-cl(λ)) ≤ δ^*-cl(F(λ)) for each fuzzy subset λ of X.

(9) δ^*-cl(F^-(μ)) ≤ F^-(δ^*-cl(μ)) for each fuzzy subset μ of Y.

Proof:

It is quite similar to the case of fuzzy lower δ-continuous which was discussed in Theorem (2.1)

Theorem 2.3:

If a fuzzy multifunction F: X → Y is fuzzy lower (resp. upper) δ-continuous and λ is a fuzzy open subset of X, then F|λ : λ → Y is fuzzy lower (resp. upper) δ-continuous.

Proof:

Let xε be a fuzzy point in λ and ξ be a fuzzy regular open in Y with (F(xε) q ξ). Then there is a fuzzy regular open set μ of X with (xε q μ) such that μ ≤ F–(ξ). By Lemma, μ ∧ λ is a fuzzy regular open set of λ. Hence (μ ∧ λ) is a fuzzy regular open set of λ with (xε q λ) such that (F|λ)(μ ∧ λ) ≤ ξ. Therefore, F|λ is fuzzy lower δ-continuous. While, the proof of the fuzzy upper case is similarly.

Definition 2.4:

A fuzzy multifunction F: X → Y is said to be fuzzy weakly almost open, if for each fuzzy regular open set λ of X, then F(cl(λ)) ≤ cl(F(λ)).

Theorem 2.5:

If a fuzzy multifunction F: X → Y is fuzzy weakly almost open and fuzzy lower weakly continuous multifunction, for each fuzzy regular closed (resp.

fuzzy regular open) set μ in Y , then $F-(\mu)$ is fuzzy regular closed (resp. $F+(\mu)$ is fuzzy regular open) in X.

Proof:

Let μ be a fuzzy regular closed set in Y, then $int(\mu)$ is a fuzzy regular open set in Y. Since F: X → Y is fuzzy weakly almost open, we have $cl(F+(\mu)) = cl(F+(cl(int(\mu)))) \leq cl(F+(int(\mu))) \leq F+(cl(int(\mu))) = F+(\mu)$. Hence $F+(\mu)$ is fuzzy closed. We know that $F+(int(\mu)) \leq int(F+(\mu))$. Thus we obtain $F+(\mu) = F+(cl(int(\mu))) \leq cl(F+(int(\mu))) \leq cl(int(F+(\mu))) \leq cl(F+(\mu)) = F+(\mu)$. Then $F+(\mu)$ is fuzzy regular closed in X. Similarly the proof of the other case.

Theorem 2.6:

If a fuzzy multifunction F: X → Y is a fuzzy weakly almost open and fuzzy lower (resp. upper) weakly continuous multifunction, then F is fuzzy lower (resp. upper) almost continuous.

Proof:

Let $x \in X$ and any fuzzy open set μ of Y such that $(F(x)\ q\ \mu)$, there exists a fuzzy regular open set λ such that $F(\lambda) \leq int(cl(\mu)$. Since F is fuzzy weakly almost open, we have $F(\lambda) \leq cl(F(\lambda)) \leq int(cl(\mu)$. Hence F is fuzzy lower almost continuous. Also the other case follows similarly.

Corollary 2.7:

If a fuzzy multifunction F: X → Y is a fuzzy weakly almost open and fuzzy lower (resp. upper) weakly continuous multifunction, then F is fuzzy lower (resp. upper) δ-continuous.

Corollary 2.8:

If a fuzzy multifunction F: X → Y is a fuzzy almost open and fuzzy lower (resp. upper) continuous multifunction, then F is fuzzy lower (resp. upper) δ-continuous.

Definition 2.9:

A fuzzy multifunction F: X → Y is said to be fuzzy weakly open if $F(\lambda) \leq int(cl(F(\lambda)))$, for each fuzzy open set λ of X.

Theorem 2.10:

If a fuzzy multifunction F: X → Y is fuzzy weakly open and fuzzy lower almost continuous multifunction, for each fuzzy regular open (resp. fuzzy regular closed) set μ in Y , then $F-(\mu)$ is fuzzy regular open (resp. $F+(\mu)$ is fuzzy regular closed) in X.

Proof:

Let μ be a fuzzy regular open set in Y. and F be fuzzy lower almost continuous, then $F-(\mu)$ is fuzzy open in X and $F-(\mu) \leq int(cl(F-(\mu)))$. Since F is fuzzy lower almost continuous multifunction we have int(cl(F−(μ))) ≤ cl(F−(μ)) ≤ F−(cl(μ)). Moreover, F is fuzzy weakly open, this means F(int(cl(F−(μ)))) ≤ int(F(cl(F−(μ)))) ≤ int(cl(μ)) = μ. Which implies, int(cl(F−(μ))) ≤ F−(μ) and hence int(cl(F−(μ))) = F−(μ). Thus F−(μ) is fuzzy regular open in X. But the other case is similar.

Corollary 2.11:

If a fuzzy multifunction F: X → Y is a fuzzy weakly open and fuzzy lower (resp. upper) almost continuous multifunction, then F is fuzzy lower (resp. upper) δ-continuous.

Corollary 2.12:

If a fuzzy multifunction F: X → Y is a fuzzy almost open S. and fuzzy lower (resp. upper) almost continuous multifunction, then F is fuzzy lower (resp. upper) δ-continuous.

Definition 2.13:

A fuzzy multifunction F: X → Y is called fuzzy lower (resp. upper) θ-weakly continuous at any point xε ∈ X, iff for every fuzzy open set μ of Y with xε ∈ F−(μ) (resp. xε ∈ F+(μ)), there exists a fuzzy open set λ of X with (xε q λ) such that int(cl(λ)) ≤ F−(cl(μ)) (resp. int(cl(λ)) ≤ F+(cl(μ))).

Remark 2.14:

One may notice that: Fuzzy upper almost continuous ⇒ Fuzzy upper θ-weakly continuous ⇒ Fuzzy upper weakly continuous. But the converses need not true in general as the following examples show.

Example 2.15:

Let X = {x, y} with topologies

τ1 = {X, ϕ, λ, μ, λ ∧ μ, λ ∨ μ } and τ2 = {X, ϕ, η}, where the fuzzy sets η, ρ, μ are defined as :

λ(x) = 0.3, λ(y) = 0.6, μ(x) = 0.5 , μ(y) = 0.5, η(x) = 0.3 and η(y) = 0.4

A fuzzy multifunction F: (X,τ1) → (Y,τ2) given by xε → F(xε) = {xε} is upper weakly θ-continuous, but it is not upper almost continuous.

Example 2.16:

Let X = {x, y} with topologies τ1 = {X, ϕ, λ} and τ2 = {X, ϕ, μ}, where the fuzzy sets ρ, μ are defined as :

λ(x) = 0.6, λ(y) = 0.6, μ(x) = 0.4 and μ(y) = 0.4

A fuzzy multifunction F: (X,τ1) → (Y,τ2) given by xε → F(xε) = {xε} is upper weakly continuous, but it is not upper weakly θ-continuous.

Theorem 2.17:

If a fuzzy multifunction F: X → Y is fuzzy lower (resp. upper) weakly θ-continuous and fuzzy almost open S., then F is fuzzy lower (resp. upper) δ-continuous.

Proof:

Let xε be a fuzzy point in X and μ be a fuzzy open set of Y with (F(xε) q μ). Since F is fuzzy lower weakly θ-continuous, then there is a fuzzy open set λ of X with (xε q λ) such that F(int(cl(λ))) ≤ cl(μ). Since F is fuzzy almost open S. and int(cl(ρ)) is fuzzy regular open in X, then F(int(cl(λ))) is fuzzy open in Y and hence, F(int(cl(λ))) ≤ int(cl(μ)). This shows that F is fuzzy lower δ-continuous. Also, the proof of fuzzy upper δ-continuous case is similar to the above one.

Theorem 2.18:

If a fuzzy multifunction F: X → Y is fuzzy lower weakly θ-continuous and fuzzy almost open S., then F–(μ) is fuzzy regular open (resp. F+(μ) is fuzzy regular closed) in X, for each fuzzy regular open set (resp. fuzzy regular closed set) μ in Y.

Proof:

It follows from Theorems (2.10), (2.17) and Remark (2.14).

Recall that; (X,τ) is fuzzy extremlly disconnected (abbreviated by FED), if the fuzzy closure of each fuzzy open set in it is also fuzzy open.

Theorem 2.19:

Let Y be a FED-space. If a fuzzy multifunction F: X → Y is fuzzy lower (resp. upper) weakly θ-continuous, then F is fuzzy lower (resp. upper) δ-continuous.

Proof:

Let F be a fuzzy lower weakly θ-continuous multifunction, for each fuzzy point xε in X and each fuzzy open set μ of Y with (F(xε) q μ), there is a fuzzy open set λ of X with (xε q λ) such that F(int(cl(λ))) ≤ cl(μ). Since Y is an FED-space, we have cl(μ) = int(cl(λ)) and hence F(int(cl(λ))) ≤ int(cl(μ)). Therefore, F is fuzzy lower δ-continuous. So, the proof of fuzzy upper δ-continuous of F is similar to the above.

3. δ-CONTINUOUS MULTIFUNCTIONS AND SOME FUZZY TOPOLOGICAL SPACES.

Theorem 3.1:

If a fuzzy multifunction $F: X \to Y$ is fuzzy lower δ-continuous and ζ is fuzzy α-nearly compact relative to X, then F(ζ) is fuzzy α-nearly compact relative to Y.

Proof:

Let {μ : i ∈ I} be a fuzzy regular open α-shading of S(F(ζ)), then {F–(μ) : i ∈ I} is an α-shading of S(ζ) by fuzzy δ*-open of X, for if x ∈ S(ζ) this implies F(x) ∈ F(S(ζ)) = S(F(ζ)) and hence there is γ such that μγ (F(x) > α, which implies (F–(μγ))(x) > α. Since ζ is fuzzy α-nearly compact relative to X, then by Theorem (1.6), there is a finite subfamily {F–(μi): i = 1, 2, ..., n} which are α-shading of S(ζ). For y ∈ S(F(ζ)) that, F(x) = y for some x ∈ S(ζ), then (F–(μγ))(x) > α which implies μγ (F(x)) = μγ (y) > α. Hence {μi : i = 1, 2, ..., n} is an α-shading of S(F(ζ)). Therefore, F(ζ) is α-nearly compact relative to Y

Theorem 3.2:

If a fuzzy multifunction $F: X \to Y$ is fuzzy upper δ-continuous and ζ is fuzzy α-nearly compact relative to X, then F(ζ) is fuzzy α-nearly compact relative to Y .

Proof:

Similar to Theorem (3.1).

Corollary 3.3:

If a fuzzy multifunction $F: X \to Y$ is a fuzzy lower (resp. upper) δ-continuous surjective multifunction and X is fuzzy α-nearly compact, then so is Y.

Corollary 3.4:

If a fuzzy multifunction $F: X \to Y$ is a fuzzy weakly open, fuzzy lower (resp. upper) almost continuous surjective multifunction and X is α-nearly compact, then so is Y.

Corollary 3.5:

If a fuzzy multifunction $F: X \to Y$ is a fuzzy weakly almost open, fuzzy lower (resp. upper) weakly θ-continuous surjective multifunction and X is fuzzy α-nearly compact, then so is Y.

Corollary 3.6:

If a fuzzy multifunction $F: X \to Y$ is a fuzzy almost open, fuzzy lower (resp. upper) weakly continuous surjective multifunction and X is fuzzy α-nearly compact, then so is Y.

Corollary 3.7:

If a fuzzy multifunction F: X → Y is a fuzzy almost open S., fuzzy lower (resp. upper) weakly continuous surjective multifunction and X is fuzzy α-nearly compact, then so is Y.

Definition 3.8:

A fuzzy multifunction F: X → Y is said to be fuzzy δ^*-closed if F(ξ) is fuzzy δ^*-closed in Y for each fuzzy δ^*-closed set ξ of X.

Theorem 3.9:

Let X be a fuzzy α-nearly compact space and Y be a fuzzy Hausdorff space. If F: X → Y is a fuzzy lower (resp. upper) almost continuous multifunction, then F is fuzzy δ^*-closed.

Proof:

Let ξ be a fuzzy δ^*-closed set of X. By Theorem (1.9), ξ is α-nearly compact relative to X. We shows that F(ξ) is fuzzy δ^*-closed set in Y. Let yε $\notin$ F(ξ) with yε $\notin$ S(F(ξ)). For x $\in$ S(ξ), then F(ξ) $\neq$ y. Since Y is fuzzy Hausdorff, then there are fuzzy regular open sets μx, ρx in Y such that μx(F(x)) = ρx(y) = 1 and cl(μx) $\leq$ 1 − ρx, by Theorem (1.8). Since F is fuzzy lower almost continuous, then the family {F−(μx): x $\in$ S(ξ)} is a fuzzy open α-shading of S(ξ). Hence there is a finite subfamily {F−(μxj): j = 1, 2, ..., n} such that {int(cl(F−(μxj))) : j = 1, 2, ..., n} is an α-shading of S(ξ). Let z $\in$ S(F(ξ)) = F(S(ξ)), there is x $\in$ S(ξ) such that F(X) = z. Hence there is n such that (int(cl(F−(μxj))))(x) > α, which implies (cl(F−(μxj)))(x) > α. Since F is fuzzy lower almost continuous, then F(cl(F−(μxj))) $\leq$ cl(μxj). But F(cl(F−(μxj)))(z) = sup(cl(F−(μxj)))(x) > α, then (cl(μxj))(z) > α. Therefore {cl(μxj) : j = 1, 2, ..., n} is an α-shading of S(F(ξ)) and hence F(ξ) $\leq$ S(F(ξ)) $\leq$ S(cl(μxj)). Put ρ = S{ρxj : j = 1, 2, ..., n}, then ρ is a fuzzy regular open set in Y with (yε q ρ) and (F(ξ) q ρ). Hence F(ξ) is fuzzy δ^*-closed.

The following two corollaries are consequences results of Theorem (2.6) and Remark (7.4) of [5] and the proofs are thus omitted.

Corollary 3.10:

If a fuzzy multifunction F: X → Y is a fuzzy lower (resp. upper) δ-continuous multifunction, X is fuzzy α-nearly compact and Y is fuzzy Hausdorff space, then F is fuzzy δ^*-closed.

Corollary 3.11:

If a fuzzy multifunction F: X → Y is a fuzzy lower (resp. upper) continuous multifunction, X is fuzzy α-nearly compact and Y is fuzzy Hausdorff space, then F is fuzzy δ^*-closed.

REFERENCES

[1] M. E. Abd El-Monsef and M. H. Ghanim: *Almost compact fuzzy topological spaces*, Delta Journal of Science, 5(1981), 19-29.

[2] K. K. Azad: *On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity*, J. Math. Anal. Appl. 82 (1981), 14-32.

[3] A. S. BinShahna: *On fuzzy strong semi continuity and fuzzy precontinuity*, Fuzzy sets and systems, 44 (1991), 303-308.

[4] C. L. Chang: *Fuzzy topological spaces*, J. Math. Appl., 24 (1968), 182-190.

[5] D. E. Cameron: *Properties of S-closed spaces*, Proc. Amer. Math. Soc., 72(1978), 581-586.

[6] A. D. Concilio and G. Gerla: *Almost compactness in fuzzy topological spaces*, Fuzzy sets and systems, 13(1984), 187-192.

[7] M. A. Fath Alla: *On fuzzy topological spaces*, Ph.D. Thesis, Assuit Unvi. (1984).

[8] T.E. Ganter, R. S. Steinlage and R. H. Warren: *Compactness in fuzzy topological space*, J. Math. Anal. Appl., 62(1986), 547-562.

[9] E. S. A. Haydar: *Almost compactness and near compactness in fuzzy topological space*, Fuzzy sets and systems, 22(1987), 289-295.

[10] Y. Kucuk: *On some characterizations of continuous δ-multifunctions*, Demonstration Math. 1995, 28(3), 587-595.

[11] A. S. Mashhour, M. H. Ghanim and M.A. Fath Alla: *α-separation axioms and α-compactness in fuzzy topological spaces*, Rocky Mountain J. Math., 16(3) (1986), 591-600.

[12] M. N. Mukherjee and S. Malakar: *On almost continuous and weakly continuous fuzzy multifunctions*, Fuzzy Sets and Systems, 41(1991), 113-125.

[13] R. A. Mahmoud: *An application of continuous fuzzy multifunctions*, Chaos Solitons and Fractals, 17(2003), 833-841.

[14] C. K. Wong: *Fuzzy topology: Product and quotient theorems*, J. Math. Anal. Appl. 45 (1974), 512-521.

[15] H. T. Yalvac, *Fuzzy Sets and functions*, J. Math. Anal. Appl. 126 (1987), 409-423.

[16] A. M. Zahran, *Some results on fuzzy topological spaces*, Ph.D. Thesis, Assuit Unvi. (1989).