EFFECTS OF STATES WEIGHTING MATRIX ON THE OPTIMAL CONTROLLER DESIGN PART II : STOCHASTIC CASE

By

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Abstract :

The presence of noise affects the design of feedback controller. Therefore a best estimate of the states should be used to obtain the feedback structure. A filtering algorithm Based on the minimum error variance criterion is coded to provide the best estimate. The effect of the states weighting matrix Q, on the estimated values is then investigated. The study reveals that proper choice of Q, in addition to the proper representation of noise, may result in a good controller which derive the system savely in a noisy inviroment.

Introduction :

In deterministic control systems it is usually assumed that exact measurments of the states are available. Then the design becomes somewhat easier. However the uncertainty which accomplish the measuring process may result in unproper behaviour of the controller. Therefore it is usually better that the design is obtained based on the best estimate of the system states.

One important reason, in control systems design for obtaining the estimate of states is that unstable plants can often be effectively stabilised by state variable feedback. This desirable propety is retained when instead of feedingback the observed states directly, the best estimates are used in the feedback loop (1).

The estimation of states represent ... additional factor which complicates the design ... in linear dynamical systems with white ... plant and observation, and quadratic c... known separation theorm holds (2). The ... imal control is determined by first e... the system, then the resulting estima... ministic optimal control problem.

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This paper is intended to study the effect of noise on the behaviour of an armature controlled D-C motor. First, problem formulation is given . A method of efficient computation of the estimation and control problemsis then presented. Results and discussions conclude the stated problem.

Problem Formulation :

Consider a linear time invarient dynamic system described by;

$$\dot{X}(t) = A\,x(t) + B\,(u(t) + w(t)) \quad \ldots\ldots \qquad (1)$$

and a measurment vector

$$z(t) = H\,x\,(t) + v(t) \quad \ldots\ldots \qquad (2)$$

The matrices A,B,H have the proper demensions. The initial state is assumed to be a random vector with x_o as the mean and P_o as the covarience.

P_o is symmetric non-negative definite matrix. The control-input is assumed to be known and non-random. w(t) and v(t) are white Gasussian zero mean stochastic processes uncorrelated with each other or with x_o and having the following characteristics;

$$\begin{aligned} &E\,(w(t)) = 0 \quad , E\,(v(t)) = 0 \\ &\mathrm{cov}\,(w(t), w(s)) = R_1\,\delta\,(t-s) \quad \ldots\ldots \qquad (3) \\ &\mathrm{cov}\,(v(t), v(s)) = R_2\,\delta\,(t-s) \end{aligned}$$

R_1 is positive definite and R_2 is positive semi definite.
The quadratic cost function to be optimised is;

$$J = \lim_{T\to\infty} \frac{1}{T}\, E\,\left(\int_{-T}^{T} (x^*(t) \;\; Q\,x(t) \;\; + \;\; U^*(t)\,R\,u(t)\,dt \quad \ldots\ldots \qquad (4)$$

It can be easily shown that the optimal control law is given by;

$$u^o = -\,R^{-1}B^*P_1\hat{x}(t) = K_1\,\hat{x}(t) \quad \ldots\ldots\ldots \qquad (5)$$

where P_1 is the positive definite solution of the matrix equation

$$P_1A + A^*P_1 - P_1B\,R^{-1}B^*P_1 + Q = 0 \quad \ldots\ldots\ldots \qquad (6)$$

and $\hat{x}(t)$ is the states of the system as estimated by a Kalman filter .

The best estimate $\hat{x}(t)$ can be written as the n-dimensional ·ear dynamical system (1);

$$\dot{\hat{x}}(t) = A\,\hat{x}(t) + B\,u(t) + K_2\,(\,z(t) - H\,\hat{x}(t)\,) \quad \ldots\ldots \qquad (7)$$

with initial condition $\hat{x}(0) = \hat{x}_o = E(x(0))$

K_2 is the Kalman gain matrix and is given by;

$$K_2 = P_2 H^* R_2^{-1} \quad \ldots\ldots\ldots \quad (8)$$

These equations are valid on the assumption of uncorrelated noise.

P_2 is the positive definite covariance matrix of the error $(\hat{x}(t) - x(t))$ and is the solution of the matrix Ricatti equation;

$$P_2 A^* + A P_2 + B R_1 B^* - P_2 H^* R_2^{-1} H P_2 = 0 \quad \ldots\ldots \quad (9)$$

This filter has several features :

(1) A unity gain negative feedback loop which generates at each time step the updated value $(z(t) - H \hat{x}(t))$

(2) The filter gain, given in eq. (8), which operates on the updated value is constant.

(3) The model of the deterministic part of the estimator is given by $A \hat{x}(t) + B u(t)$.

(4) P_2 is independent of control input and may be precomputed from the Ricatti equation.

Figure (1) shows the block diagram for the optimum compensator using combined estimation and control.

Algorithm for calculating the states trajectory :

The major computation burden in the combined estimation and control problem is the computation of Ricatti equations [3]. Simplifications that have been attempted include precomputed gains using approximations, piecewise gains and use of steady state gains.

In practice considerable enginneering judgement is required in selecting the covariance matrices R_1 and R_2 ; otherwise the filter can exhibit possible divergence. The selection of R_1 can be interpreted as one way of controlling the bandwidth of the filter.

Close examination of equations (6) and (9) shows a certain correspondance between them. This can be exploited to improve the computation by adopting a modified version of the algorithm given by the authers of [4].

An algorithm is written which has excellent convergence characteristics and moreover is efficient. Some of its features is stated next.

The algorithm performs the following tasks ;

(1) Construct eq. (9) from the system input data and solve it for P_2, calculate the feedforward gain K_2 as given in eq.(8).

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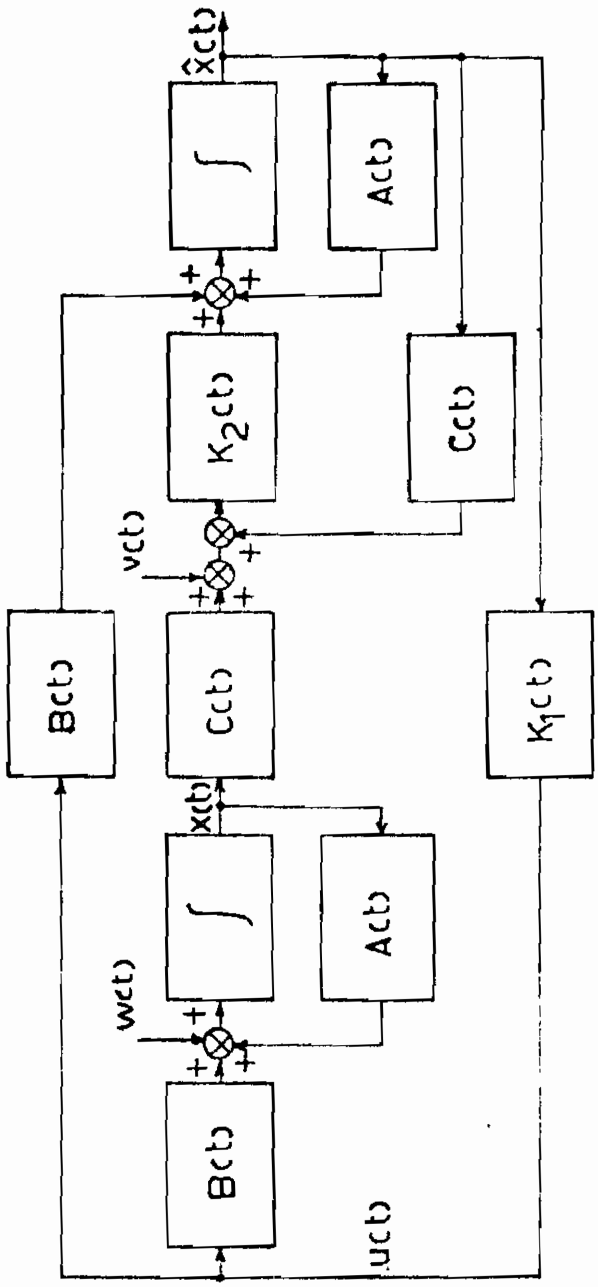

Fig. (1) Optimum linear estimation and control system

(2) Construct and solve eq. (6) and calculate the feedback gain K_1 from eq. (5)

(3) Construct the augumented system;

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \left[\begin{array}{c|c} A & -B\,K_1 \\ \hline K_2H & A-B\,K_1-k_2H \end{array}\right] \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} B & 0 \\ 0 & K_2 \end{bmatrix} \begin{bmatrix} w(t) \\ v(t) \end{bmatrix}, \begin{bmatrix} x\ (0) \\ \hat{x}\ (0) \end{bmatrix} = \begin{bmatrix} x_o \\ \hat{x}_o \end{bmatrix} \quad \ldots(10)$$

Equation (10) represents the observed as well as the estimated states as a function of time in terms of the precalculated gains and the system data. This augumented system can be simplified to the form;

$$\dot{y}(t) = F\,y(t) + B_1u(t) \quad \ldots\ldots \quad (11)$$

(4) Calculate the state transition matrix $\phi(t) = e^{Fh}$ as a step in the course of solving eq. (11) using the truncated series;

$$\phi(t) = I + Fh + (Fh)^2/2 + (Fh)^3/6 \quad , h \text{ is the step size.}$$

(5) The input of system (11) is calculated from the Gaussian denisties given by;

$$u_i = 1/\ 2\sqrt{2}\ R_{i,i}\left[\exp\ (-t/2\ R^2_{ii}) + \exp(-(t+h)/2\ R^2_{ii})\right]$$

(6) Calculate and plot the observed states and the estimated ones as time goes.

Application, Results and comments:

The following data are fed as inputs to the described algorithm;

$$A = \begin{bmatrix} -.119 & 1.119 \\ -1.01 & -1.069 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1,18 \end{bmatrix} \qquad H = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$R = [1]$, Q is the design parameter. $R_1 = [.2]$, $R_2 = [.5]$ these values represent a practical Gaussian noise in controlling the D - C motor which has the above matrices. the states are chosen to be the motor velocity and the armature current.

Fig.(2) gives the computer output for the above given data. Fig.(3) and (4) show the observed and the estimated values for motor velocity and armature current. Observe that the initial error in the estimated values decays exponentially as time elapsed. However, the decaying time increases as Q goes smaller. This fact is clear from plots of fig.(5) and (6).

THE MATRIX A

-0.11900	1.11900
-1.01000	-1.09600

THE MATRIX B

0.00000	1.08000

THE C MATRIX

1.00000	0.00000

THE D MATRIX

0.00000

THE INIIAL STATEXOIS

0.0000 L.0000 0.0000 1.0000

THE R MATRIX

1.00000

VARIANCE

.2000E 000.5000E 00

THE OPTIMAL FEEDFORWARDMATRIX

0.16466	-0.15246

THE Q MATRIX

1.00000

THE OPTIMAL FEEDBACK MATRIX

-0.36750	-0.32361

STATES ARE

TIME	x_1 ob	x_2 ob	x_1 est	x_2 est
0.000	0.00000	1.00000	0.00000	1.00000
0.050	0.05511	1.97607	0.05825	0.92408
0.100	0.10844	0.94842	0.11192	0.84978
0.150	0.15991	0.91974	0.16113	0.77705
0.200	0.20941	0.88877	0.2060L	0.70631
0.250	0.25682	0.85533	0.24669	0.63765
0.300	0.30200	0.81937	0.28332	0.57120
0.350	0.34433	0.78070	0.31603	0.50708
0.400	0.33517	0.73958	0.34499	0.44539
0.450	0.42291	0.69611	0.37034	0.38622
0.500	0.45794	0.66033	0.39225	0.32963
0.550	0.49017	0.60300	0.47088	0.27558
0.600	0.5L952	0.55435	0.42638	0.22440
0.700	0.56044	0.	0.43893	0.17583

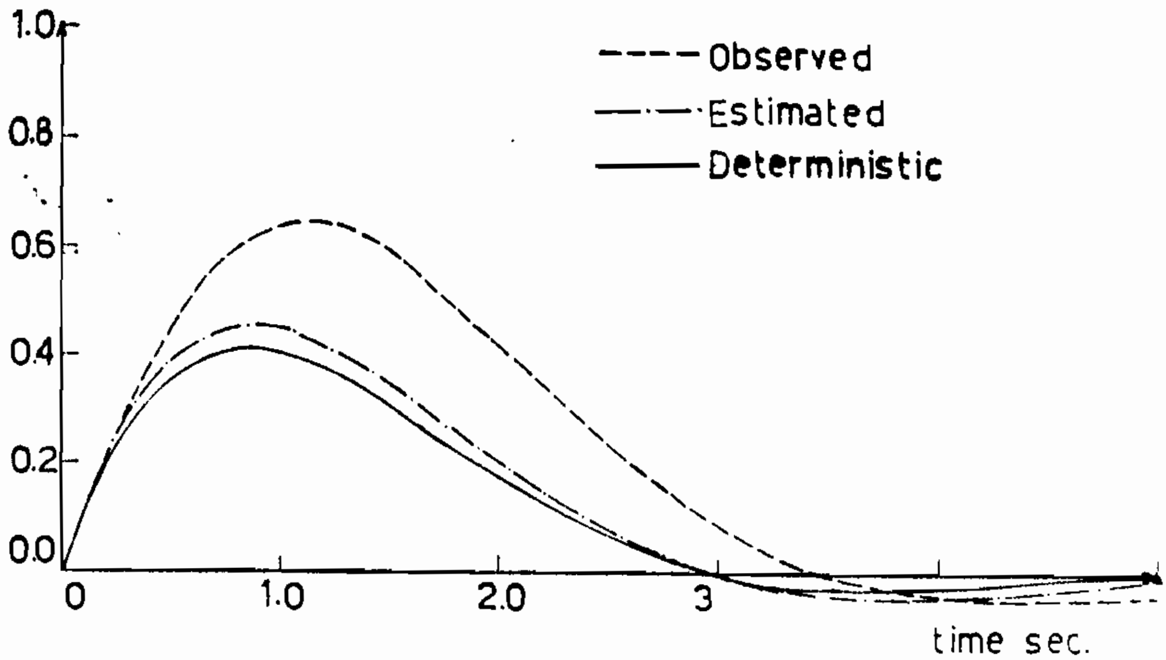

Fig. (3) Observed,estimated and deterministic velocity when Q=1.0

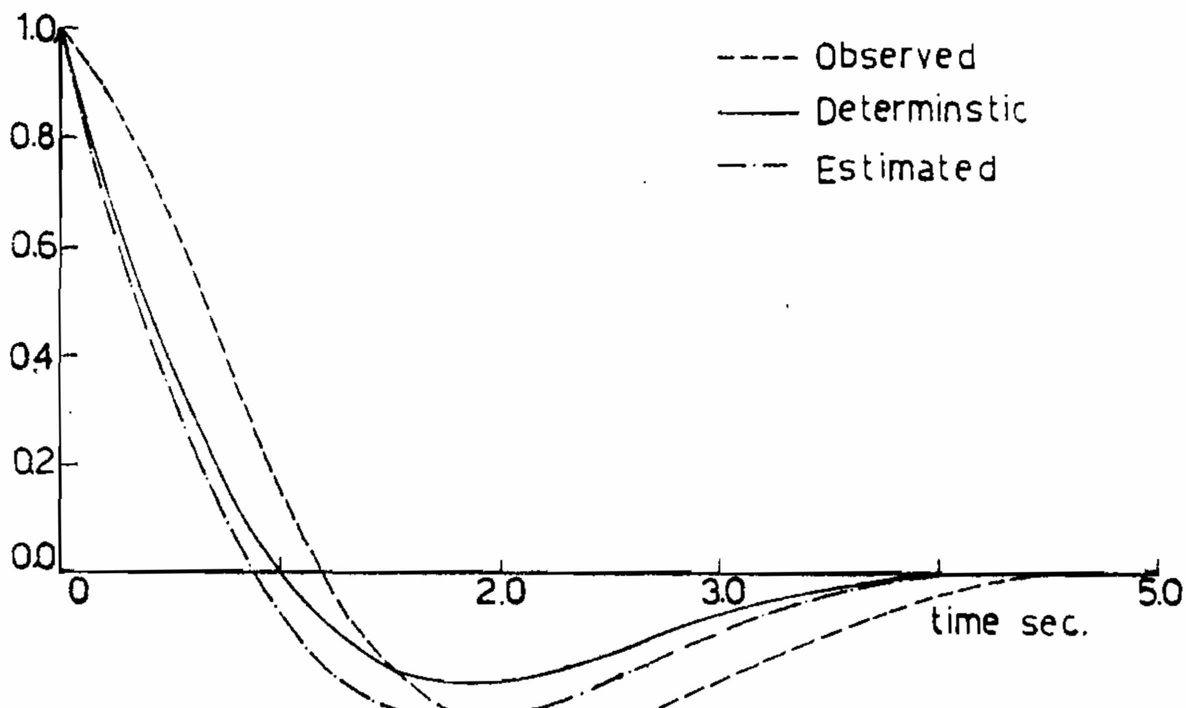

Fig. (4) Observed,determinstic and estimated armature current when Q=1.0

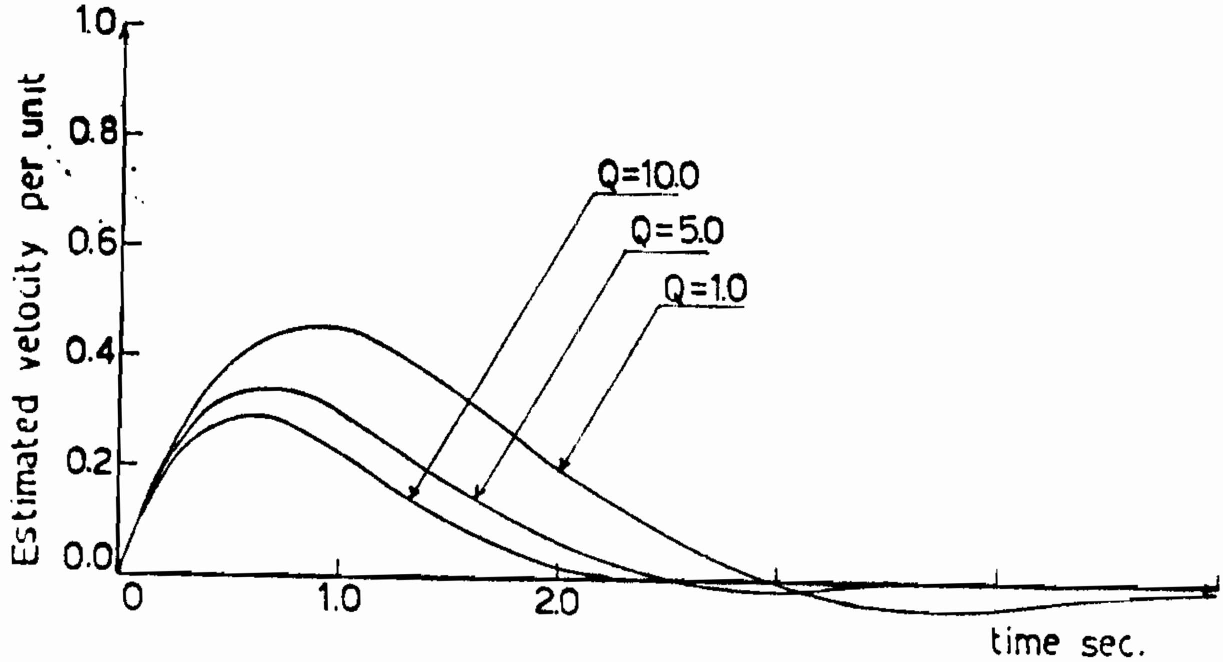

Fig. (5) Estimated velocity for different Q

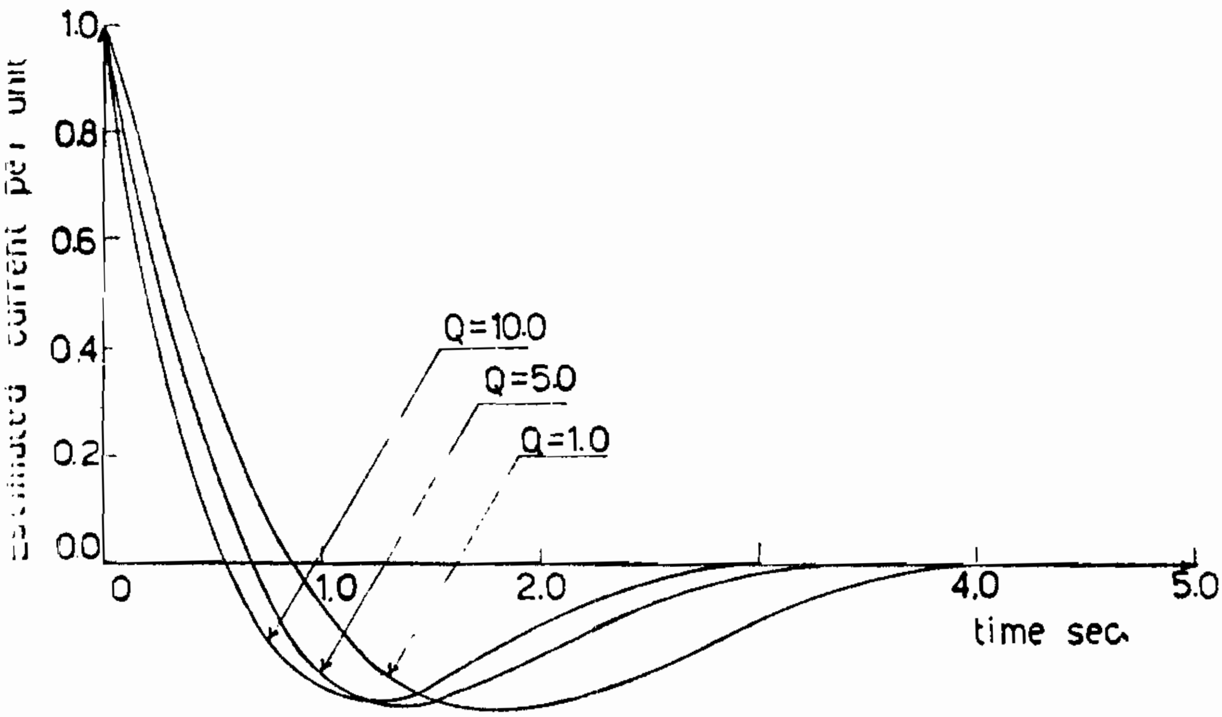

Fig. (6) Estimated armature current for different Q

Close examination of the given results reveals the following;

1- The feedforward gain of the estimator offsets the increase of response due to disturbance. The feedback gain directly affects the estimates in a manner that the increase of this gain results in decrease in the amplitude and reachable time of the response.

2- The observed states depend on the optimal control law which is directly proportional to the estimated states. Thus, the increase in the feedback gains reduces the amplitude of the observed states trajectory.

3- For stochastic control systems, The increase in feedback gains results in a better overall performance of the compensator.

Conclusions:

A computer program is written to solve the combined control and estimation problem. The program has excellent convergence charateristics with proper initialization. It requires moderate storage and minimum exegution time.

The results of studies performed on an armature controlled D - C motor reveals that better performance can be achieved by properly adjusting the states weighting matrix Q to arrive at the proper design of the feedback controller. Proper choice of the noise covarience matrices results in a feedforward gains which effectively balance the effects of noise on the contoller performance.

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(3) W.M. Wonham," On a Matrix Ricatti Equation of Stochastic Control", SIAM Journal on Control 6, No 4, 1968.

(4) E.H. Elkonyaly et al., " Effects of States Weigting Matrix on the Optimal Controller Design", Companion Paper.