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DESIGN OF STABLE CONTROLLERS FOR MODEL FOLLOWING DISCRETE TIME SYSTEMS USING APPROXIMATE INVERSE SYSTEM

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تصميم المتحكمات الديناميكية المستقرة للنظم الرقمية باستخدام النظام العكسي التقريبي

جمال البيومي

خلاصة

تصميم المتحكمات الديناميكية عن طريق تحديد مواقع الجذور للنظام المغلق استحوذ على الاهتمام خلال الفترة السابقة ، ومن المحتمل أن يكون المتحكم الناتج باستخدام هذه الطريقة أو الطرق الأخرى غير مستقر في ذاته رغم كونه محققا الاستقرار للنظام المغلق. و حيث أن التنفيذ العملي للمتحكمات الغير مستقره يعتبر في غاية الصعوبة ، فانه من الجدير الاهتمام بالبحث عن متحكمات ديناميكية مستقرة في ذاتها و تحققه أيضا الاستقرار للنظام الكلي. و لذلك فان هذا البحث يقدم طريقة جديده لتصميم المتحكمات الديناميكية المستقرة و ذلك باستخدام النظام العكسي التقريبي ، و هذا الأسلوب في التصميم يستخدم طريقه تعتمد على تحقيق اقل قيمه لمربع الخطأ في الحسابات و لذلك فانه مناسب للتطبيق العملي في نظم التحكم و خاصة تلك التي تحتاج إلي تخمين قيم معاملات النظام في التحكم المتضابط.

ABSTRACT

A formulation of stable dynamical controllers is proposed for discrete time systems. Based on polynomial pole placement, the resulting controller may be unstable. Despite the fact that controller stability is often overlooked in the design strategy, it is of fundamental importance since the practical implementation of an unstable controller is extremely difficult. Using approximate inverse systems obtained from least square approximation, we show that unstable controllers can be avoided. One of the major points in this method is the use of least square approximation t determine an approximate inverse system easily, which is suitable for practical applications in control systems. The results of computer simulations are presented to illustrate the effectiveness of the proposed method.

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KEY WORDS : Discrete time, Model following, Pole placement, Stable controllers, Approximate inverse systems, Least squares approximation.

1. INTRODUCTION

During the past couple of decades, a lot of attention has been given to the problem of designing pole placement controllers. The fundamental result on pole placement in linear time invariant controllable systems states that the closed loop eigen values of any controllable system may be arbitrarily assigned by state feedback control [1-4]. Most of the early work on pole placement utilizes state feedback methods. For some systems in which the states are not measurable, full state feedback is not practicable. Thus, a number of methods for pure gain pole placement by output feedback have been developed [5]. If the number of inputs and outputs are less than the order of the plant, the pure gain output feedback controller can not arbitrarily assign the closed loop poles. For this case, the dynamical controller is very helpful for pole placement via output feedback because it not only enables arbitrary assignment of poles but also provides additional design freedom [6]. A number of techniques for dynamical controller design using pole placement have been developed in recent years. However, the resulting controllers, although internally stabilizing the system, may themselves not be stable. This problem is not unique to the pole assignment approach and can also occur in other major controller design strategies. Despite the fact that controller stability is often overlooked in the design strategy, it is of fundamental importance since the practical implementation of an unstable controller is extremely difficult. As a result a method which exploits design freedom to guarantee both closed loop stability and controller stability is sought. Based on a generic controller form for polynomial pole placement, a formulation of stable dynamical controllers was introduced in [7]. In this paper, another method for stable dynamical controllers is presented. The proposed method is based on approximate inverse systems obtained from least square approximation [8]. The least square approximation is used to find the approximate inverse system. One of the major points in this method is the use of least square approximation to determine an approximate inverse system easily, which is suitable for practical applications in control systems.

The paper is organized as follows. In section 2, classical polynomial pole placement is summarized. Section 3, introduces the concept and analysis of stable pole placement. The algorithm of approximate inverse system using least square approximation is found in section 4. The results of computer simulations for some examples are presented in section 5, to illustrate the effectiveness of the proposed method. Finally, the main conclusions are formulated in section 6.

2. POLYNOMIAL POLE PLACEMENT

Survey of polynomial pole placement for linear time invariant systems is considered. The input output characteristics of a general plant $P(z^{-1})$ are described by

$$P(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} \tag{1}$$

d is the time delay, $A(z^{-1})$, and $B(z^{-1})$ are polynomials of order na, and nb respectively, and has the form

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \dots\dots\dots + a_{na} z^{-na} \tag{2}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots\dots\dots + b_{nb} z^{-nb} \tag{3}$$

It is assumed that the (nt) fixed desired poles of the closed loop system are given by the roots of the polynomial $T(z^{-1})$ which has the form

$$T(z^{-1}) = 1 + t_1 z^{-1} + t_2 z^{-2} + \dots\dots\dots + t_{nt} z^{-nt} \tag{4}$$

As a result there exists the following Diophantine polynomial identity for polynomials $A(z^{-1})$, $B(z^{-1})$, and $T(z^{-1})$:

$$A(z^{-1})G_0(z^{-1}) + z^{-d}B(z^{-1})F_0(z^{-1}) = T(z^{-1}) \tag{5}$$

$F_o(z^{-1})$ and $G_o(z^{-1})$ defines the minimum order controller $C(z^{-1})$ which assigns the nt fixed desired poles determined by the polynomial $T(z^{-1})$.

$$C(z^{-1}) = \frac{G_o(z^{-1})}{F_o(z^{-1})} \tag{6}$$

Both $F_o(z^{-1})$ and $G_o(z^{-1})$ have the form

$$F_o(z^{-1}) = 1 + f_1 z^{-1} + f_2 z^{-2} + \dots\dots\dots + f_{nf} z^{-nf} \tag{7}$$

$$G_o(z^{-1}) = g_o + g_1 z^{-1} + g_2 z^{-2} + \dots\dots\dots + g_{ng} z^{-ng} \tag{8}$$

where

$$nf = nb + d - 1 \tag{9}$$

$$ng = na - 1 \tag{10}$$

The closed loop transfer function of the system will be given by

$$T.F. = \frac{z^{-d}B(z^{-1})G_o(z^{-1})}{A(z^{-1})F_o(z^{-1}) + z^{-d}B(z^{-1})G_o(z^{-1})} = \frac{z^{-d}B(z^{-1})G_o(z^{-1})}{T(z^{-1})} \tag{11}$$

The roots of the polynomial $F_o(z^{-1})$ may lie outside the unit circle, and the controller is not stable. In the next section, the stable pole placement is introduced.

3. STABLE POLE PLACEMENT

It can be shown that if $F_o(z^{-1})$ and $G_o(z^{-1})$ satisfy the Diophantine equation given by Eq.(5), then all $F(z^{-1})$ and $G(z^{-1})$ are given by :

$$F(z^{-1}) = F_o(z^{-1}) + z^{-d}B(z^{-1})Q(z^{-1}) \tag{12}$$

$$G(z^{-1}) = G_o(z^{-1}) - A(z^{-1})Q(z^{-1}) \tag{13}$$

must satisfy Eq.(5). If $F_o(z^{-1})$ was unstable, $F(z^{-1})$ is chosen to be stable and Eq.(12) is solved for $Q(z^{-1})$ using least square estimation. Eq.(12) is rewritten as

$$F(z^{-1}) - F_o(z^{-1}) = z^{-d}B(z^{-1})Q(z^{-1}) \tag{14}$$

$Q(z^{-1})$ is a polynomial of order p. of the form

$$Q(z^{-1}) = q_o + q_1 z^{-1} + q_2 z^{-2} + \dots\dots\dots + q_p z^{-p} \tag{15}$$

Eq.(14) has a solution iff the first d coefficients of both $F(z^{-1})$ and $F_o(z^{-1})$ are the same. This condition can be satisfied by proper choice of the roots of $F(z^{-1})$. The closed loop transfer function in this case will be

$$T.F. = \frac{z^{-d}B(z^{-1})[G_o(z^{-1}) - A(z^{-1})Q(z^{-1})}{T(z^{-1})} \tag{16}$$

4. ALGORITH FOR APPROXIMATE INVERSE SYSTEM

The problem is reduced to finding $Q(z^{-1})$ which satisfies the relation

$$z^{-d}B(z^{-1})Q(z^{-1}) = F(z^{-1}) - F_o(z^{-1}) = z^{-d}H(z^{-1}) \tag{17}$$

Eq.(17), can be rewritten as

$$[B]\,[q] = [h] \tag{18}$$

$$\text{where } B = \begin{bmatrix} b_o & 0 & \dots & & o \\ b_1 & b_o & 0 & \dots & 0 \\ b_2 & b_1 & & & \\ . & b_2 & & b_o & \\ & . & & b_1 & b_o \\ b_m & & & . & b_1 \\ & b_m & & & . \\ & & & & b_m \end{bmatrix},\ q = \begin{bmatrix} q_o \\ q_1 \\ q_2 \\ \\ \\ \\ q_{p-1} \\ q_p \end{bmatrix},\ h = \begin{bmatrix} h_o \\ h_1 \\ h_2 \\ \\ \\ \\ h_{nb+p-1} \\ h_{nb+p} \end{bmatrix} \tag{19}$$

B is the (nb+p+1)x(p+1) matrix, q is the (p+1)x1 vector, and h is the (nb+p+1) vector. Consider the following cost function :

$$J = (Bq - h)^T (Bq - h) \tag{20}$$

Minimizing the cost function J with respect to q leads to

$$q = (B^T B)^{-1} B^T h \tag{21}$$

5. SIMULATION

In this section, the results of simulation studies are presented to give an indication of the adaptive scheme.

Example 1 : Consider the design for a rotary hydraulic test rig [7] with d=3. The polynomials $A(z^{-1})$, and $B(z^{-1})$ have been identified as

$$A(z^{-1}) = 1 - 2.8805z^{-1} + 3.7827z^{-2} - 2.8269z^{-3} + 1.1785z^{-4} - 0.2116z^{-5}$$

$$B(z^{-1}) = -.0036 + 0.1718z^{-1} + 0.3029z^{-2} - 0.0438z^{-3} - 0.0775z^{-4}$$

The desired closed characteristic polynomial is assumed to be :

$$T(z^{-1}) = (1 - 0.3z^{-1})(1 - 0.4z^{-1}) = 1 - 0.7z^{-1} + 0.12z^{-2}$$

Solving the Diophantine identity Eq.(5), yields :

$$F_0(z^{-1}) = 1 + 2.1805z^{-1} + 2.6182z^{-2} + 2.131z^{-3} +$$
$$0.6969z^{-4} - 0.6995z^{-5} - 0.3725z^{-6}$$

$$G_0(z^{-1}) = 2.9023 - 6.7682z^{-1} + 7.467z^{-2} - 4.3287z^{-3} + 1.0169z^{-4}$$

The roots of the polynomial $F_0(z^{-1})$ are $-0.1225 \pm 1.1099i$, $-0.9691 \pm 0.4698i$, -0.5061, and 0.5088. Since four roots of the polynomial $F_0(z^{-1})$ are outside the unit circle, the controller is not stable. But, if we choose $F(z^{-1})$ as

$$F(z^{-1}) = 1 + 2.1805z^{-1} + 2.6182z^{-2} + 2.1266z^{-3} + 0.9076z^{-4} - 0.36245z^{-5}$$
$$-0.5097z^{-6} - 0.0317z^{-7} + 0.3416z^{-8} + 0.3496z^{-9} + 0.0895z^{-10} - 0.0931z^{-11}$$
$$-0.044z^{-12}$$

The solution of Eq.(21) for p=5, gives $Q(z^{-1})$ as

$$Q(z^{-1}) = 1.2222 - 0.1952z^{-1} - 0.131z^{-2} + 0.663z^{-3} + 0.8805z^{-4} + 0.5677z^{-5}$$

The corrosponding $G(z^{-1})$ for this $Q(z^{-1})$ will be

$$G(z^{-1}) = 1.68 - 3.0524z^{-1} + 2.4123z^{-2} - 1.0787z^{-3} + 0.271z^{-4} - 0.0553z^{-5}$$
$$+0.0187z^{-6} - 0.3536z^{-7} + 0.6871z^{-8} - 0.4827z^{-9} + 0.1201z^{-10}$$

The controller poles are all stable with their magnitude less than 0.9798. Comparison of both controllers is found in Table 1. The combarison includes maximum value of the output (y), maximum value of the control action (u), maximum value of the error (e), average of summation of the square of the error (se2), and the average of summation of the square of the control action (su2). It is clear that the suggested controller is advantageous than the ordinary pole placement controller. The only disadvantage of the proposed controller is its higher order, since the pole placement controller is of order six, while the proposed controller is of order twelve. Simulation results for the system using both controllers are shown in Fig. 1 and Fig. 2 respectively.

Table 1 Summary of results of example 1.

	y	U	e	Se2	Su2
Pole placement controller	0.957	39.92	1.936	100.1603	22852.73
Proposed controller	0.8065	15.226	1.1674	98.42	3454.479

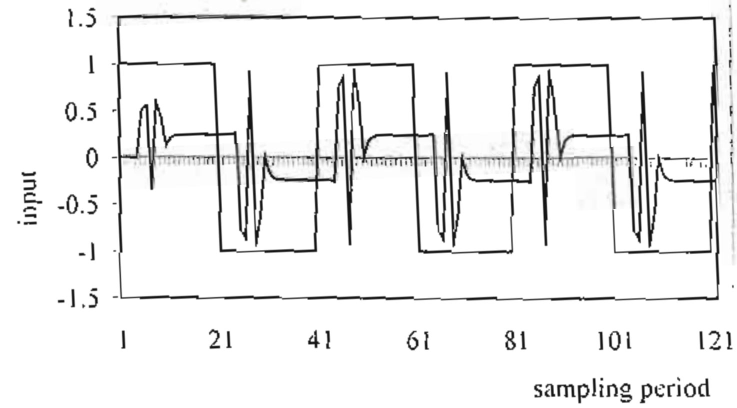
1.5
1
0.5
0
-0.5
-1
-1.5
input
1
21
41
61
81
101
121
sampling period

Fig. 1(a)

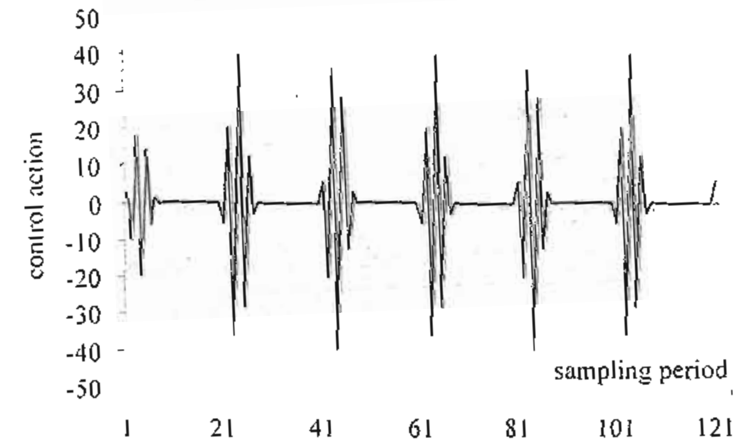
50
40
30
20
10
0
-10
-20
-30
-40
-50
control action
sampling period
1
21
41
61
81
101
121

Fig. 1(b)

$$F(z^{-1}) = 1 + 2.1805z^{-1} + 2.6182z^{-2} + 2.1266z^{-3} + 0.9076z^{-4} - 0.36245z^{-5}$$
$$-0.5097z^{-6} - 0.0317z^{-7} + 0.3416z^{-8} + 0.3496z^{-9} + 0.0895z^{-10} - 0.0931z^{-11}$$
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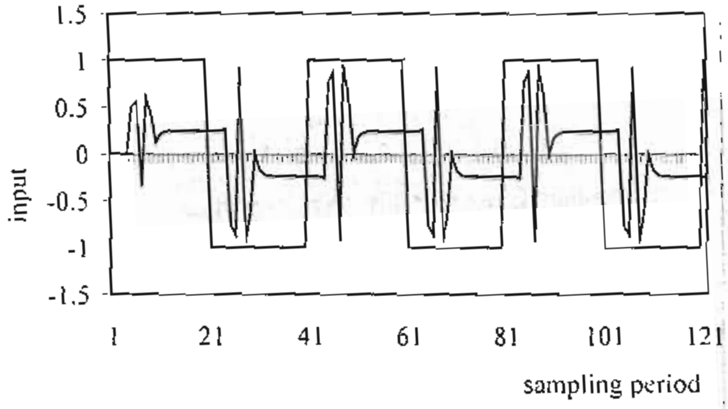
1.5
1
0.5
0
-0.5
-1
-1.5
input
1
21
41
61
81
101
121
sampling period

Fig. 1(a)

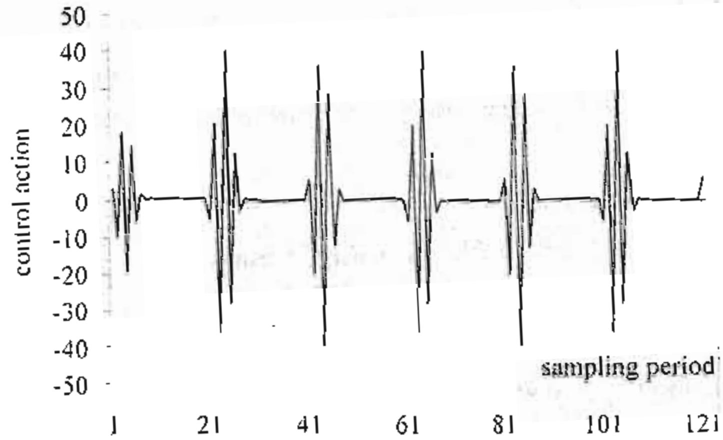
50
40
30
20
10
0
-10
-20
-30
-40
-50
control action
sampling period
1
21
41
61
81
101
121

Fig. 1(b)

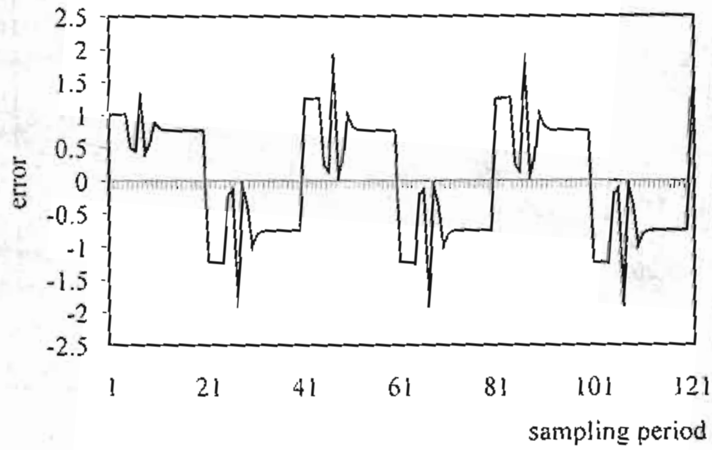

Fig. 1(c)

Fig. 1 Simulation Results of Example 1 using pole placement controller.

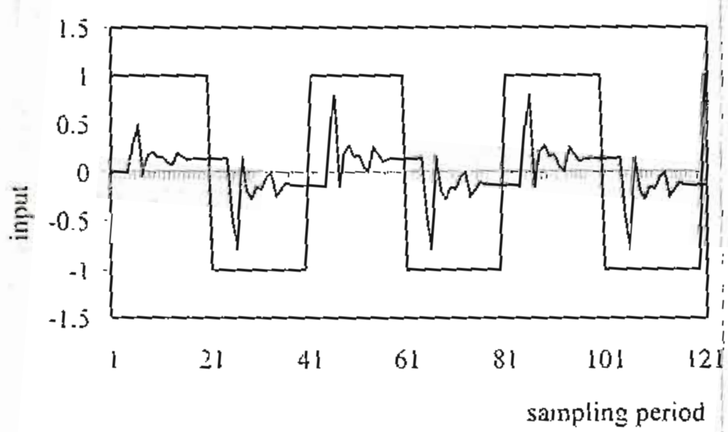

Fig. 2(a)

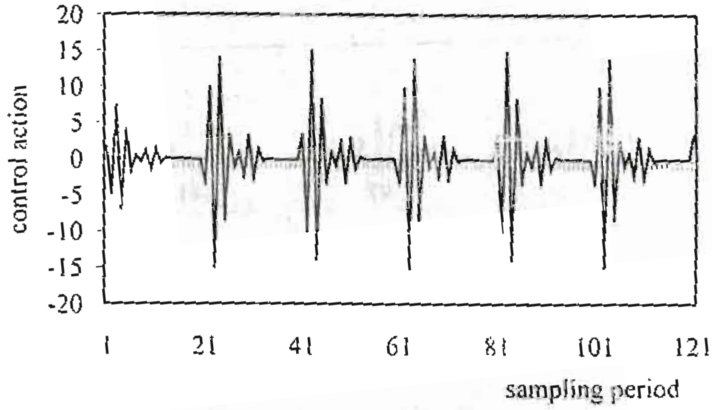

Fig. 2(b)

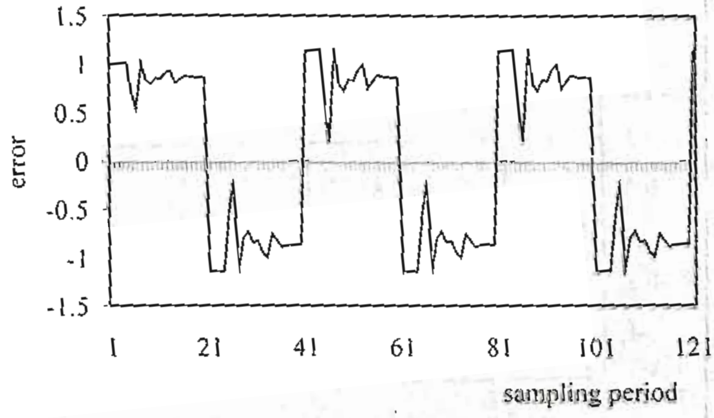

Fig. 2(c)

Fig. 2 Simulation Results of Example 1 using proposed controller.

Example 2 : Consider a system with d=2. The polynomials $A(z^{-1})$, and $B(z^{-1})$ are given by

$$A(z^{-1}) = 1 - 0.0431z^{-1} + 0.7852z^{-2}$$

$$B(z^{-1}) = -7.383 + 5.4949z^{-1}$$

The desired closed characteristic polynomial is assumed to be :

$$T(z^{-1}) = 1 - 0.8891z^{-1} + 1.1844z^{-2} - 0.2791z^{-3} - 0.022z^{-4}$$

Solving the Diophantine identity Eq.(5), yields :

$$F_o(z^{-1}) = 1 - 1.846z^{-1} + 0.8187z^{-2}$$

$$G_o(z^{-1}) = 0.0676 - 0.113z^{-1}$$

The roots of the polynomial $F_0(z^{-1})$ are 1.1053, -0.7407. Since one root of the polynomial $F_0(z^{-1})$ is outside the unit circle, the controller is not stable. But, if we choose $F(z^{-1})$ as

$$F(z^{-1}) = 1 - 1.846z^{-1} + 1.2779z^{-2} - 0.3932z^{-3} + 0.0454$$

This choice ensures that the controller poles are at 0.4615. The solution of Eq.(21) for p=5, gives $Q(z^{-1})$ as

$$Q(z^{-1}) = -0.06217 + 0.007013z^{-1} - 0.0008628z^{-2} - 0.0005884z^{-3} - 0.0003657z^{-4} - 0.0001752z^{-5}$$

The corrosponding $G(z^{-1})$ for this $Q(z^{-1})$ will be

$$G(z^{-1}) = 0.1298 - 0.1227z^{-1} + 0.05z^{-2} - 0.005z^{-3} + 0.001z^{-4} + 0.0006z^{-5} + 0.00028z^{-6} + 0.00013z^{-7}$$

Summary of the simulation results for the system using both controllers is found in Table 2. It is also clear that the suggested controller is advantageous than the ordinary pole placement controller.

Table 2 Summary of results of example 2.

	Y	u	e	Se2	Su2
Pole placement controller	3.2053	2.1514	4.2053	3.1616	1.9585
Proposed controller	2.535	2.237	3.535	8.239	2.7684

6. CONCLUSIONS

A new design strategy for stable dynamical controllers via polynomial pole placement has been presented. Based on approximate inverse systems obtained from least square approximation, a formulation of stable dynamical controllers was detailed. One of the major points in this method is the use of least square approximation to determine easily an approximate inverse system. This is suitable for practical applications in control systems. Two examples demonstrate the significance of the controller design strategy presented in this paper. Though a new stable controller design by polynomial pole placement was discussed, the design strategy can also be applied to other control design approaches which result in unstable controllers.

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