ثانية قوى

Menofia University	Department: Mech. Power Eng.
Faculty of Engineering	Year : 2nd
Shebien El-kom	Subject : Eng. Mathematics
First Semester Examination	Time Allowed : 3 hours
Academic Year : 2015-2016	Date : 13 / 1 / 2016

Allowed Tables and Charts : None

Answer all the following questions: [100 Marks]

Q.1 [50]

(A) Let $\phi(x, y, z) = xe^{y+z}$, and $\overline{F} = grad\phi$, find **div** $\overline{F}$, **curl** $\overline{F}$

(B) Evaluate using Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the area bounded by **x = 0** , **y = 0** , and **x + y = 2.**

(C) Show that $\nabla\phi$ is a vector perpendicular to the surface $\phi(x, y, z) = c$, where c is constant.

(D) If $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$ a) Find $\nabla\phi$ if $\phi = \frac{1}{r}$, b) Find $\nabla\phi$ if $\phi = \ln r$

(E) Show by Green's theorem that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint_c xdy - ydx$, then compute the area of ellipse whose parametric equations are $(x = a\cos\theta \;, y = b\sin\theta)$

(F) Determine whether the vector field $\overline{F} = \cosh x\,\overline{i} + 6yz^2\,\overline{j} + 6y^2z$ is conservative. If it is conservative, find its scalar potential. Then, Evaluate $\oint_C \overline{F}.d\overline{r}$ along any simple closed curve. Also, Evaluate $\int_C \overline{F}.d\overline{r}$ between the points (0, 0, 0) and (2, 4, 2) along the curve given by the parametric equations

$$x = t^2 + 1, \quad y = 3t^2 + \sqrt{t}, \quad z = t^3 + t.$$

Q2.	(A) Evaluate $\oint_C (x^2y\cos x + 2xy\sin x - y^2e^x)dx + (x^2\sin x - 2ye^x)dy$ around the hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$ (B) Verify Stokes' theorem for $\overline{F} = \left(x^3 + \frac{yz^2}{2}\right)\overline{i} + \left(y^2 + \frac{xz^2}{2}\right)\overline{j} + xyz\overline{k}$ where S is the surface of the cube $x=0,\ y=0,\ z=0,\ x=3,\ y=3,\ z=3$ above the x-y plane. (C) Use divergence theorem to evaluate the surface integral $\iint_S \overline{A}.\overline{n}\,dS$ where $\overline{A} = x^2\overline{i} + y^2\overline{j} + z^2\overline{k}$ and the surface S is the surface of the cube $0 \le x \le 1,\quad 0 \le y \le 1,\quad 0 \le z \le 1.$ (D) **Solve** the following L.P.P. using **Simplex** method *Maximize* $Z(\$) = 3x_1 + 5x_2$ *Subject to* $5x_1 + 5x_2 \le 25$ $9x_1 + 13x_2 \ge 117$ $x_1,\ x_2 \ge 0$ Then check your answer using **graphical method.**	[50]
	(E) **Evaluate** 1) i) $\Gamma(-5/2)$ ii) $\int_0^1 \frac{dx}{\sqrt{-\ln x}}$ iii) $\int_0^{\pi/2} \sin^6\theta\,d\theta$ iv) $\int_0^{\pi/2} \sqrt{\tan\theta}\,d\theta$ 2) **Prove that** $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta\cos^{2n-1}\theta\,d\theta$ ii) $\int_0^{2\pi} \sin^8\theta\,d\theta = \frac{35\pi}{64}$	

With my best wishes

Dr. osama N.Saleh