IMPROVED MODEL OF VIBRATORY DAMPED GLASS REINFORCED PLASTIC (GRP)

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ABSTRACT

An improved model of modal parameters of glass reinforced plastic structures (GRP) is presented. The extensive analysis of the fitted experimental results proves that the quasi uniform mass damping is the main feature of vibratory GRP structures. Consistent with the analysis it is stated that the number of boundary degrees of freedom, code number, volume fraction and typical order of natural mode have significant effects respectively on controlling the type of each set of quasi rectangular hyperbolic relations associated with the vibration damping of GRP structures.

The close agreement between the numerical results of the finite element model and the fitted experimental results shows the efficiency and applicability of the present modelling techniques, resulting in significant simplifications in solutions of idiosyncratic composite systems with lowest residual errors.

1. INTRODUCTION :

Model updating has been a subject of study in literature for many years [1, 2]. Most of the procedures try to minimize the deviation between the analytical and experimental models by adjusting the analytical and experimental model on the basis of experimental measurements. The development of mathematical models has required best memory to be more accurate and has high computational speed which plays an important role in vibration analysis of composite complex structures in the recent decades.

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At the present time, it is still difficult to determine accurately the model characteristics of composite structures particularly the damping capacity by an analytical approach. The experimental confirmation prediction is therefore at very least desirable and can be used to build up the mathematical model. This in turn leads to more clearly understand the effect of parameters for controlling the dynamic nature of composite complex structure.

In the present work, an attempt has been made to build up an efficient simulation of modal parameters with an improvement of convergence characteristics of modelling process within a wide range of frequencies for different stacking sequences at two levels of volume fraction taking into account the influences of constraints subjected to the composite structural beams.

For the sake of minimization the residual errors and raising the confidence levels for building up the model, weight factors (α, a) have been introduced for correlating and updating the construction of the mathematical model to the experimental data through the utilization of the curve fitting response functions [3]. This has resulted generalized quasi rectangular hyperbolic relationships between the loss factors and the natural frequencies for various degrees of constraints within the confidence level 99.2 % at least. This in turn permits the uncoupling of simultaneous equations of motion of composite structures of large number of degrees of freedom with the lowest residual errors.

In that way the curvilinear fitting techniques are reutilized to generate other quasi hyperbolic relationships concerned with the loss factors in various set of natural modes with confidence level 98.6 % at least.

Other object of the present work is to analyze quantitatively the influences of the degree of constraints, to be considered, on the nature of modal parameters of composite structures.

In the experimental work, Fig. (1b), four types of boundary conditions have been subjected to composite beams made from glass reinforced plastic GRP. Various specimens made from three plies (L X b X t) Fig. (1a), are tested for two levels of volume fraction (a) a weakly composite 15 % and (b) an average composite 45 % in each type of constraints.

Experimentally the first four natural frequencies and the corresponding loss factors are listed in the third and fourth columns in tables 1 to 8, respectively. For the sake of verification with the experimental measurements the first four natural frequencies at two levels of volume fraction are computed by the use of the modified formula MFM [3] and listed in the second column of these tables.

To high light the nature of the damping parameters, family of curves representing mutual relationships of modal parameters are plotted in Figs. (2 & 3) in terms of the results listed in Tables (1 to 8).

The close agreement of the results of the proposed mathematical and experimental models proves the efficient applicability of the proposed model and enriches the understanding of dynamic nature of vibrating damping composite structures. This in turn leads to uncoupling the dynamic equations of motion particularly of large complex composite structures with minimum time of computation and residual errors.

2. MATHEMATICAL MODEL OF VIBRATION DAMPING OF COMPOSITE STRUCTURAL BEAM

In the last decade, it was mentioned in Ref. [5] that the hyperbolic relations between η and ω of composite plates, vibrating at the first mode and subjected to different boundary conditions, provide a more reliable prediction throughout the utilization of the uniform mass damping model. The developed mathematical model was established by the utilization of the student distribution approximation with confidence level at 95%.

To improve the convergence characteristics of the mathematical model within a wide range of frequency spectrum at various code numbers, modified quasi hyperbolic relations are developed by introducing the proper weight factors (α, a), [3] throughout utilization of the least square technique.

In the present work a generalized vibratory damped model made from glass reinforced plastic is developed. The interrelations between the loss factors and the natural frequencies from one side and the order of natural modes from the other side are obtained by applying the fitting response function on the experimental results of composite structural beam specimens. The models are subjected to different types of boundary conditions, for various stacking sequences, and for two levels of volume fraction. As stated in [3] the quasi rectangular hyperbolic relations between loss factors η and natural frequencies ω are recast as

$$\eta_i = a_i (\omega_i)^{-\alpha i} \qquad i = 1, 2, ...n \qquad \text{.................................... (1)}$$

In addition the same relations between loss factor η and order of natural modes (i) is deduced by the same technique and the form

$$\eta_i = B_i (i)^{-\beta i} \quad . \qquad i = 1, 2, ...n \qquad \text{.. (2)}$$

where n=the number of natural frequencies in the selected frequency spectrum.

With the help of logarithm forms these relations can be transferred to linear forms here as.

$$l_n \eta_i + \alpha_i l_n \omega_i = l_n a_i \qquad \text{.. (3)}$$

$$l_n \eta_i + \beta_i l_n (i) = l_n B_i \text{ for } i = 1, 2, \ldots r \qquad (4)$$

The advantages of logarithm form is mainly for facilitation the quantitative analysis and the applications of extrapolation and interpolation techniques for controlling the loss factors and frequencies at the selected boundary conditions within the required frequency spectrum.

For the sake of verification of the measured natural frequencies at various states, the first four natural frequencies were numerically computed by the modified developed formula [4] and the form

$$\omega_i = \frac{\lambda_i}{2\Pi L^2} \left(\frac{D^*}{\rho_c \cdot t} \right)^{\frac{1}{2}} \qquad (5)$$

where D^* = the condensed bending stiffness modulus of composite structural beam,

ρ_c = the equivalent specific mass of composite beam [3].

3. EXPERIMENTAL MODEL OF VIBRATION DMPING O F COMPOSITE STRUCTURAL BEAM

3.1. Experimental specimens :

The frequency response tests were performed on four types of fixation on composite beams made from three layers glass fiber reinforced plastic GRP of various orientations at the first four modes and at two level of volume fractions. A typical specimens GRP composite beam of dimension (210 x 20 x 3 mm) made of three plies with 1 mm thickness for each ply for V_f 15 % and V_f 45 % is shown in Fig. (1a). To study the effects of degree of constraint, lamina orientations and stacking sequences on the modal parameters, six code numbers of specimens were fabricated by hand lay up technique and stated as (0/0/0), (0/30/0), (0/45/0), (0/90/0), (45/-45/0) and (45/0/45) for the two volume fraction 15 % and 45%.

3.2. Instrumentation Layout :

The experimental apparatus is shown in Fig. (1b). The four boundary conditions of the six specimens stated as fixed-free (3D), hinged - hinged (4D), fixed-hinged (5D), and fixed-fixed (6D) could be maintained by the clamping fixture.

The specimens are excited by impact hammer to determine the resonance frequency [6]. The excitation signal is fed to the dual channel analyzer through conditioning amplifier and a light weight accelerometer.

Signal analysis is carried out by the analyzer linked to a computer with structure measurement system as shown in Fig. (1b). The analyzer having a frequency range of 25 KHz can zoom in various selected frequency ranges.

The damping parameters for each specimen are calculated by using the peak method within the selected frequency spectrum.

4. RESULTS AND DISCUSSIONS :

Refer to [3] the least square technique is utilized and the quasi rectangular hyperbolic curve fitting is plotted for the measured values (η_i, ω_i) within the ranged of confidence level 98% to 99.99% as shown in Figs. (2, 3,).

To study the effects of the number of boundary degrees of freedom stated as 3D, 4D, 5D and 6D on the damping capacity of the samples, Fig. (2) indicates that the damping capacities increase and the natural frequencies decrease in monotonically manner with increasing degrees of freedoms.

Without loss the generality the quasi hyperbolic interrelations of damped natural frequencies ω_d and damping capacities are permanently valied for the four cases of boundary conditions subjected to the six specimen lamina orientations for the first four modes, and for the two volume of fractions V_f as depicted in Figs. (2 and 3) and the form

$$\eta_i = a(\omega_i)^{-\alpha} \quad \text{for } i = 1, 2, 3, 4 \ (1>\alpha > 0,\ a > 0) \qquad \text{................(6)}$$

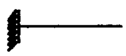
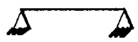
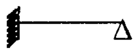
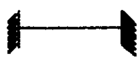
With the help of the least square technique concerned with the curve fitting response functions, the generalized quasi hyperbolic quantitative relations between damping capacities ($\eta_{av.}$) and frequencies ($\omega_{av.}$) for the various types of fixation at the two volume fraction are correlated with confidence level 98.4%, at least, as listed in Table (9) and plotted in Figs. (4a and 4b).

To facilitate the quantitative analysis of the weight factors (α, a) for updating the mathematical model logarithmic forms of the quasi-hyperbolic relations (η_i - ω_i), for various orientations and for different types of fixation for all specimens are plotted in linear forms as represented in Figs. (5 & 6) and (7 & 8). It is noticed that the slopes assigned by the weight factor (α) are mainly depending on the degree of isotropical state while the damping constant (a) depends mainly on the flexibility of the specimens.

From the computational and experimental values depicted in the previous tables and figures it is shown that the weight factors increase as the volume fraction increases and as the lamina orientation leading to a low stiff composite structural beams as expected.

Also it is shown that boundary conditions (types of fixation) have significant effects on the damping parameters associated with the weight factors (α, a) compared either with the influences of lamina orientations or with the volume fraction V_f. As an example the damping constant in the fixed free state is increased by about 24 times of the correspondant in the fixed fixed state as shown in Table (9).

Table (9) : Generalized forms of interrelations between damping capacities, η and natural frequencies, ω for various fixations of LCB at two levls of fiber volume fractions. V_f = 15 % and 45 %. (Ref. Tables 1 to 4 for V_f = 15 % and Tables 5 to 8 foi V_f=45%).

Fixation	Fiber volume fraction, Vf	
	15 %	45 %
	$\eta_i = 3.097479503\ (\omega_i)^{-0.5637788911}$ Confidence level = 98.79 %	$\eta_i = 2.852583936\ (\omega_i)^{-0.5567207156}$ Confidence level = 99.25 %
	$\eta_i = 13.67496044\ (\omega_i)^{-0.7156483416}$ Confidence level = 98.97 %	$\eta_i = 12.48529197\ (\omega_i)^{-0.704074391}$ Confidence level = 99.41 %
	$\eta_i = 31.375641\ (\omega_i)^{-0.799730281}$ Confidence level = 98.75 %	$\eta_i = 26.87749517\ (\omega_i)^{-0.7792277878}$ Confidence level = 98.96 %
	$\eta_i = 74.56746475\ (\omega_i)^{-0.8867527246}$ Confidence level = 98.40 %	$\eta_i = 63.95389482\ (\omega_i)^{-0.8660440344}$ Confidence level = 98.46 %

In view of the experimental measurements listed in Tables 1 to 8 it is obvious that the loss factors are monotonically decrease in hyperbolic feature as the mode number increases.

For quantitative estimation of these relations, the curve fitting response techniques is utilized and the form

$$\eta_i = B\ (i)^{-1.4} \quad \text{for} \quad i = 1, 2, 3, 4 \qquad \text{(7)}$$

It is of interesting to note that the loss factors decrease by the same rate by about 1.4 for all cases while the damping constants B; are strongly affected by the volume fraction.

By visual inspection of the measured values of loss factors and the corresponding hyperbolic fitting relations in Tables 1 to 8 it is noticed that the damping constant B is nearly equal to the loss factor of the first natural mode. This remark exists for all types of fixations and for various code number such that one can suggest that the loss factor at any number of mode can be related to the correspond 1st mode in the following empirical serial form.

$$\eta_i = \eta_1\ (i)^{-1.4} \quad i = 2, 3 \ldots n \qquad \text{(8)}$$

The correspond logarithmic form is then given by

$$l_n\ \eta_i + 1.4\ l_n\ i = l_n\ \eta_1 \qquad \text{(9)}$$

and as shown in Fig. (9).

Figs. (10) and (11) show the quasilinear relations between the number of degrees of boundary freedom and the equivalent damping loss factor for the two levels of volume fractions.

It is worth to mention here that the use of logarithmic form is not only, for facilitation the quantitative analysis and for computing the loss factors either in terms of the natural frequency or in terms of the mode number by extrapolation or interpolation techniques, but also for control the magnitudes of natural frequencies and damping capacity by proper choice of boundary conditions and types of fixations in the selected range of the frequency spectrum. As an example from the results listed in Tables (1 and 4) it is noticed that the third natural frequency at (0/30/0) orientation of the fixed-fixed beam is nearly equal to the fourth natural frequency at (0/30/0) of the fixed free for V_f 15%. Similarly the fourth natural frequency at (0/0/0) orientation of the fixed free beam is nearly equal to the third natural frequency (0/0/0) at fixed fixed for volume V_f 15%. Also for $V_f = 45\%$ at fixed free it is obvious that the third natural frequency at (0/30/0) is nearly equal to the second natural frequency at the fixed fixed beam and the damping capacities are almost the same.

By the inspection of experimental results listed in Tables (1 to 8) it is evident that the changes of outer orientations have permenant significant effects on the damping capacity, and stiffness, of the specimens compared with the changes of the inner orientation regardless to the degrees of constraint and of the degrees of isotropism at different mode shapes as stated in [3].

For the sake of verification of measurements of frequencies listed in column three, the modified formula MFM is utilized to compute the first four undamped natural frequencies and listed in the second column of Tables 1 to 8. The comparison between the numerical results and experimental measurements shows the good agreement and the efficiency of the modified MFM to be utilized for computing the natural frequencies of composite structures with a wide range of frequencies and at different degrees of fixations.

5. CONCLUSION :

The present work is focused on the development of a generalized model of vibratory damped glass reinforced plastic (GRP) structures. The analysis of the fitted results of measurements indicates the following conclusion remarks.

1. Without loss the generality the quasi uniform mass damping is the main feature of damping behavior of GRP structures in various states.

2. The families of quasi rectangular hyperbolic relations between the damping loss factors and natural frequencies are stated by the lowest residual errors by using the mathematical fitting

convenience of the two weight factors α and a.

3. The properties of each family relating loss factors and resonant frequencies are controlled mainly by the following four parameters :
* number of boundary degrees of freedom,
* volume fraction,
* lamina orientations and stacking sequences and
* typical order of natural mode.

In contrast to the limited variations of the weight factor (α), the damping constant (a) is strongly affected by the type of fixation compared with the other controlling parameters as shown in the curves of figures.

4. To disregard the influences of controlling parameters, the logarithmic trend of the uniform mass damping behavior of GRP is characterized by the linear decreasing of loss factors and by nearly constant rate (α) against the monotonic increasing of order of the natural modes.

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NOMENCLATURE :

V_f	Volume fraction	GRP	Glass reinforced plastic
i	Mode number	D	The degree of constraint
α, a, B. β	weight factors	η	Damping loss factor
η_{av}.	Average damping capacities	ω_{av}.	Average natural frequencies

Table (1) : The numerical and experimental modal parameters of the first four modes of the fixed-free GRP beam of volume fraction 15 %

Table 1.1 [0 / 0 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	211.109	188.496	13.334
2	1322.981	1294.336	4.370
3	3704.384	3694.513	2.552
4	7259.115	7250.796	2.080

$\eta_i = 2.248738345\,(\omega_i)^{-0.53724390}$
confidence level = 99.4803 %
$\eta_i = 0.1248960229\,(i)^{-1.373564051}$
confidence level = 99.21 %

Table 1.2 [0 / 30 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	209.544	185.354	13.898
2	1313.186	1244.071	4.546
3	3676.958	3606.548	2.614
4	7205.362	7200.530	2.094

$\eta_i = 2.463872500\,(\omega_i)^{-0.5463537063}$
confidence level = 99.5838 %
$\eta_i = 0.1308671564\,(i)^{-1.397219936}$
confidence level = 99.34 %

Table 1.3 [0 / 45 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	208.936	179.071	14.912
2	1307.506	1237.788	6.076
3	3661.049	3581.416	2.632
4	7174.191	6754.424	2.280

$\eta_i = 2.743609311\,(\omega_i)^{-0.55319680819}$
confidence level = 99.3578 %
$\eta_i = 0.1416141987\,(i)^{-1.410761555}$
confidence level = 99.10 %

Table 1.4 [0 / 90 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	208.482	175.929	16.072
2	1306.526	1231.504	5.612
3	3658.309	3568.849	2.728
4	7168.825	6572.212	2.486

$\eta_i = 3.011008077\,(\omega_i)^{-0.5545636963}$
confidence level = 99.0864 %
$\eta_i = 0.1534061781\,(i)^{-1.41778176}$
confidence level = 98.82 %

Table 1.5 [45 / -45 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	171.160	163.363	20.000
2	1072.640	1043.009	6.024
3	3003.425	2978.230	3.798
4	5885.510	5811.946	2.648

$\eta_i = 3.5631542\,(\omega_i)^{-0.5694874495}$
confidence level = 99.6619 %
$\eta_i = 0.1878084785\,(i)^{-1.457365957}$
confidence level = 99.49 %

Table 1.6 [45 / 0 / 45]

mode No (i)	ω_i	ω_{di}	η_i %
1	123.615	119.381	23.684
2	774.666	766.549	8.196
3	2169.094	2161.416	4.534
4	4250.556	4234.867	3.086

$\eta_i = 3.797399516\,(\omega_i)^{-0.5762650855}$
confidence level = 99.9998 %
$\eta_i = 0.2333795608\,(i)^{-1.476815122}$
confidence level = 99.97 %

ω_i = Frequency, (rad / sec)
ω_{di} = Damped frequency,(rad / sec)
η_i = Damping (%)

Table (3) : The numerical and experimental modal parameters of the first four modes of the fixed-hinged GRP beam of volume fraction 15 %

Table 3.1 [0 / 0 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	925.742	918.733	12.756
2	2999.959	2960.321	4.267
3	6259.172	6240.411	2.489
4	10703.556	10694.240	2.020

$\eta_i = 22.80581266\,(\omega_i)^{-0.7705142447}$
confidence level = 99.01 %
$\eta_i = 0.1200086107\,(i)^{-1.361846764}$
confidence level = 99.29 %

Table 3.2 [0 / 30 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	918.879	910.324	13.600
2	2977.748	2960.422	4.460
3	6212.832	6207.430	2.580
4	10624.298	10600.911	2.087

$\eta_i = 28.40687783\,(\omega_i)^{-0.793418371}$
confidence level = 99.16 %
$\eta_i = 0.1277672409\,(i)^{-1.385050408}$
confidence level = 99.28 %

Table 3.3 [0 / 45 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	916.213	910.732	14.291
2	2964.868	2950.310	4.762
3	6185.951	6179.211	2.624
4	10578.336	10520.902	2.220

$\eta_i = 28.6286925\,(\omega_i)^{-0.7962969359}$
confidence level = 98.86 %
$\eta_i = 0.1345336958\,(i)^{-1.389030475}$
confidence level = 99.14 %

Table 3.4 [0 / 90 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	914.222	907.423	15.613
2	2962.646	2950.310	5.342
3	6181.321	6168.422	2.644
4	10570.424	10520.400	2.441

$\eta_i = 33.55103939\,(\omega_i)^{-0.7981386971}$
confidence level = 99.41 %
$\eta_i = 0.1477456184\,(i)^{-1.410681864}$
confidence level = 98.69 %

Table 3.5 [45 / -45 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	750.560	732.410	18.462
2	2432.292	2422.934	5.943
3	5074.786	5052.412	3.243
4	8678.177	8650.932	2.594

$\eta_i = 39.82579689\,(\omega_i)^{-0.8223842554}$
confidence level = 99.17 %
$\eta_i = 0.1748510369\,(i)^{-1.453017763}$
confidence level = 99.41 %

Table 3.6 [45 / 0 / 45]

mode No (i)	ω_i	ω_{di}	η_i %
1	542.069	539.702	22.301
2	1756.613	1732.322	7.213
3	3665.045	3651.634	3.973
4	6267.440	6260.124	2.981

$\eta_i = 40.62264216\,(\omega_i)^{-0.836224538}$
confidence level = 99.49 %
$\eta_i = 0.2135632769\,(i)^{-1.476719962}$
confidence level = 99.68 %

ω_i = Frequency, (rad / sec)
ω_{di} = Damped frequency,(rad / sec)
η_i = Damping (%)

Table (2) : The numerical and experimental modal parameters of the first four modes of the hingd-hinged GRP beam of volume fraction 15 %

Table 2.1 [0 / 0 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	593.069	582.932	12.924
2	2372.247	2360.756	4.300
3	5337.553	5329.478	2.523
4	9488.985	9479.624	2.051

$\eta_i = 9.327002654\,(\omega_i)^{-0.6800214877}$
confidence level = 99.25 %
$\eta_i = 0.1213309721\,(i)^{-1.360037027}$
confidence level = 99.25 %

Table 2.2 [0 / 30 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	588.673	580.402	13.699
2	2354.683	2349.331	4.500
3	5298.036	5289.322	2.600
4	9418.720	9400.163	2.090

$\eta_i = 10.78741662\,(\omega_i)^{-0.6941263682}$
confidence level = 99.32 %
$\eta_i = 0.1289065342\,(i)^{-1.388250332}$
confidence level = 99.32 %

Table 2.3 [0 / 45 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	586.964	577.432	14.645
2	2344.498	2339.542	5.000
3	5275.113	5269.921	2.629
4	9377.974	9369.542	2.250

$\eta_i = 12.23150901\,(\omega_i)^{-0.7021821801}$
confidence level = 99.17 %
$\eta_i = 0.1391538401\,(i)^{-1.403660715}$
confidence level = 99.17 %

Table 2.4 [0 / 90 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	585.689	574.231	15.974
2	2342.741	2331.456	5.469
3	5271.165	5260.374	2.699
4	9370.959	9365.423	2.430

$\eta_i = 14.3048158\,(\omega_i)^{-0.7131617267}$
confidence level = 99.88 %
$\eta_i = 0.1519456954\,(i)^{-1.42631996}$
confidence level = 98.88 %

Table 2.5 [45 / -45 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	480.840	470.232	19.634
2	1923.358	1920.272	6.001
3	4327.559	4320.432	3.456
4	7693.433	7686.245	2.613

$\eta_i = 16.54395142\,(\omega_i)^{-0.737840974}$
confidence level = 99.47 %
$\eta_i = 0.1846925915\,(i)^{-1.477203114}$
confidence level = 99.47 %

Table 2.6 [45 / 0 / 45]

mode No (i)	ω_i	ω_{di}	η_i %
1	347.272	340.234	22.732
2	1389.059	1379.921	7.602
3	3125.393	3120.781	4.214
4	5556.251	5549.956	3.000

$\eta_i = 17.67582368\,(\omega_i)^{-0.73860172}$
confidence level = 99.86 %
$\eta_i = 0.2208159001\,(i)^{-1.475672715}$
confidence level = 99.86 %

ω_i = Frequency, (rad / sec)
ω_{di} = Damped frequency,(rad / sec)
η_i = Damping (%)

Table (4) : The numerical and experimental modal parameters of the first four modes of the fixed-fixed GRP beam of volume fraction 15 %

Table 4.1 [0 / 0 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	1343.339	1320.421	12.334
2	3702.921	3698.343	4.222
3	7259.203	7241.722	2.460
4	11999.832	11972.345	2.000

$\eta_i = 51.28616086\,(\omega_i)^{-0.8493262489}$
confidence level = 98.79 %
$\eta_i = 0.116470374\,(i)^{-1.344920653}$
confidence level = 99.33 %

Table 4.2 [0 / 30 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	1333.381	1319.734	13.426
2	3675.505	3654.432	4.400
3	7205.458	7198.762	2.560
4	11910.975	11812.326	2.083

$\eta_i = 63.64979558\,(\omega_i)^{-0.869594388}$
confidence level = 98.66 %
$\eta_i = 0.1258691388\,(i)^{-1.377577889}$
confidence level = 99.23 %

Table 4.3 [0 / 45 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	1329.512	1320.462	14.032
2	3659.607	3640.321	4.693
3	7174.283	7163.522	2.620
4	11859.447	11850.231	2.135

$\eta_i = 73.42085231\,(\omega_i)^{-0.882331201}$
confidence level = 98.83 %
$\eta_i = 0.1328192739\,(i)^{-1.396000153}$
confidence level = 99.35 %

Table 4.4 [0 / 90 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	1326.623	1318.420	15.224
2	3656.864	3532.326	5.239
3	7168.913	7159.463	2.640
4	11850.577	11820.234	2.392

$\eta_i = 81.27553568\,(\omega_i)^{-0.8852311729}$
confidence level = 98.28 %
$\eta_i = 0.144295589\,(i)^{-1.401865082}$
confidence level = 98.82 %

Table 4.5 [45 / -45 / 0]

mode No (i)	ω_i	ω_{di}	η_i %
1	1089.134	1079.923	17.934
2	3002.236	3000.226	5.712
3	5885.586	5880.421	2.884
4	9729.166	9720.262	2.548

$\eta_i = 107.6262435\,(\omega_i)^{-0.9284367455}$
confidence level = 98.33 %
$\eta_i = 0.1685159872\,(i)^{-1.470720083}$
confidence level = 98.90 %

Table 4.6 [45 / 0 / 45]

mode No (i)	ω_i	ω_{di}	η_i %
1	786.593	780.932	21.952
2	2168.230	2153.462	6.933
3	4250.610	4230.262	3.800
4	7026.471	7019.942	2.803

$\eta_i = 117.4189045\,(\omega_i)^{-0.9534623338}$
confidence level = 99.32 %
$\eta_i = 0.210201369\,(i)^{-1.507551967}$
confidence level = 99.71 %

ω_i = Frequency, (rad / sec)
ω_{di} = Damped frequency,(rad / sec)
η_i = Damping (%)

Table (5) : The numerical and experimental modal parametrs of the first four modes of the fixed-free GRP beam of volume fraction 45 %

Table 5.1 [0 / 0 / 0]

mode No (i)	ωi	ωdi	η_i %
1	299.004	295.310	10.106
2	1873.816	1859.823	3.632
3	5246.742	5152.212	2.256
4	10281.522	10040.530	1.564

$\eta i = 1.973894157\ (\omega_i)^{-0.5243924613}$
confidence level = 99.9324 %
$\eta i = 0.09808470537\ (i)^{-1.343348129}$
confidence level = 99.86 %

Table 5.4 [0 / 90 / 0]

mode No (i)	ωi	ωdi	η_i %
1	294.631	276.460	11.932
2	1846.433	1822.124	4.396
3	5170.069	4913.451	2.430
4	10131.272	9952.566	1.704

$\eta i = 2.765085107\ (\omega_i)^{-0.5522194691}$
confidence level = 99.9926 %
$\eta i = 0.1181837326\ (i)^{-1.415279022}$
confidence level = 99.97 %

Table 5.2 [0 / 30 / 0]

mode No (i)	ωi	ωdi	η_i %
1	296.372	289.027	10.870
2	1857.310	1847.256	3.996
3	5196.766	4988.349	2.330
4	10190.975	10027.964	1.630

$\eta i = 2.288586882\ (\omega_i)^{-0.5362827918}$
confidence level = 99.9956 %
$\eta i = 0.1067766377\ (i)^{-1.374869678}$
confidence level = 99.95 %

Table 5.5 [45 / -45 / 0]

mode No (i)	ωi	ωdi	
1	230.643	213.628	15.294
2	1445.403	1432.566	5.702
3	4047.176	4021.239	2.812
4	7930.844	7929.380	2.298

$\eta i = 3.097594465\ (\omega_i)^{-0.5536083162}$
confidence level = 99.6279 %
$\eta i = 0.1502380332\ (i)^{-1.417323566}$
confidence level = 99.49 %

Table 5.3 [0 / 45 / 0]

mode No (i)	ωi	ωdi	η_i %
1	294.926	282.743	11.112
2	1848.281	1859.823	4.222
3	5175.234	4951.150	2.348
4	10141.394	10015.397	1.662

$\eta i = 2.400855191\ (\omega_i)^{-0.5394284987}$
confidence level = 99.9872 %
$\eta i = 0.1103373523\ (i)^{-1.382578696}$
confidence level = 99.97 %

Table 5.6 [45 / 0 / 45]

mode No (i)	ωi	ωdi	η_i %
1	141.529	138.230	20.910
2	886.960	841.947	7.462
3	2483.511	2475.575	3.808
4	4866.691	4863.185	2.842

$\eta i = 3.591001961\ (\omega_i)^{-0.5741157503}$
confidence level = 99.8897 %
$\eta i = 0.2062230506\ (i)^{-1.470657879}$
confidence level = 99.81 %

ω_i = Frequency, (rad / sec)
ω_{di} = Damped frequency,(rad / sec)
η_i = Damping (%)

Table (6) : The numerical and experimental modal parametrs of the first four modes of the hinged-hinged GRP beam of volume fraction 45 %

Table 6.1 [0 / 0 / 0]

mode No (i)	ωi	ωdi	η_i %
1	839.317	792.731	10.100
2	3357.250	3217.402	3.549
3	7553.816	7345.341	2.230
4	13429.009	12959.213	1.340

$\eta i = 12.2618702\ (\omega_i)^{-0.5841881622}$
confidence level = 99.82 %
$\eta i = 0.1000745595\ (i)^{-1.4283739}$
confidence level = 99.82 %

Table 6.4 [0 / 90 / 0]

mode No (i)	ωi	ωdi	η_i %
1	827.042	734.252	11.805
2	3308.189	3214.621	4.350
3	7443.429	7324.250	2.400
4	13232.763	12941.623	1.700

$\eta i = 13.36235298\ (\omega_i)^{-0.7056087517}$
confidence level = 99.95 %
$\eta i = 0.1167480526\ (i)^{-1.411221058}$
confidence level = 99.95 %

Table 6.2 [0 / 30 / 0]

mode No (i)	ωi	ωdi	η_i %
1	831.929	792.423	10.507
2	3327.677	3304.214	3.820
3	7481.865	7264.232	2.300
4	13310.742	12364.235	1.615

$\eta i = 12.705645702\ (\omega_i)^{-0.616894804}$
confidence level = 99.90 %
$\eta i = 0.1024333098\ (i)^{-1.353649578}$
confidence level = 99.90 %

Table 6.5 [45 / -45 / 0]

mode No (i)	ωi	ωdi	
1	647.425	623.231	14.621
2	2589.677	2504.921	5.622
3	5826.782	5810.234	2.731
4	10358.717	9978.721	2.200

$\eta i = 14.14468289\ (\omega_i)^{-0.7075073493}$
confidence level = 99.54 %
$\eta i = 0.1450963529\ (i)^{-1.415010707}$
confidence level = 99.53 %

Table 6.3 [0 / 45 / 0]

mode No (i)	ωi	ωdi	η_i %
1	827.870	798.624	11.100
2	3311.500	3300.423	4.220
3	7450.865	7422.372	2.346
4	13245.983	12932.421	1.650

$\eta i = 12.9114341\ (\omega_i)^{-0.6929672648}$
confidence level = 99.97 %
$\eta i = 0.1103654776\ (i)^{-1.38593684}$
confidence level = 99.97 %

Table 6.6 [45 / 0 / 45]

mode No (i)	ωi	ωdi	η_i %
1	397.278	369.412	19.345
2	1589.135	1579.413	6.293
3	3575.549	3565.321	3.478
4	6356.533	6243.413	2.620

$\eta i = 14.9544474\ (\omega_i)^{-0.7337658609}$
confidence level = 99.67 %
$\eta i = 0.1852006964\ (i)^{-1.467538494}$
confidence level = 99.67 %

ω_i = Frequency, (rad / sec)
ω_{di} = Damped frequency,(rad / sec)
η_i = Damping (%)

Table (7) : The numerical and experimental modal parametrs of the first four modes of the fixed-hinged GRP beam of volume fraction 45 %

Table 7.1 [0 / 0 / 0]

mode No (i)	ωi	ωdi	η_i %
1	1311.173	1210.233	10.070
2	4249.018	4196.204	3.400
3	8865.242	7672.410	2.210
4	15160.092	14098.341	1.290

$\eta i = 23.79972844\ (\omega_i)^{-0.7147911147}$
confidence level = 99.65 %
$\eta i = 0.09911400699\ (i)^{-1.445466973}$
confidence level = 99.70 %

Table 7.4 [0 / 90 / 0]

mode No (i)	ωi	ωdi	η_i %
1	1291.997	1021.912	11.69[illegible]
2	4186.925	4062.314	4.23[illegible]
3	8735.691	55[illegible].513	2.3[illegible]
4	14938.548	130[illegible].711	1.6[illegible]

$\eta i = 34.67443178\ (\omega_i)^{-0.79[illegible]10031}$
confidence level = 99.86 %
$\eta i = 0.1150149544\ (i)^{-1.4[illegible]957}$
confidence level = 99.95 %

Table 7.2 [0 / 30 / 0]

mode No (i)	ωi	ωdi	η_i %
1	1299.632	1130.235	10.332
2	4211.590	3394.144	3.970
3	8780.800	7586.213	2.284
4	15026.580	14011.321	1.600

$\eta i = 23.9763658\ (\omega_i)^{-0.7637993269}$
confidence level = 99.80 %
$\eta i = 0.1009072246\ (i)^{-1.347655561}$
confidence level = 99.92 %

Table 7.5 [45 / -45 / 0]

mode No (i)	ωi	ωdi	
1	1011.401	[illegible].910	13.4[illegible]
2	3277.560	3185.322	4.9[illegible]
3	6838.376	4821.933	2.42[illegible]
4	11694.020	10[illegible].211	2.1[illegible]

$\eta i = 35.96904270\ (\omega_i)^{-0.7[illegible]}$
confidence level = 99.20 %
$\eta i = 0.1297809696\ (i)^{-1.35[illegible]}$
confidence level = 99.39 %

Table 7.3 [0 / 45 / 0]

mode No (i)	ωi	ωdi	η_i %
1	1293.291	1021.444	10.990
2	4191.116	3988.303	4.215
3	8744.418	6859.523	2.343
4	14953.473	14500.231	1.640

$\eta i = 29.69637997\ (\omega_i)^{-0.7838391628}$
confidence level = 99.93 %
$\eta i = 0.1095308524\ (i)^{-1.382204698}$
confidence level = 99.98 %

Table 7.6 [45 / 0 / 45]

mode No (i)	ωi	ωdi	η_i %
1	620.624	[illegible]	18.[illegible]
2	2011.248	192[illegible].421	6.14[illegible]
3	4196.305	3012.633	3.[illegible]
4	7175.930	5572.414	2.4[illegible]

$\eta i = 38.143634\ (\omega_i)^{-0.841462[illegible]}$
confidence level = 99.22 %
$\eta i = 0.1731617549\ (i)^{-1.4[illegible]}$
confidence level = 99.41 %

ω_i = Frequency, (rad / sec)
ω_{di} = Damped frequency,(rad / sec)
η_i = Damping (%)

Table (8) : The numerical and experimental modal parametrs of the first four modes of the fixed-fixed GRP beam of volume fraction 45 %

Table 8.1 [0 / 0 / 0]

mode No (i)	ωi	ωdi	η_i %
1	1902.637	1824.301	10.049
2	5244.665	5123.914	3.239
3	10281.646	10391.313	2.199
4	16996.086	15992.412	1.290

$\eta i = 53.07860041\ (\omega_i)^{-0.8496440376}$
confidence level = 98.30 %
$\eta i = 0.09738447238\ (i)^{-1.440801284}$
confidence level = 99.54 %

Table 8.4 [0 / 90 / 0]

mode No (i)	ωi	ωdi	η_i %
1	1874.811	1793.212	11.320
2	5168.022	5109.333	4.226
3	10131.396	10123.612	2.350
4	10747.712	15932.400	1.670

$\eta i = 84.68154885\ (\omega_i)^{-0.8831281142}$
confidence level = 99.20 %
$\eta i = 0.1119917016\ (i)^{-1.393397664}$
confidence level = 99.78 %

Table 8.2 [0 / 30 / 0]

mode No (i)	ωi	ωdi	η_i %
1	1885.889	1693.621	10.200
2	5198.466	5187.322	3.741
3	10183.711	10162.911	2.270
4	16846.405	15932.440	1.590

$\eta i = 58.93749276\ (\omega_i)^{-0.8500408401}$
confidence level = 99.64 %
$\eta i = 0.09951071051\ (i)^{-1.34216299}$
confidence level = 99.91 %

Table 8.5 [45 / -45 / 0]

mode No (i)	ωi	ωdi	
1	1467.639	1279.224	12.344
2	4045.570	4023.914	4.442
3	7930.947	7033.510	2.549
4	13110.248	12195.623	1.970

$\eta i = 85.04971645\ (\omega_i)^{-0.8929326324}$
confidence level = 99.30 %
$\eta i = 0.1189287291\ (i)^{-1.348544721}$
confidence level = 99.69 %

Table 8.3 [0 / 45 / 0]

mode No (i)	ωi	ωdi	η_i %
1	1876.688	1798.213	10.920
2	5173.194	5094.231	3.999
3	10141.517	10111.732	2.340
4	16764.444	15942.344	1.630

$\eta i = 74.57958705\ (\omega_i)^{-0.8719420255}$
confidence level = 99.73 %
$\eta i = 0.1072237864\ (i)^{-1.376301682}$
confidence level = 99.95 %

Table 8.6 [45 / 0 / 45]

mode No (i)	ωi	ωdi	η_i %
1	900.584	897.410	16.345
2	2482.532	2374.234	5.994
3	4866.750	4671.603	2.991
4	8044.986	7213.422	2.370

$\eta i = 96.0953574\ (\omega_i)^{-0.9[illegible]}$
confidence level = 99.36 %
$\eta i = 0.1607317303\ (i)^{-1.43[illegible]}$
confidence level = 99.62 %

ω_i = Frequency, (rad / sec)
ω_{di} = Damped frequency,(rad / sec)
η_i = Damping (%)

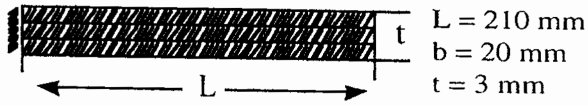
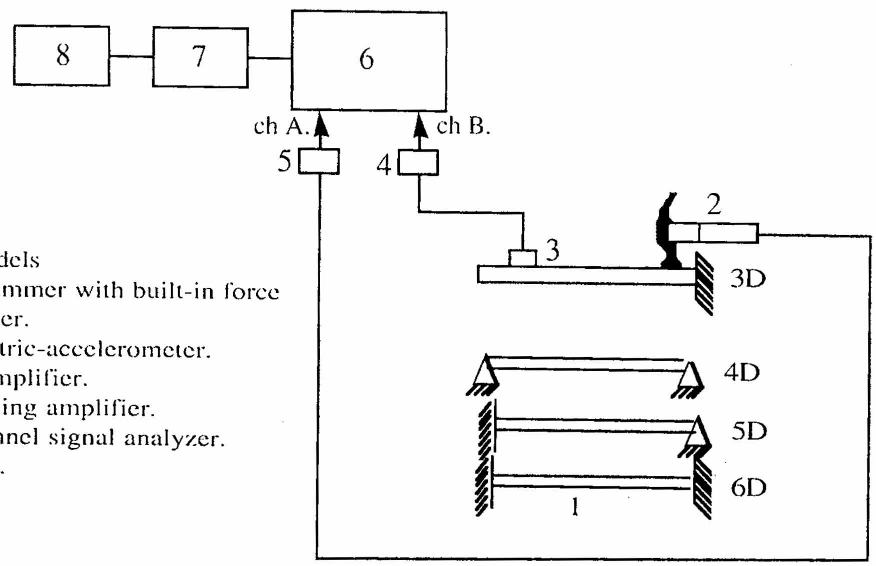


Fig. (1 a) : 3-layer beam model.



1. Beam models.
2. Impact hammer with built-in force transducer.
3. Piezoelectric-accelerometer.
4. Charge amplifier.
5. Conditioning amplifier.
6. Dual-channel signal analyzer.
7. Computer.
8. Printer.

(Fig. 1 b) : Schematic block diagram of the measuring circuit.

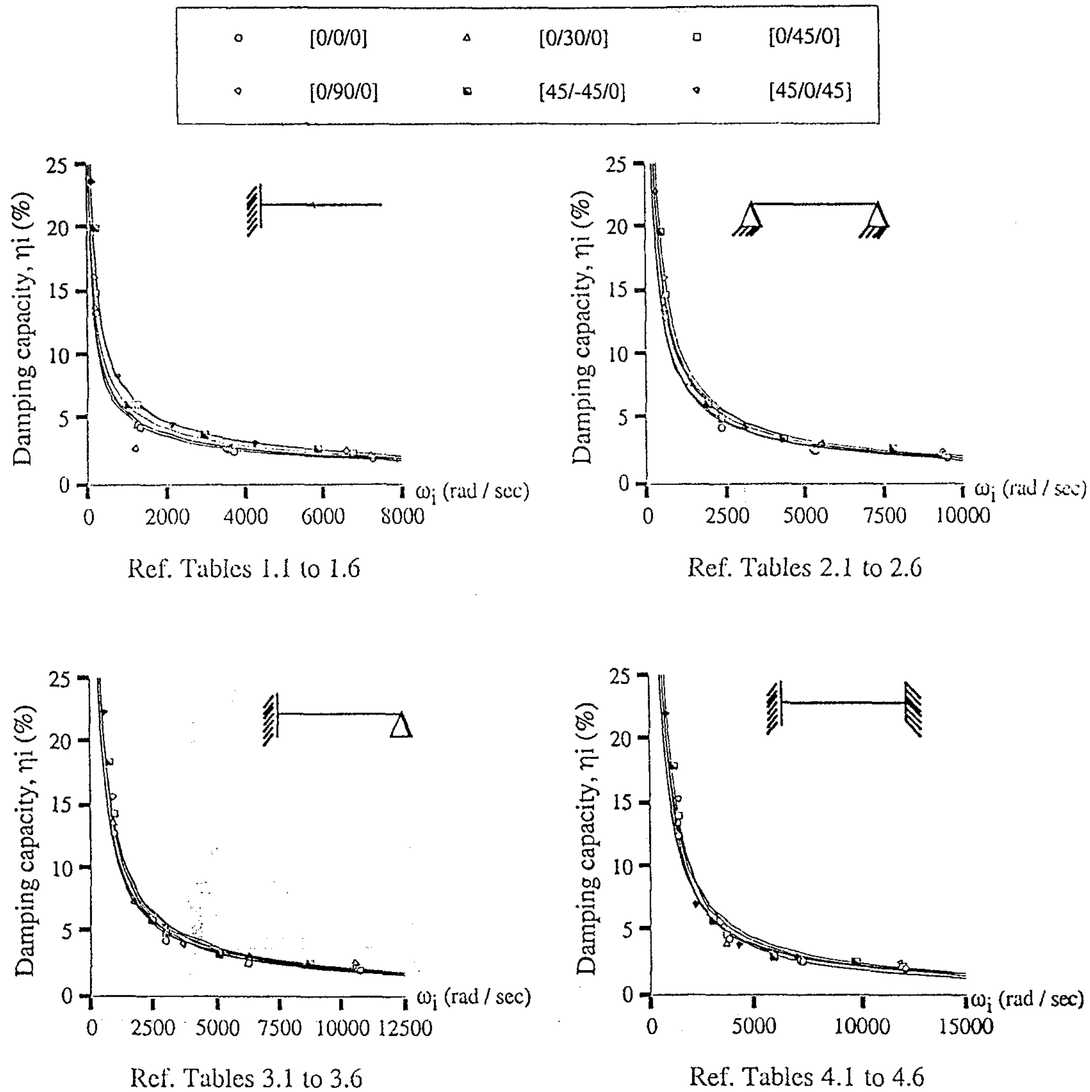


Fig. (2) : Quasihyperbolic relationship relating damping capacities ηi with natural frequencies, ω_i for LBC of various orientations for different types of fixations at $V_f = 15$ %.

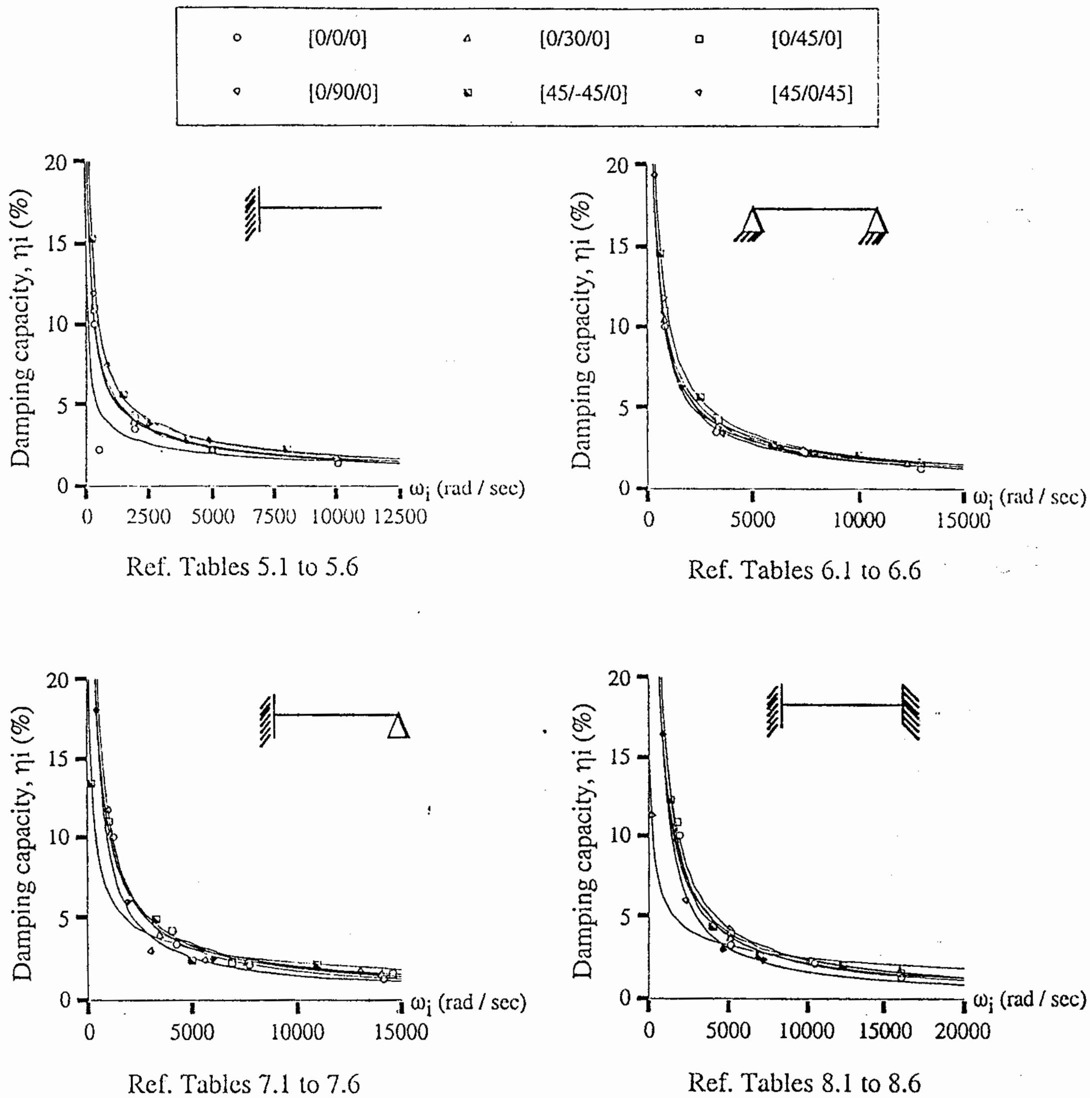


Fig. (3) : Quasihyperbolic relationship relating damping capacities ηi with natural frequencies, ω_i for LBC of various orientations for different types of fixations at $V_f = 45$ %.

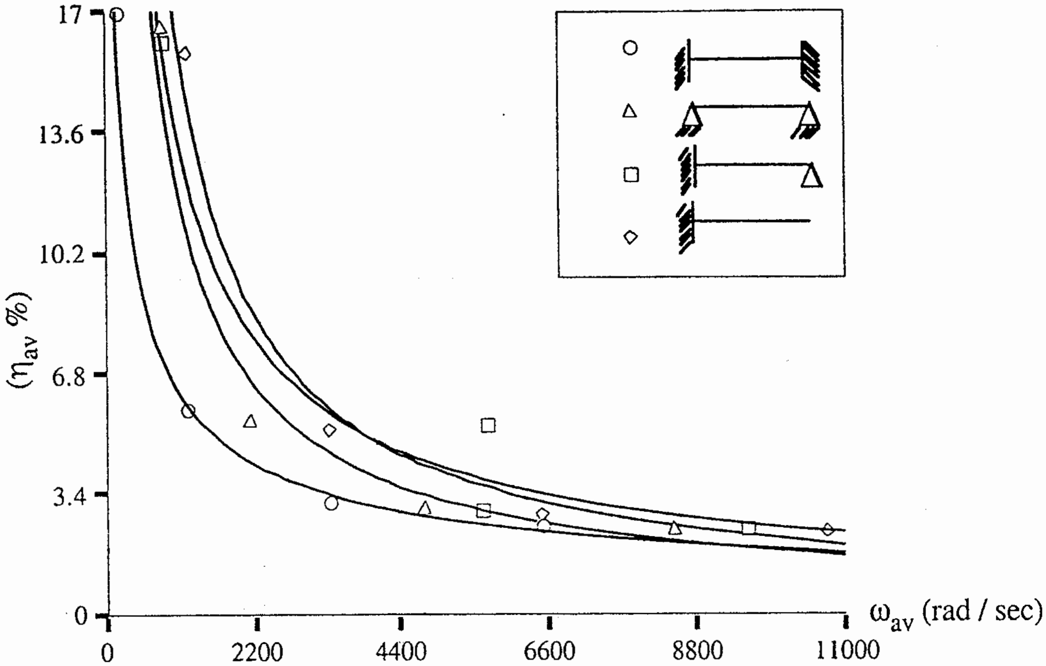


Fig. (4a) : Generalized quasihyperbolic relations between damping capacities, η_{av} and natural frequencies ω_{av} for various types of fixations at $V_f = 15$ %.

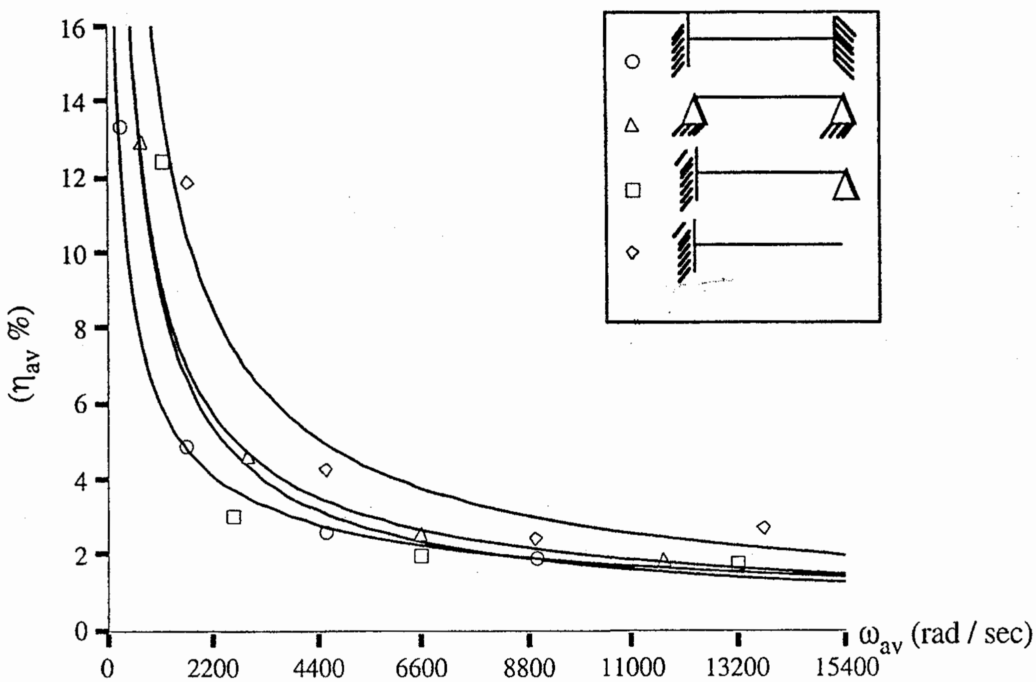


Fig. (4b) : Generalized quasihyperbolic relations between damping capacities, η_{av} and natural frequencies ω_{av} for various types of fixations at $V_f = 45$ %.

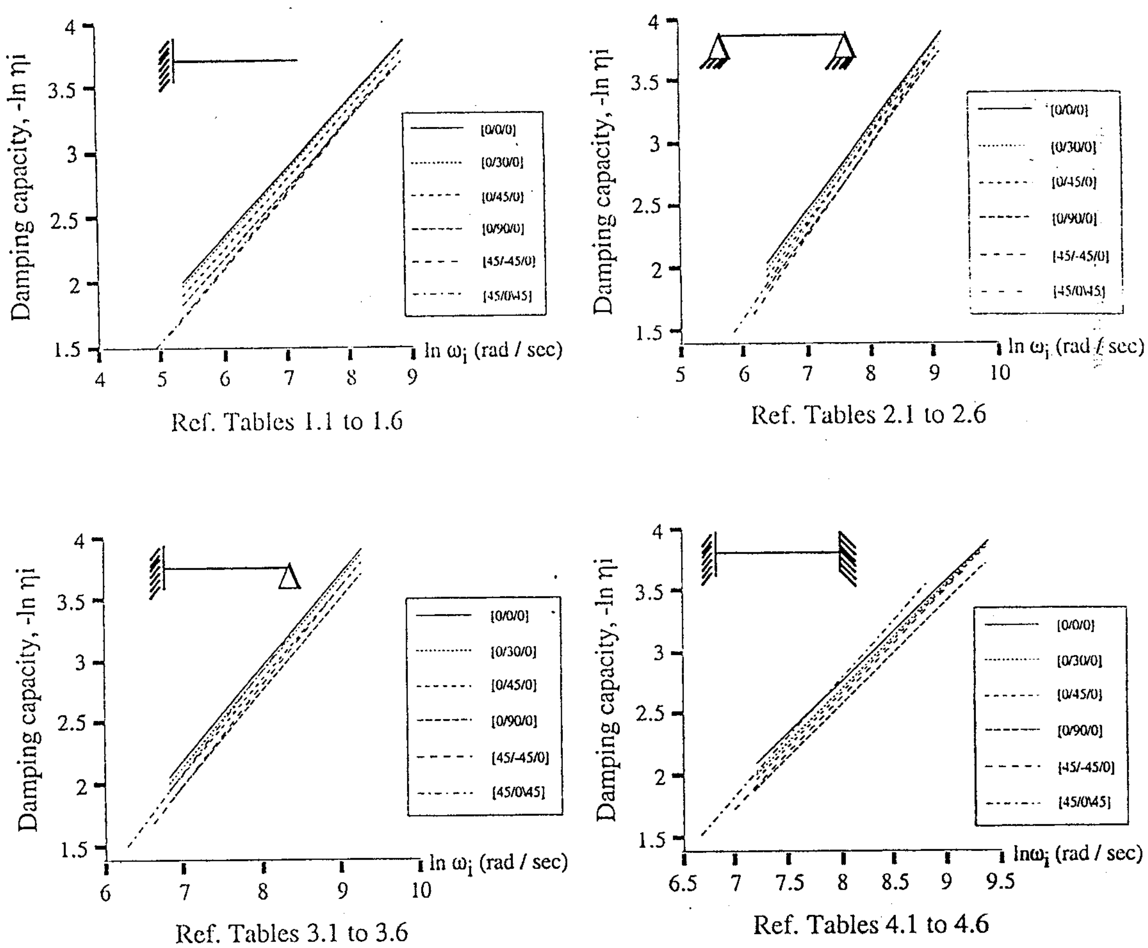


Fig. (5) : Logarithmic forms relating damping capacities, η_i with natural frequencies, ω_i for LCB of various orientations for different types of fixations at Vf = 15 %.

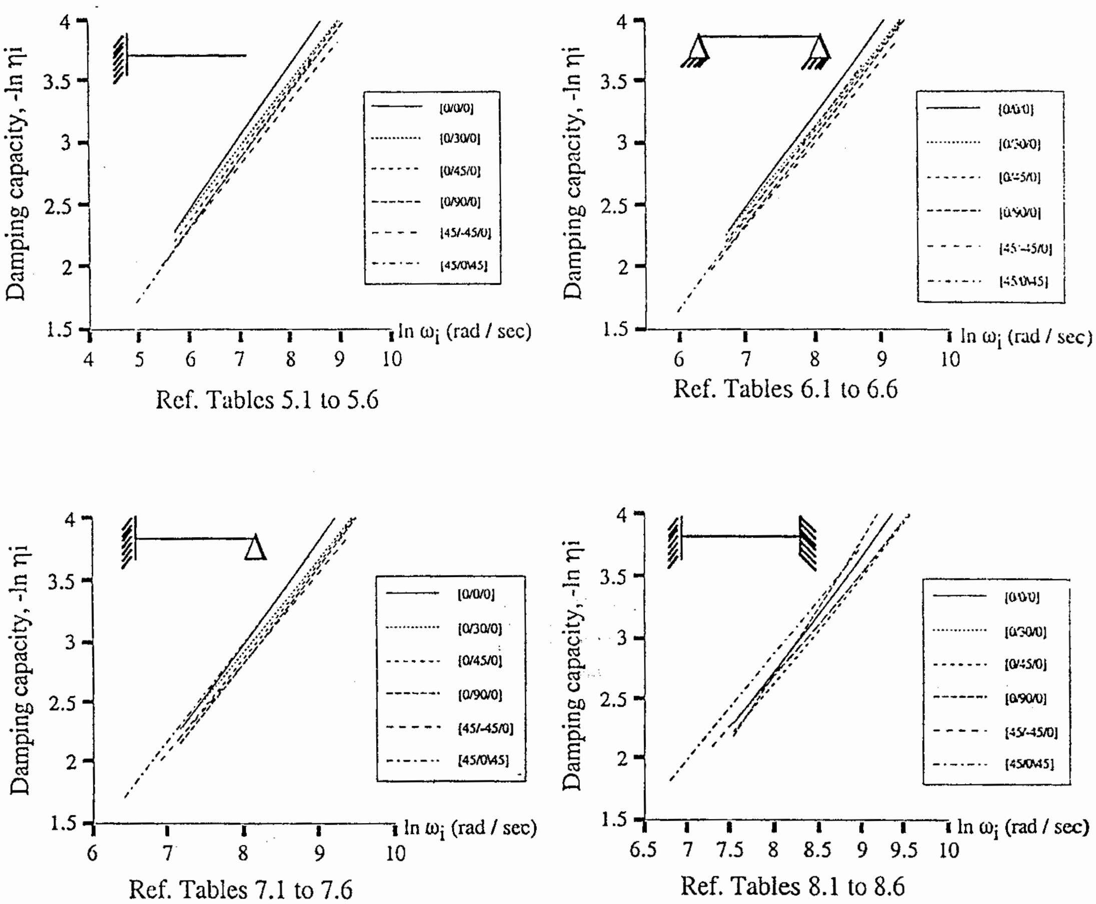


Fig. (6) : Logarithmic forms relating damping capacities, ηi with natural frequencies, ω_i for LCB of various orientations for different types of fixations at V_f = 45 %.

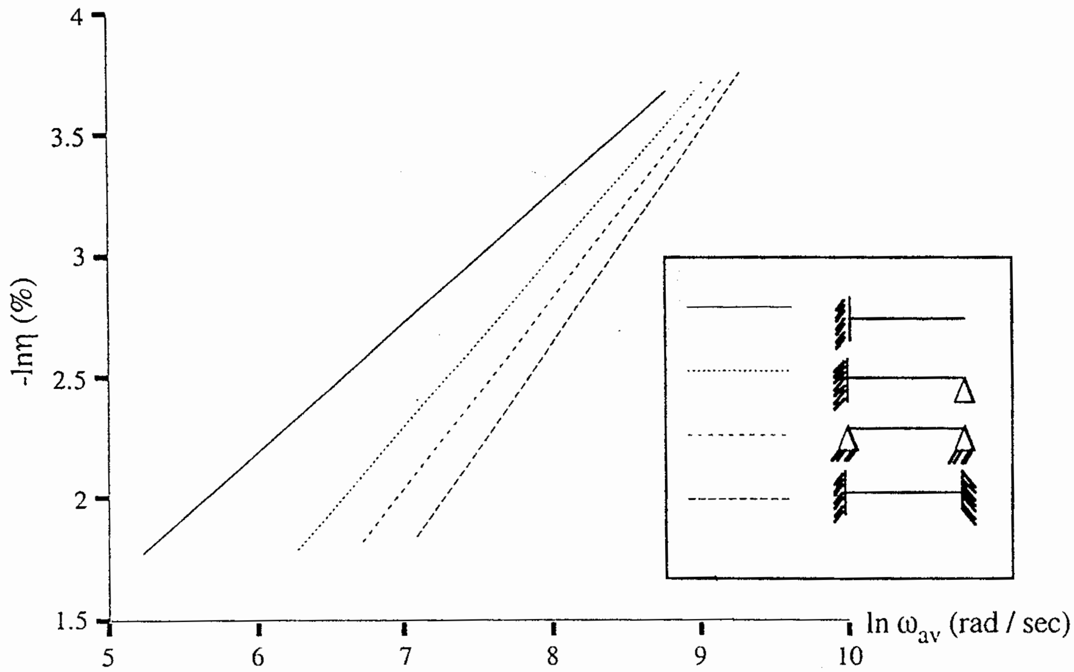


Fig. (7) : Generalized logarthmic quasilinear forms relating damping capacities, η_i with natural frequencies ω_i for various types fixations at $V_f = 15$ % (ref. Table 9).

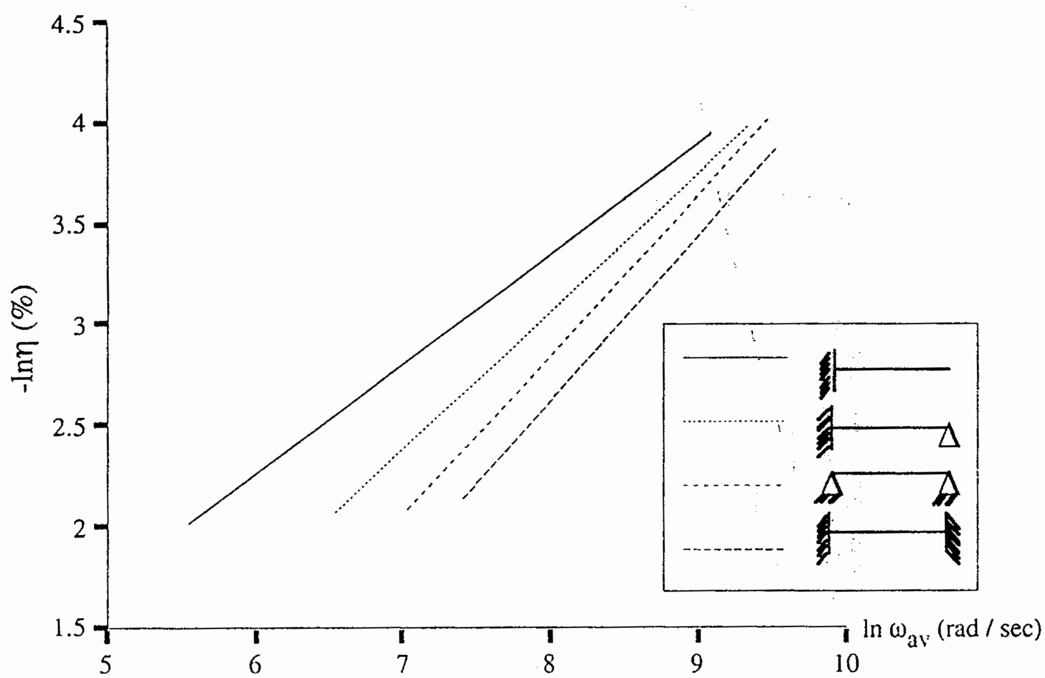


Fig. (8) : Generalized logarithmic quasilinear forms relating damping capacities, η with natural frequencies ω_i for various types of fixations at $V_f = 45$ % (ref. Table 9).

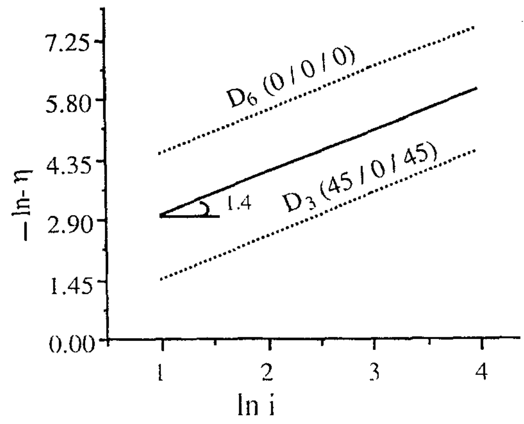


Fig. (9) : Logarithmic representation of dmping loss factors in various natural modes.

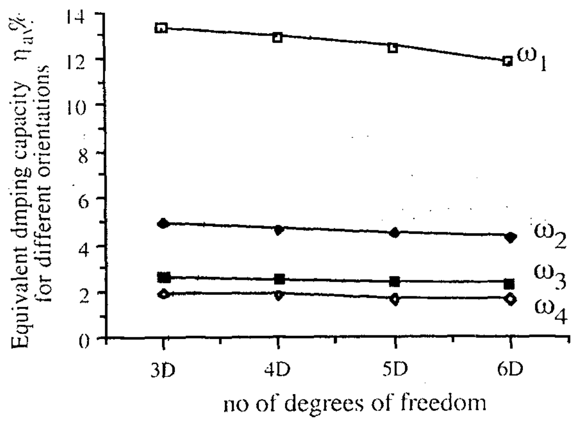


Fig. (10) : Equivalent damping capacity η_{eq} % for various degree of constraint (D) at V_f = 45%.

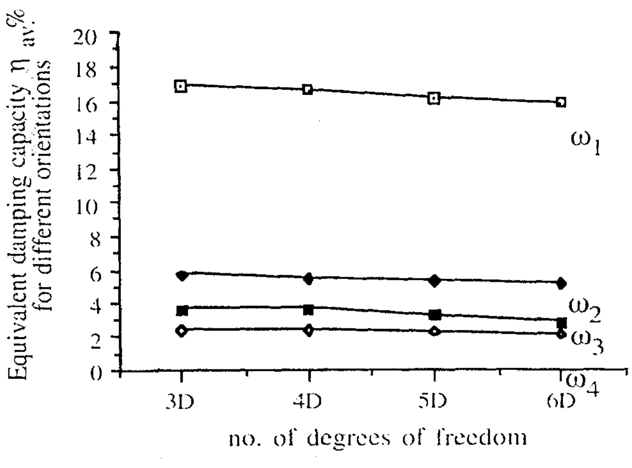


Fig. (11) : Equivalent damping capacity η_{eq} % for the degree of constraint (D) at V_f = 15%.

نموذج معدل للسلوك الإخمادى فى المنشآت المهتزة المصنعة من ألياف الزجاج المعزز بالبلاستك

د. أحمد ماهر، د. فوقية رمضان

ملخص البحث

يهدف البحث إلى إعداد نموذج معدل وكفء للسلوك الإخمادى فى المنشآت المهتزة المصنعة من ألياف الزجاج المعزز بالبلاستك وذلك بتحسين خصائص التقارب بين القياسات المعملية والتقنية التجميعية بهدف تقليل أى أخطاء جانبية ورفع مستوى الإعتماد فى بناء النموذج المعدل وذلك بحسابات عوامل الوزن (α , a) مع مراعاة مدى واسع من الترددات – ودرجات مختلفة من الرقم الكودى – ونسبة الألياف – ودرجات القيد المعرض لها المنشأ وتمت المقارنة بينها وبين النموذج الرياضى باستخدام طريقة العناصر المحددة . وتم التوصل إلى تمثيل السلوك الإخمادى لألياف الزجاج المعزز بالبلاستك بالإخماد ذو الكتلة المنتظمة فى جميع الحالات .

وتم التوصل أيضاً إلى أن العلاقة بين معامل بالإخماد والترددات الطبيعية للمنشأ قطع زائدى المقطع .

وتم إنشاء علاقة لوغاريتمية يسهل التعامل معها قبل وبعد قيم الترددات التى تمت الدراسة عندها .