Answer all the following questions:

Question 1 (20 MARKS)

(A) *The joint density function of two random variables X and Y is given by:*

$$f(x,y) = \begin{cases} \dfrac{xy}{96}, & 0 < x < 4, 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$

Find (a) $E(X)$, (b) $E(Y)$, (c) $E(XY)$, (d) $E(2X + 3Y)$.   **(10 Marks)**

(B) (i)
Prove that the mean and variance of a binomially distributed random variable are, respectively,
$\mu = np$ *and* $\sigma^2 = npq$.

(ii) *Out of 2000 families with 4 children each, how many would you expect to have*
(a) *at least 1 boy,*
(b) *2 boys,*
(c) *1 or 2 girls,*
(d) *no girls?*

(10 Marks)

Question 2 (20 MARKS)

(A)
The mean weight of 500 male students at a certain college is 151 lb and the standard deviation is 15 lb.
Assuming that the weights are normally distributed, find how many students weigh
(a) *between 120 and 155 lb,*
(b) *more than 185 lb.*   *(10 Marks)*

(B)
The mean inside diameter of a sample of 200 washers produced by a machine is 0.502 inches and

the standard deviation is 0.005 inches. The purpose for which these washers are intended allows

a maximum tolerance in the diameter of 0.496 to 0.508 inches, otherwise the washers are considered

defective. Determine the percentage of defective washers produced by the machine, assuming the

diameters are normally distributed.   (10 Marks)

Question 3 (20 MARKS)

(A) *Establish the validity of the Poisson approximation to the binomial distribution.*

(B)
If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.001,
determine the probability that out of 2000 individuals, (a) exactly 3, (b) more than 2, individuals will
suffer a bad reaction.

Question 4 (20 MARKS)

(A)

A population consists of the five numbers $2, 3, 6, 8, 11$. Consider all possible samples of size two which can be drawn with replacement from this population. Find

(a) the mean of the population,

(b) the standard deviation of the population,

(c) the mean of the sampling distribution of means,

(d) the standard deviation of the sampling distribution of means, i.e., the standard error of means.

(10 Marks)

(B)

The table gives experimental values of the pressure P of a given mass of gas corresponding to various values of the volume V. According to thermodynamic principles, a relationship having the form

$PV^{\gamma} = C$ where γ and C are constants, should exist between the variables.

(a) Find the values of and C.

(b) Write the equation connecting P and V.

(c) Estimate P when in $V = 100.0 \ in^3$

Volume V (in^3)	54.3	61.8	72.4	88.7	118.6	194.0
Pressure P (lb/in^2)	61.2	49.5	37.6	28.4	19.2	10.1

(10 Marks)

Question 5 (20 MARKS)

(A) Show that the linear correlation coefficient is given by:

$$r = \frac{n \Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{[n \Sigma x^2 - (\Sigma x)^2][n \Sigma y^2 - (\Sigma y)^2]}}$$

(10 Marks)

(B)

The mean inside diameter of a sample of 200 washers produced by a machine is 0.502 inches and the standard deviation is 0.005 inches. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 0.496 to 0.508 inches, otherwise the washers are considered defective. Determine the percentage of defective washers produced by the machine, assuming the diameters are normally distributed.

(10 Marks)

With my best wishes

Associate Prof. Dr. Islam M. Eldesoky