University	:Menoufia		Date	: 25/ 12/2019
Faculty	:Electronic Engineering		Time	: 3 Hours
Department	:Physics and Engineering Mathematics		No. of pages	: 2 pages
Academic level	:Preparatory year		Full Mark	: 100 Marks
Course Name	:Mathematics (1)		Exam	: Final Exam
Course Code	:PEM (1)		Examiner	: Prof. Dr. Ramadan Dr. Hany El-gohary

Part 1

Answer all the following questions:

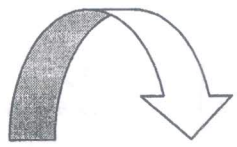**Question No 1:**   (15 Marks)

    a) Solve the inequality $\left|3x-5\right|-\left|2x+3\right|>0$.

    b) Find the inverse of $y=\dfrac{x}{4}+3$.

    c) Discuss the continuity of $f(x)=\dfrac{x^2-2}{\left|x-2\right|}$.

Question No 2:   (15 Marks)

    a) Graph the function $y=x^3-3x^2$.

    b) Find $\dfrac{dy}{dx}$ if $y=\log_x(x^2+1)$.

    c) Prove that $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$.

Question No 3:   (20 Marks)

    a) Evaluate $(i)\ \lim\limits_{x\to 1}\dfrac{\cos\left(\dfrac{\pi x}{2}\right)}{1-x}$, $(ii)\ \dfrac{d}{dx}\left(\tan^{-1}\sqrt{x}\right)^3$

    b) Prove that $\sec^{-1}x=\cos^{-1}\left(\dfrac{1}{x}\right)$, and obtain the logarithmic expression for $\sinh^{-1}x$.

    c) Find the Maclaurin's series for $f(x)=\sin x$ and show that it represents f for all real x.

Part 2

<u>Answer all the following questions:</u>

<u>Question No 1:</u>   (30 Marks)

a) **Find a formula for the** nth **term of the sequences and find** $\lim\limits_{n \to \infty} a_n$ **(Converges**

   or diverges).   (9 Marks)

   (1) $-\dfrac{4}{2}, -\dfrac{7}{5}, -\dfrac{10}{8}, -\dfrac{13}{11}, \ldots$ (2) $-\dfrac{3}{5}, 0, \dfrac{5}{9}, \dfrac{12}{11}, \ldots$

   (3) $(\sqrt{2}-1), 2(\sqrt{5}-2), 3(\sqrt{10}-3), 4(\sqrt{17}-4), \ldots$

b) **Find a formula for the** nth **partial sum of each series and use it to find the**

   series sum if the series converges.   (6 Marks)

   (1) $2 + \dfrac{2}{3} + \dfrac{2}{9} + \dfrac{2}{27} + \cdots$ (2) $\dfrac{1}{2.3} + \dfrac{1}{3.4} + \dfrac{1}{4.5} + \dfrac{1}{5.6} + \cdots$

c) **Test the series.**   (9 Marks)

   (1) $\displaystyle\sum_{n=1}^{\infty} \dfrac{2n}{n^2 + 2n + 1}$ (2) $\displaystyle\sum_{n=2}^{\infty} (-1)^{n-1} \dfrac{1}{n \ln^2(n)}$ (3) $\displaystyle\sum_{n=1}^{\infty} \dfrac{4^n n! n!}{(2n)!}$

d) **Find the series radius and interval of convergence. For what values of** x **does**

   the series converge and absolutely or conditionally.   (6 Marks)

   $$\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}(x+2)^n}{n \, 2^n}$$

<u>Question No 2:</u>   (20 Marks)

a) **A concrete bridge is designed with an arch in the shape of a Parabola. The road**
   over the bridge is 120 **feet long and the maximum height of the arch is** 50 **feet.**
   Write an equation for the parabolic arch.   (6 Marks)

b) **Prove that the locus of the point from which we can draw perpendicular**
   tangents to the Parabola $y^2 = 4ax$ **is the directrix.**   (4 Marks)

c) **Prove that this equation is a Hyperbola section and find vertices, foci,**
   directrices, Asymptotes and focal length.   (10 Marks)

   $$9x^2 - 2y^2 - 18x - 4y + 25 = 0$$

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Good Luck