Eng. Research Bull.
Faculty of Eng. & Tech.,
Menoufia University
Vol. VIII Part II, 1986.

PP: 31-42.

A GENERALIZED DYNAMIC LINEAR MODEL OF A CONVERTER - FED DC MOTOR SYSTEM

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ABSTRACT:

This paper deals with an AC-DC power conversion unit connected to a separately excited DC motor drive system, and supplied by an AC feeder line. A generalized linear mathematical model of such a system is developed, which represents the equations describing the system dynamic behaviour. The converter current controller and the DC motor speed controller are presented, and the model is valid for any type of such controllers. The developed model represents different configurations of the converter operation; it is also applicable to any type of AC - DC converter supplying a separately excited DC motor, and it is suitable for any mode of converter operation.

The analysis is based on perturbation technique method, and the system with their controllers are represented by a signal flow graph. This permits the evaluation of transfer functions of the overall system for a step change in the control reference parameters (speed reference and load torque). Then, the time responses of the motor speed and the armature current can be determined.

An application is considered for a separately excited DC motor fed by a single phase bridge rectifier. A comparison is made between the experimental and theorotical results.

1. INTRODUCTION:

Direct current drives are extensively used in industry. The outstanding advantages of DC drives, such as ease of control, precise and continuous

control of speed over a wide range ensure their popularity. When designing a high response thyristor drive with internal multicontrol loops, the transient behaviour must be taken into consideration to achieve the required dynamic performance.

The design, construction and testing of a closed-loop system for the speed control of a separately excited DC motor fed from dual converter are well discussed in literature [1]. The perturbation technique has been used for the transient analysis of a controlled bridge rectifier with a DC motor load [2-4]. A microprocessor implementation is proposed to study the effect of discontinuous conduction on a converter-fed DC drive [5]. The application of dynamic optimal control theory on typical industrial speed-controlled DC machine drives reveals the manner in which the control should operate [6]. Three different methods to calculate the current and speed controller parameters for a DC motor fed from a single-phase thyristor bridge have been presented [7,8].

In this paper, a generalized mathematical linear model of a separately excited DC motor supplied from an AC-DC converter is presented. The dynamic model allows us to obtain the time responses of the motor speed and armature current for a step change in the control reference parameters.

In the model, different configurations of converter operation are considered: the load current continuity, the presence of freewheeling diode across the motor terminals, the type of the AC feeder, the type of rectifier elements and the mode of rectification.

The analysis is based on perturbation technique method; and the overall system is represented in the form of signal flow graph. Experimental and theoretical results are compared for a single phase bridge rectifier with a DC motor load.

2. SIGNAL FLOW GRAPHS OF SYSTEM ELEMENTS:

The dynamic simulation of a separately excited DC motor fed by a thyristorized converter is presented. With the application of perturbation technique method on the equations representing the steady state operation of the system elements, the incremental variations of the variables of each element can be obtained.

2.1 DC motor:

The armature voltage and the torque balance equations are:

$$v_d = k_e \phi n_m + R_a i_a + L_m (di_a/dt) \quad (1)$$

$$k_t \phi i_a = T_L + B i_a + J (dn_m/dt) \quad (2)$$

consider that the motor mechanical time constant is greater than the electrical time constant; and for small perturbations in the motor variables, the incremental equations are:

$$\Delta I_a = A_1 \Delta V_d + B_1 \Delta N_m \quad (3)$$

$$\Delta N_m = C \Delta I_a + D \Delta T_L \quad (4)$$

where the constants A_1, B_1, C and D are functions of the nominal values I_a, N_m, T_L, V_d and T_e.

The signal flow graph is shown in Figure (1).

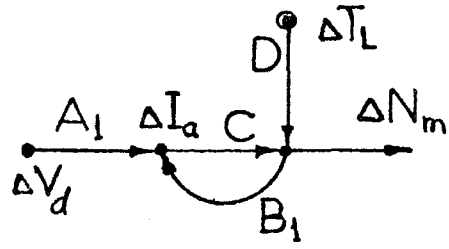
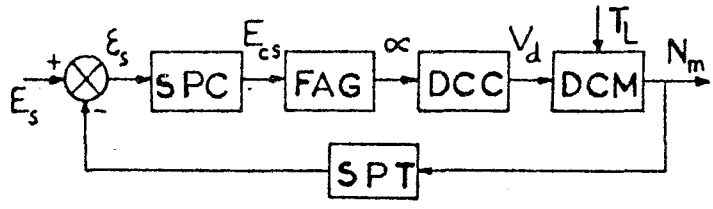


Figure (1): Motor signal flow graph.

2.2 Closed loop speed controller:

In most applications, the armature voltage of the motor is controlled in a closed loop feedback system. The schematic diagram of the motor speed controller associated with the converter-motor is shown in Figure (2).



SPC = Speed controller , DCC = Direct current converter,
FAG = Firing angle pulse generator, DCM = DC motor
SPT = Motor speed transducer.

Figure (2): Speed Controller diagram.

The speed controller is represented by the generalized transfer function G(s), which can be a proportional type or proportional-integral type. The firing angle pulse generator is represented by a cosine function related to the output voltage of the DC converter.

The incremental variations of the speed control circuit are defined by:

$$\Delta E_{cs} = F \Delta E_s + F_N \Delta N_m \quad (5)$$

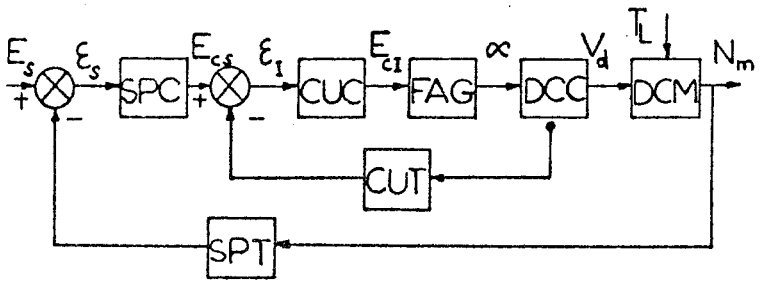
$$\Delta \alpha = T_1 \Delta E_{cs} \quad (6)$$

where, F and F_N are the transfer functions of the speed controller and speed transducer, respectively.

T_1 represents the pulse generator transfer function.

2.3 Closed loop current controller:

An inner current limiting control loop is incorporated as a protective feature, and the clamped speed error is used as a current reference. The schematic diagram of the overall system is shown in Figure (3).



CUC = Current controller , CUT = Current transducer

Figure (3): Control loop diagram.

The current controller is represented by its generalized transfer function $G_I(s)$, which may be simulated as a proportional type or proportional-integral type.

The incremental variations of the current control loop are defined by:

$$\Delta E_{cI} = F_1 \cdot \Delta E_{cs} + F_{1_I} \cdot \Delta I_a \quad (7)$$

$$\Delta \alpha = T_2 \cdot \Delta E_{cI} \quad (8)$$

where, F_1 and F_{1_I} are the transfer functions of the current controller and current transducer.

T_2 represents the pulse generator transfer function.

The signal flow graph of the system is shown in Figure (4).

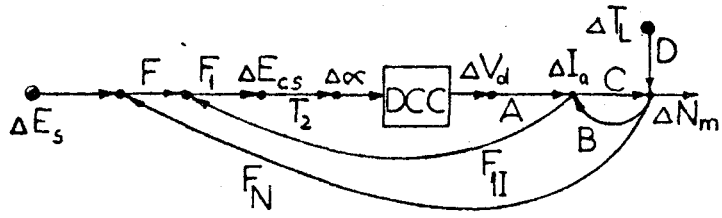


Figure (4): System signal flow graph.

2.4 AC-DC converter:

In this section, the signal flow graph which shows the relation between the incremental variations ΔV_d and $\Delta\alpha$, will be determined.
An attempt is made to analyse all possible modes of steady state operation for different types of AC-DC converters; this permits a general form definition of the average motor terminal voltage:

$$V_d = (/\overline{2}\ V\ /\ 2\pi/qh_w)\,[/\overline{h_\alpha q}\ \cos(\alpha+h_q\,\tfrac{\pi}{6})+h_\pi\ \cos\{\pi+h_f h_d\ (\beta+h_q\tfrac{\pi}{6})\}\ +\ h_d m\ \{(2\pi/qh_w)+(\alpha-\beta)\}] \quad (9)$$

and the general form of the equation which permits evaluation of the extinction angle β is given by:

$$\sin\{\pi+h_f(\beta-\pi)+h_q\tfrac{\pi}{6}-\phi\} \frac{(\beta+h_q.\pi/6-\pi)(1-h_f)}{\tan\phi} \frac{\alpha-\pi-h_f(\beta-\pi)}{\tan} = \frac{m}{\cos\phi}[e - e \frac{\alpha-\pi-h_f(\beta-\pi)}{\tan\phi} + \sin(\alpha+h_q\tfrac{\pi}{6}-\phi)\ e \quad (10)$$

where, q defines the type of the AC feeder (single or three phase).

h_w defines the type of the output DC voltage wavefeform (half wave or fullwave).

h_d defines the armature current continuity.

h_f defines the presence of the freewheeling diode across the motor trerminals.

h_α, h_π and h_q define some different modes of converter operation for the previous cases.

The following table summerizes the different types and modes of converter operation, and the values of the definition parameters are indicated.

	Single phase														Three phase							
	half wave				full wave										half wave					full wave		
	fully controlled FC				half controlled NF					fully,controlled EC					fully comtrolled.EC					HC		FC
	TF		WF		TF		WF			TF		WF			TF		WF			WF		TF
	D	D	DN	C	D	C	D	DN	C	D	C	D	DN	C	D	C	D	DN	C	D	C	C
q	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	3	3	3	3	3	3	3
h_w	1	1	1	1	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	2
h_d	1	1	1	0	1	0	1	1	0	1	0	1	1	0	1	0	1	1	0	1	0	0
h_f	1	0	1	0	0	0	0	1	0	1	1	0	1	0	1	1	0	1	0	0	0	1
h_α	1	1	1	1	1	1	1	1	1	1	4	1	1	1	1/3	1	1/3	1/3	1/3	1	1	1
h_π	1	-1	1	-1	-1	-1	-1	1	-1	1	0	-1	1	-1	1	0	-1	1	-1	-1	-1	0
h_q	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	0	0	0

HC half controlled , FC fully controlled
TF without freewheeling diode, WF with freewheeling diode
D discontinuous current C continuous current,
DN discontinuous and the freewheeling diod not conducting.

The general form of the incremental variations of the the variables describing the operation of the converter is:

$$\Delta V_d = W_1 \Delta\alpha + W_2 \Delta N_m + W_3 \Delta\beta \qquad (11)$$

$$\Delta \beta = W_4 \Delta\alpha + W_5 \Delta N_m \qquad (12)$$

where,

$$W_1 = \frac{h_d E_a - \sqrt{2}\sqrt{q h_\alpha} V \sin(\alpha + h_q \frac{\pi}{6})}{V_d} \frac{\alpha}{\sqrt{2\pi/q\, h_w}}$$

$$W_3 = h_d \frac{-h_\pi h_f/\overline{2V}\ \sin\{\pi+h_f h_d(\beta+h_q\frac{\pi}{6})\} - E_a}{V_d} \quad \frac{\beta}{2\pi/qh_w}$$

$$W_2 = h_d \frac{\{(2\pi/q\ h_w) + (\alpha-\beta)\}}{(2\pi/q\ h_w)} \frac{E_a}{V_d}$$

$$W_4 = h_d \frac{\cos\{\alpha + h_q \frac{\pi}{6} - \phi\}}{\cos\{\pi+h_f(\beta-\pi)+h_q\frac{\pi}{6} - \phi\}} \frac{\alpha}{\beta} e^{\frac{\alpha-\pi-h_f(\beta-\pi)}{\tan\ \phi}}$$

$$W_5 = h_d \frac{e^{(\beta+h_q\frac{\pi}{6}-\pi)(1-h_f)} - e^{\frac{\alpha-\pi-h_f(\beta-\pi)}{\tan\ \phi}}}{\cos\{\pi+h_f(\beta-\pi)+h_q\frac{\pi}{6} - \phi\}} \frac{E_a}{\beta/\overline{2}V\cos\phi}$$

The general form of the signal flow graph is shown in Figure (5):

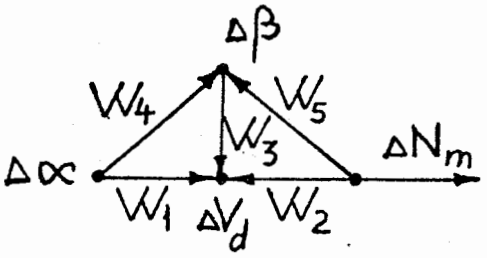


Figure (5): Signal flow graph of converter.

The values of W_1 : W_5 are determined, depending on the converter type and the nature of its mode of operation.

3. CONVERTER-MOTOR SYSTEM RESPONSES:

The signal flow graph representing the dynamic operation of the AC-DC converter and the separately excited DC motor load with the associated control loops is shown in Figure (6).

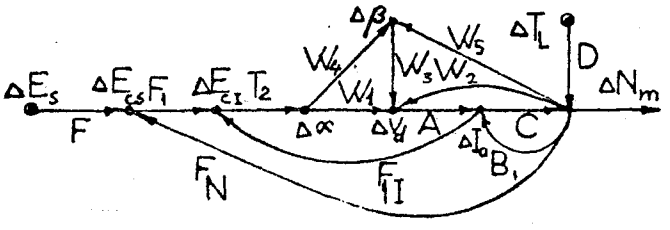


Figure (6): System signal flow graph.

The signal flow graph theory permits eliminating the intermediate nodes in order to obtain a direct relation between the output and input nodes [9]. Then, the transfer functions $\Delta N_m / \Delta E_s$, $\Delta N_m/\Delta T_L$, $\Delta I_a/\Delta E_s$ and $\Delta I_a/\Delta T_L$ are determined; and the time responses can be evaluated when either a step change in the speed reference or in the load torque.

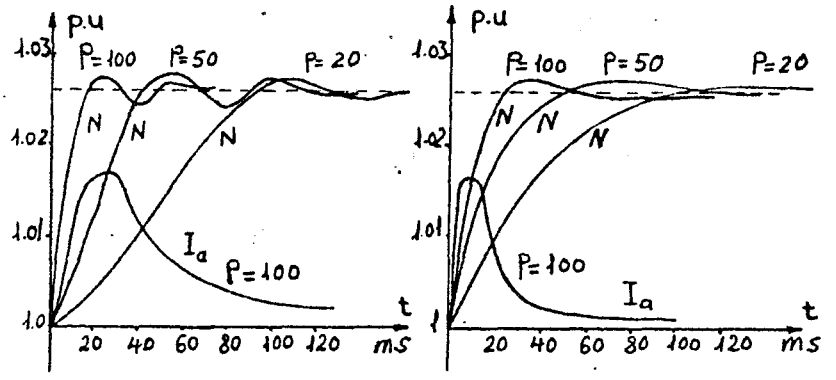
4. RESULTS:

The model responses for a step change in the speed reference of a single phase converter connected to a separately excited DC motor are obtained.

Figure (7) shows the speed and current responses for continuous and discontinuous current operation.

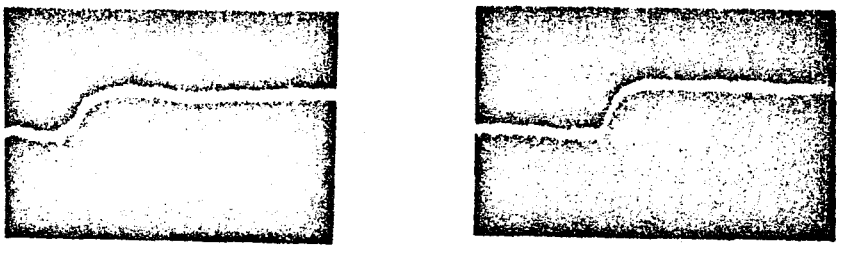
The experimental responses of current and speed have been recorded in the linear operating mode, by giving a small step change in the speed reference signal of the speed controller. Figure (8) shows photographs of the speed response for two different values of the controller gain.

Figure (9) shows the experimental time response of the armature current.
The experimental results verify the validity of the proposed mathematical model representing the system dynamic operation.



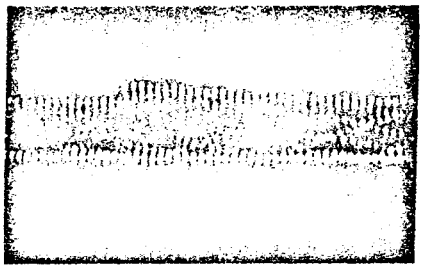
a- Continuous current. b- Discontinuous current.

Figure (7): Time responses of current and speed for step change in speed reference.



a- K = 0.0209 b- K = 0.1239

Figure (8): Experimental speed response.



K = 0.0209

Figure (9): Experimental current response.

5. CONCLUSION:

The generalized dynamic linear model given in this paper is adaptable for different types of converter with different modes of operation. Good correlation between computed and experimental responses confirms that the model is valid for predicting the behaviour of a given system.

6. LIST OF SYMPOLS:

i_a	Instantaneous armature current (A)
I_a	Average value of armature current (A)
ν_d	Instantaneous converter output voltage (V)
V_d	Average value of converter output voltage (V)
E_{cs}	Control voltage of the speed (V)
E_{cI}	Control voltage of the current (V)
E_s	Speed reference voltage (V)
n_m	Instantaneous motor speed (RPM)
N_m	Average value of motor speed. (RPM)
T_L	Load torque (M.m)
$K_t \phi$	Torque constant (N.m/A)
$K_e \phi$	Motor E.M.F constant (V/RPM)
R_a, L_m	Armature resistance and inductance
J	Inertia of the motor and coupled system K_{gm}^2
B	Viscuss friction coefficient N.m/red. Sec^{-1}
α	Thyristor firing angle
β	Thyristor extinction angle
Δ	Incremental variation
ϵ_s, ϵ_I	Speed and current error.

7. SYSTEM PARAMETERS:

The system parameters in this work had the following values:

P = 2.2 KW ; N = 1500 rpm.

$K_e\phi$= 0.518 (2 /60) ; $K_t\phi$ = 0.518

	a- continuous current		b- Discontinuous	
L_m	0.09	H	0.08	H
R_a	1.65	ohm	3.45	ohm
J	0.01985	Kg m^2	3.46	Kg.m^2
B	0.0055	Nm/rad.Sec^{-1}	0.029	Nm/rad.$Sec.^{-1}$
I_a	11.32	A	8.944	A
V_d	100	V	112.2	V
T_L	5	Nm	0.08	Nm
α	58°		128°	
β	238°		198°	

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9. APPENDIX:

The transfer function of the signal flow graph

a- DC motor:

$A_1 = (V_d/I_a\ R_a).(1/1+\tau_e s)$; $B_1 = (-E_a/I_a\ R_a).(1/1+\tau_e s)$

$C = (T_e/B\ N_m).(1/1+\tau_m s)$; $D = (-T_L/B\ N_m).(1/1+\tau_m s)$

$\tau_e = L_m / R_a$; $\tau_m = J/B$

b- Closed loop speed controller:

$$F = \frac{E_s}{E_{cs}} G_s(P) \quad ; \quad F_N = \frac{-A_2\ N}{E_{cs}} G_s(P)$$

$$T_1 = \frac{-E_{cs}}{E_o \sin\alpha} \left(\frac{1}{\alpha} \right)$$

c- Closed loop current controller:

$$F_1 = \frac{E_{cs}}{E_{cI}} G_I(P) \quad ; \quad F_{1I} = \frac{-A_3\ I_4}{E_{cI}} G_I(P)$$

$$T_2 = \frac{-E_{cI}}{E_o \sin\alpha} \left(\frac{1}{\alpha} \right)$$