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REGRESSION APPROACH TO CALCULATE THE EFFECTIVE MUD ANNULAR VISCOSITY DURING DRILLING OIL WELLS

صيغة رياضية-احصائية لحساب درجة اللزوجة المؤثرة لسائل الحفر فى الفراغ البينى أثناء حفر آبار البترول

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الخلاصة :

فى هذا البحث تم سياق صيغة رياضية-احصائية غير خطية و عديدة الإتجاهات و التى يمكن تطبيقها لحساب درجة اللزوجة لسائل الحفر المستخدم أثناء حفر آبار البترول . و لقد تم استخدام الطريقة الجبرية الخاصة (Rayleigh) للتحليل الإتجاهى فى سياق هذه الصيغة (Approach) . و هذه الصيغة تعكس تاثيرات الخواص الإنسيابية لسائل الحفر بشروط أى Plastic Viscosity و Yield Point و هيدروليكية سائل الحفر بشروط اى متوسط السرعة الإسمية لسائل الحفر و كذلك مقدار الفراغ البينى (الفراغ الذى ينحصر بين عمود وجدار البئر) على درجة اللزوجة المؤثرة لسائل الحفر فى الفراغ البينى.

و لقد تم تقييم هذه الصيغة (الموديل الرياضى) أولا بايجاد ثابت و أس المعادلة الرياضية و الذى اضيف بواسطة استخدام طريقة التحليل الإتجاهى. و لقد أستخدم فى هذا الطريقة الإحصائية الغير الخطية Least Squares لتثبيت المعلومات الحقلية التى قام باعدادها الباحث من ١٢١ فترة محفورة (Bit Runs) فى ستة ابار بترول حفرت فى منطقة حفر فى بحر الشمال و للحصول على data base المطلوبة لسياق ثابت و اس معادلة الصيغة التى تم الحصول عليها فى هذا البحث. و ايضا تم تقييم اخر لمعادلة الموديل او الصيغة الرياضية بواسطة التقييم البارامي‌ترى (Parametric) . و هذه الدراسة البارامي‌ترية قد بينت ان قيمة اللزوجة المؤثرة لسائل الحفر تزداد بزيادة Plastic Viscosity و Yield Point لسائل الحفر. وأيضا لقد وجد ان قيمة اللزوجة المؤثرة لسائل الحفر فى الفراغ البينى تزداد بزيادة الفراغ بين عمود الحفر و جدار البئر. و يمكن اضافة ان درجة اللزوجة المؤثرة لسائل الحفر فى الفراغ البينى تقل عندما يزداد متوسط السرعة الإسمية لسائل الحفر فى الفراغ البينى.

ايضا لقد تم تقييم معادلة الموديل الرياضى بواسطة استخدام معلومات حقلية حقيقية و كذلك مقارنتها بالموديلات الرياضية الموجودة مثل Bingham plastic model . و لهذا فان هذا الموديل يمكن تسميته Regression model مثل Bingham plastic model, Power-law model, and Polynomial model . وطبقا لهذا فان الموديل الرياضى فى هذا البحث يمكن اعتباره رابع موديل رياضى للتنبؤ او لحساب قيم درجة اللزوجة لسائل الحفر فى الفراغات البينية أثناء حفر آبار البترول. و لهذا فانه يوصى بان هذا الموديل او الصيغة الرياضية المساقة فى هذا البحث يمكن استخدامها فى صناعة حفر آبار البترول..

ABSTRACT

In this paper, a non-linear multidimensional, dimensionless mathematical drilling mud annular viscosity approach for oil well drilling applications, consisting of one equation, was developed.

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This utilizes the Rayleigh algebraic method for dimensional analysis. Also, this approach reflects the effects of mud rheological properties in terms of plastic viscosity and yield point; mud hydraulics in terms of the nominal average velocity of that mud; and geometry of the well-bore annulus on the effective mud viscosity in the annulus.

This approach was evaluated by first deriving the unknown proportionality constant and one exponent added by dimensional analysis technique. This utilizes the statistical regressional curve of the least-squares fitting method and using the prepared input field data by the author from 121 drilled intervals in six wells in the North Sea drilling region, so as to provide data base required to derive the approach's equation constant and its exponent. A further evaluation of the approach was involved parametrically. This parametric study shows that the effective mud annular viscosity increases when the yield point and plastic viscosity of that mud increase. Also, it was found that the mud annular viscosity increases as the annular space between the wall of hole and drill-pipe increases. In addition, the mud annular viscosity decreases when the nominal average velocity of drilling mud increases.

Also, the approach's equation was verified using actual field data and was compared with existing models such as Bingham plastic model. Thus, this approach may be called as "Regression Model" similar to Bingham plastic model, Power-law model, and Polynomial model. Accordingly, this approach is considered as the fourth model for predicting the values of viscosity of drilling fluid in the annuli of well-bores during drilling. Thus, it was concluded that the approach's predictions were more realistic and reasonable which may give a recommendation that the approach could be used by the oil-well drilling industry.

1. INTRODUCTION

The ratio of shear stress to shear rate is the viscosity. The shear stress-shear rate behavior of a Newtonian fluid can be determined by measuring the shear stress at one shear rate. Water for example will have a viscosity of 1.0 cps in the drill-pipe, through the bit nozzles, in the annulus and in the mud tanks. Conversely, the viscosity of a non-Newtonian fluid varies with shear rates. This means that it is impossible to define a viscosity of a non-Newtonian fluid without specifying a shear rate. Under pumping conditions, an example of a non-Newtonian drilling fluid might have the following viscosities: 20 cps inside the drill-pipe, 3 cps at the bit nozzles, 50 cps opposite the drill collars, 120 cps opposite the drill-pipe and 1200 cps in the mud tanks. That is the reason why the minimum mud rheological/hydraulics program must be flexible during drilling out of gauge sections, so as to avoid the hole cleaning problems. Thus, the rheological properties and/or pump rate can be raised without over stressing the well-bore and leaving up the hole around the drill collars. In other words, when drilling in areas where relatively old, dry sloughing shale is a problem.

Thus, the basic objectives of a rheology/hydraulics program should be to reduce annular pressure losses and ensure laminar flow by minimizing oversized drill collars and stabilizing equipment. Also, by the use of very non-Newtonian low-solids drilling fluids, minimum bit nozzles viscosities can be obtained with maximum effective mud annular viscosities for hole cleaning. Mud flow properties and/or pump rates can be adjusted to keep the adequate hole cleaning.

Accordingly, it is necessary to raise the viscosity of the drilling fluid by increasing the yield point or ratio of yield point to plastic viscosity which are considered the most significant rheological properties of the drilling fluid. that affect the cleaning-up of the bit cuttings and cavings from the hole during drilling. Field experience indicated that during very fast upper-hole drilling or soft formation drilling, raising mud viscosity, or annular mud viscosities considerably in excess of the maximum slip velocity of drilled-cuttings or cavings are necessary to prevent balling the bit and drill collars. These variations in drilling fluid properties and circulation rates of that fluid are required to ensure adequate hole cleaning during drilling.

Because this effective mud annular viscosity is dependent on the velocity of the mud and the pattern of flow, whether laminar or turbulent, this viscosity is difficult to measure. In oil-well drilling engineering only changes in the annular viscosity are of great concern to the drilling engineer, as such changes directly affect cuttings removal (hole cleaning) and annular pressure losses, which in turn affect the actual bottom-hole pressure of circulating drilling fluid. In practice, three models [1,2] are used to describe the flow of non-Newtonian fluids which form the most of drilling fluids, these models are:

1. Bingham plastic model: $\tau = A + BY$
2. Power-Law model: $\tau = mY^N$
3. Polynomial model: $\tau = A + BY + CY^3 + EY^5 + \dots$.

But in oil-well drilling industry, two models [3,4] are only used to describe the flow behavior of drilling fluids; the Bingham plastic model and Power-Law model.

The objective of this paper was to develop a field-oriented regression effective mud annular viscosity approach for oil-well drilling applications, in order to design optimum drilling operations with adequate hole cleaning. In turn, an increase in drilling rate is obtained at the lowest possible drilling coast. This study was planned due to the inaccuracies of both Bingham plastic model and Power-Law model predictions at the lower shear rate ranges. However, this regression approach or the proposed mud rheological model provides more consistently accurate descriptions of the rheology of non-Newtonian fluids (drilling fluids) used during drilling oil-wells. In other word, this regression approach may improve the predictions accuracy of effective mud annular viscosities more than the Bingham plastic model and Power-Law model predictions. This was due to the mathematical formulation of the regression approach, that was generated from the fitting actual field data by statistical regression analysis which gives more precise constants for this approach more the constants of both Bingham plastic and Power-Law models.

Therefore, this study was planned to investigate the most significant factors which affect the physical situation of effective mud annular viscosity predictions. The factors under study were generally classified into the following:

A. Rheological properties of drilling fluid in terms of:
   1. Plastic viscosity.
   2. Yield point.
B. Drilling fluid hydraulics in terms of the nominal average velocity of that fluid in the annulus.
C. Well-bore annulus geometry in terms of the annular space between the drill-pipe and the wall of the hole.

Then, applying the Rayleigh algebraic method of dimensional analysis to develop a mathematical formula which relates the effective mud annular viscosity parameter with the above-mentioned factors (independent parameters). To determine the unknown constant and one exponent added by dimensional analysis technique, this can be determined by the best fitting curve through the prepared field data by the author from 121 drilled intervals in six wells in the North Sea drilling region. The developed regression approach of effective mud annular viscosity was evaluated parametrically by applying the partial differentiation principles, so as to investigate the effect of changing one of the following : yield point, plastic viscosity, nominal average velocity of drilling fluid in the annulus and annular space on the effective viscosity of that fluid in the annulus during drilling.

Finally, to determine the validity and generality of the developed regression approach for application in oil-well drilling industry, the approach's equation was verified using actual field data and was compared with existing models such as Bingham plastic model.

2. MATHEMATICAL FORMULATION OF EFFECTIVE MUD ANNULAR VISCOSITY APPROACH

The effective annular viscosity of drilling fluid is defined as the viscosity of drilling fluid which surrounds the drilled-cuttings in the annulus without consideration of the shear thinning of drilling fluid, which is caused by the drill-pipe rotation during drilling. Perhaps this condition represents the drilling process with using " drilling downhole motors " where no rotation for drill-pipe during drilling operations. The study of the effect of drill-pipe rotation on mud annular viscosity may complicate the problem, therefore, this effect is neglected for saking the simplicity of the problem. It was found that [1-4] the effective viscosity of the drilling fluid in the annulus during drilling depends on the following parameters:

A. Physical or flow properties of drilling fluid in terms of rheological properties of that fluid , namely:
   1. Plastic viscosity, μ_p.
   2. Yield point, Y_P.

B. The hydraulic effect of drilling fluid in terms of the nominal average velocity of that fluid in the annulus, V_m.

C. Geometry of the well-bore annulus in terms of the annular space between the drill-pipe and the wall of the hole, $D_H - D_P$.

Thus, the physical relationship between effective annular viscosity of drilling fluid and the above-mentioned parameters can be expressed as follows:

$$\mu_e = f(\mu_P , Y_P , V_m , D_H - D_P) \qquad (1)$$

Mathematically, Equ. (1) can be equated to zero by including the dependent parameter, μ_e , as independent parameter.
Therefore,

$$f(\mu_e , \mu_P , Y_P , V_m , D_H - D_P) = 0 \qquad (2)$$

Applying the Rayleigh method of dimensional analysis [5,6] ,each parameter can be raised to an exponent. Therefore, Equ. (2) becomes:

$$f((\mu_e)^a, (\mu_P)^b, (Y_P)^c, (V_m)^d, (D_H - D_P)^e) = 0 \qquad (3)$$

Equ. (3) may be expressed in terms of its dimensions for each parameter in the M, L, and T system as follows:

$$f((ML^{-1}T^{-1})^a, (ML^{-1}T^{-1})^b, (ML^{-1}T^{-2})^c, (LT^{-1})^d, (L)^e) = M^0 L^0 T^0 \qquad (4)$$

In Equ. (4), the right hand side has zero exponents. equating the powers of M, L, and T. Therefore,

$$\sum_{1}^{3} M = 0, \quad \text{then, } a + b + c = 0 \qquad (5)$$

$$\sum_{1}^{5} L = 0, \quad \text{then, } -a - b - c + d + e = 0 \qquad (6)$$

$$\sum_{1}^{4} T = 0, \quad \text{then, } -a - b - 2c - d = 0 \qquad (7)$$

To derive the required expression, it may be noted that the variables appearing more than once in it. Hence, eliminate b and e. This gives the following:

$$b = -a - c \qquad (8)$$

$$e = a + b + c - d \qquad (9)$$

Substituting the value of b from Equ. (8) into Equ. (9), Equ. (9) gives:

$$e = -d \qquad (10)$$

Substituting the value of b and d in Equ. (4), Equ. (4) becomes:

$$f((ML^{-1}T^{-1})^a, (ML^{-1}T^{-1})^{-a-c}, (ML^{-1}T^{-2})^c, (LT^{-1})^d, (L)^{-d}) = M^0 L^0 T^0 \qquad (11)$$

Now, collecting the terms raised to remaining powers, thus:

$$\left[\left[\frac{\mu_e}{\mu_p}\right]^a \left[\frac{Y_p}{\mu_p}\right]^c \left[\frac{V_m}{D_H - D_p}\right]^d \right] = 0 \qquad (12)$$

Therefore, the appropriate expression obtained by the Rayleigh algebraic method for solving this problem is given as :

$$\phi\left[\frac{\mu_e}{\mu_p},\ \frac{Y_p}{\mu_p},\ \frac{D_H - D_p}{V_m}\right] = 0 \qquad (13)$$

Moving the independent parameter μ_e as original parameter (dependent parameter) with its dimensionless group to the right hand side and form remaining expression. Then, renaming the exponents and adding a proportionality constant to the renaming expression. Equ.(13) becomes :

$$\frac{\mu_e}{\mu_p} = c\left[\frac{Y_p}{\mu_p}\ \frac{D_H - D_p}{V_m}\right]^n \qquad (14)$$

where

$\left[\frac{Y_p}{\mu_p}\ \frac{D_H - D_p}{V_m}\right]$ is dimensionless group similar to the right hand side.

c is proportionality constant added by Rayleigh algebraic method of dimensional analysis technique for solving the problem in the final analysis.

n raised exponent by Rayleigh algebraic method of dimensional analysis technique to the dimensionless group.

Equ. (14) is the final form of mathematical formulation for the effective annular viscosity of drilling fluid during drilling oil wells.

3. LINEAR REGRESSION ANALYSIS FOR THE MATHEMATICAL FORMULA OF EFFECTIVE MUD ANNULAR VISCOSITY

Taking the logarithms of Equ.(14). Thus.

$$\mathrm{Ln}\left[\frac{\mu_e}{\mu_p}\right] = \mathrm{Ln}\,c + n\,\mathrm{Ln}\left[\frac{Y_p}{\mu_p}\ \frac{D_H - D_p}{V_m}\right] \qquad (15)$$

Now, Equ.(14) is an intrinsically linear model regression analysis when "Ln c" is the intercept of the fitted line by the least- squares method and "n" is the parameter estimate for the independent term included in that equation. Equ.(15) may be simplified as follows:

$$\mathrm{Ln}\,A_1 = \mathrm{Ln}\,c + n\,\mathrm{Ln}\,A_2 \qquad (16)$$

where:

$$A_1 = \frac{\mu_e}{\mu_p} \qquad\qquad A_2 = \frac{Y_p}{\mu_p}\ \frac{D_H - D_p}{V_m}$$

The input values of Ln A_1 and Ln A_2 have been computed from the prepared field data which were done by the author.

Regression analysis using the statistical Minitab computing system, was applied to regress these input values ($\mathrm{Ln}\,A_1$, $\mathrm{Ln}\,A_2$), using the following command [7,8]:
Regress Y in C1 on 1 Predictor in C2.
Where:

C1 = Column 1 contains the input values of $\mathrm{Ln}\,A_1$.
C2 = Column 2 contains the input values of $\mathrm{Ln}\,A_2$.

The regression output was obtained by the author for developing Equation (14). The values of intercept and regression coefficient were rounded off as follows:

$$\mathrm{Ln}\,c = 5.69200 \cong 5.69 \qquad n = 0.974567 \cong 0.975$$

Therefore, the regression equation is:

$$C_1 = 5.69 + 0.975\,C_2 \tag{17}$$

Where:

$\mathrm{Ln}\,c$ = Ln intercept of best fitted line.
Ln = natural logarithm.
n = estimated regression coefficient of C_1 (Y) upon C_2 (X).
Y = predicted value of $\mathrm{Ln}\,(\mu_e / \mu_p)$ given by a regression equation.
X = Ln value of designed dimensionless group of

$$\frac{Y_p}{\mu_p}\;\frac{D_H - D_p}{V_m}$$

Substituting the anti-Ln (anti-logarithm) value of the proportionality constant (295.9) and the value of "n" into Equation (14). Therefore,

$$\mu_e = 295.9\,\mu_p \left[\frac{Y_p}{\mu_p}\;\frac{D_H - D_p}{V_m} \right]^{0.975} \tag{18}$$

Equation (18) is the regression approach for predicting the effective annular viscosity of drilling fluid during drilling an oil well.

4. PARAMETRIC STUDY OF THE REGRESSION APPROACH OF EFFECTIVE ANNULAR MUD VISCOSITY DURING DRILLING OIL WELLS

The resulting solutions of partial differentiation of Equation (14) for the effective annular viscosity of drilling fluid may be expressed as follows:

$$\frac{\partial \mu_e}{\partial Y_p} = \mu_e \left(\frac{n}{Y_p} \right) \qquad (19)$$

$$\frac{\partial \mu_e}{\partial \mu_p} = \mu_e \left(\frac{1 - n}{\mu_p} \right) \qquad (20)$$

$$\frac{\partial \mu_e}{\partial (D_H - D_p)} = \mu_e \left(\frac{n}{D_H - D_p} \right) \qquad (21)$$

$$\frac{\partial \mu_e}{\partial V_m} = \mu_e \left(\frac{-n}{V_m} \right) \qquad (22)$$

The lowest and highest values of Y_P , μ_P , D_H - D_P and V_m parameters being studied, and the values of μ_e at minimum and maximum values of parameters under study were prepared by the author. Also, the value of $n = 0.975$. Then, the input program of CSMP [9] was run to simulate the parametric study of $\partial\mu_e/\partial Y_P$, $\partial\mu_e/\partial\mu_P$, $\partial\mu_e/\partial V_m$ and $\partial\mu_e/\partial(D_H-D_P)$. The numerical simulation processes from CSMP have been initialized at the minimum values of Y_P , μ_P , D_H-D_P and V_m parameters, and have been ended at the maximum values of these parameters. The simulation values of parametric study was plotted against the minimum to the maximum values of the above-mentioned parameters. These plots are shown in Figures 1, 2, 3, and 4.

5. DISCUSSION AND SIGNIFICANCE OF THE PARAMETRIC STUDY OF THE REGRESSION APPROACH FOR " μ_e "

The simulation results of parametric study of Equ. (18) of effective annular viscosity of drilling fluid, are shown in Figures 1, 2, 3, and 4.

Figure 1: is the plot of the partial derivative of the effective annular viscosity of drilling fluid with respect to the yield point of that fluid, versus the yield point. The slope of the associated curve is negative, which means that an increase in the effective annular viscosity of drilling fluid when the yield point of the drilling fluid increases during drilling. This result agrees with the Power-law model and Bingham plastic model results [3] and assists the mud engineer to control the yield point of the drilling fluid to obtain the required effective viscosity of that fluid in the annuli of wellbores during drilling.

Figure 2: gives the plot of the partial derivative of the effective annular viscosity of drilling fluid during drilling oil wells with respect to the plastic viscosity of that fluid, versus the plastic viscosity. The associated curve in this plot has a negative slope, which means that an increase in the effective annular viscosity of drilling fluid will occur as the plastic viscosity of that fluid increases. Also, this result confirms the Bingham plastic model results [3]. The physical significance of plots 1 and 2 indicates that the yield point of drilling fluid increases the effective annular viscosity of that fluid more than the plastic viscosity of that fluid. Hence, it is

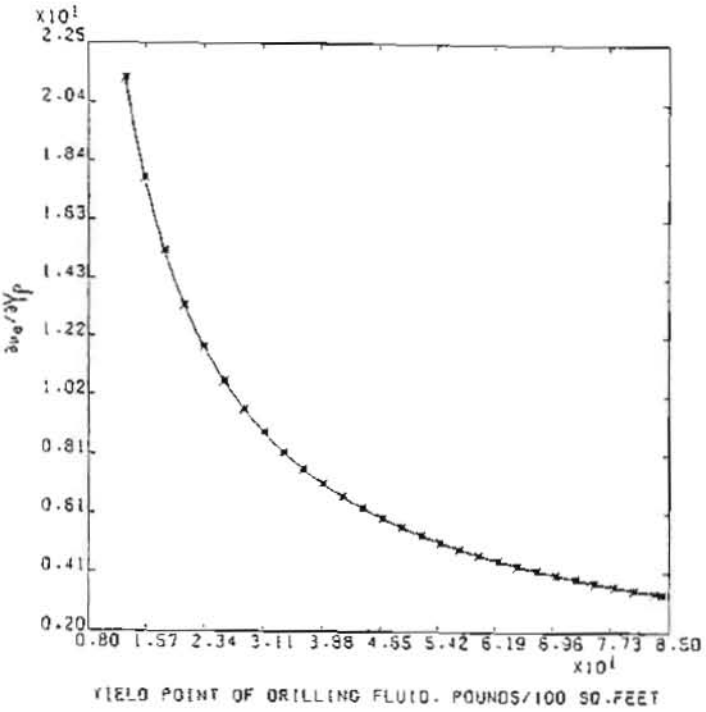

Fig 1: $\partial\mu_e/\partial Y_p$ Versus. Yield Point of Drilling Fluid.

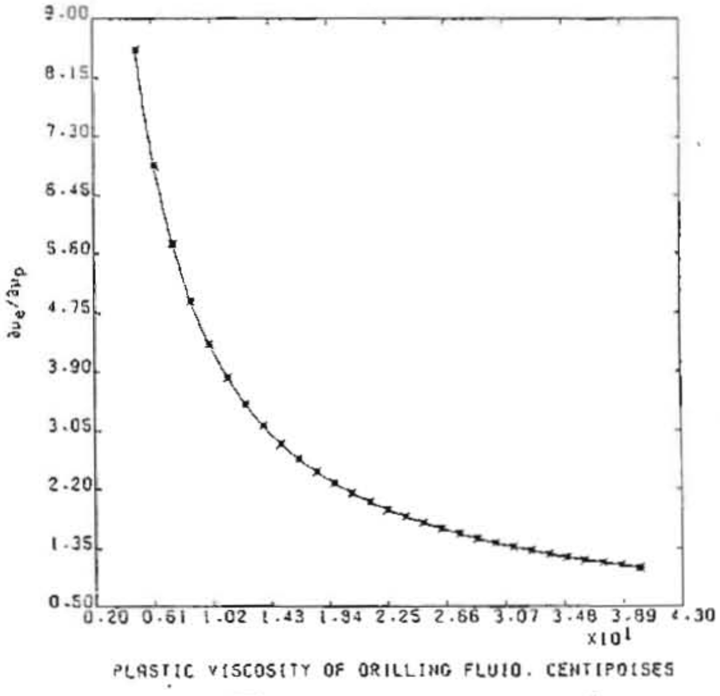

Fig. 2: $\partial\mu_e/\partial\mu_p$ Versus. Plastic Viscosity of Drilling Fluid.

recommended to use a mud with inverted rheology where the yield point is higher than the plastic viscosity.

Figure 3: represents the plot of the partial derivative of the effective annular viscosity of drilling fluid during drilling oil wells with respect to the annular space between the wall of the hole and the drill-pipe, versus the annular space. This plot shows an increase in the effective annular viscosity of drilling fluid with an increase in the annular space between the wall of the hole and the drill-pipe during drilling. As the annular space between the wall of the hole and the drill-pipe increases, the annular velocity of drilling fluid decreases at constant circulating flow rate of that fluid. This moves the flow regime in the annulus to laminar or viscous flow, and the curve shows a continuous increase. This result agrees with the Power-law model and Bingham plastic model results [3].

Figure 4: is the plot of the partial derivative of the effective annular viscosity of drilling fluid with respect to the nominal average velocity of that fluid in the annulus, versus the nominal average velocity. The resultant slope of the associated curve in this plot is positive, which means that a reduction of the effective annular viscosity of drilling fluid as the nominal average velocity of that fluid in the annulus increases during drilling. Theoretically, the increase in the nominal average velocity of drilling fluid moves the flow regime in the annulus to the turbulent flow, and this is why the curve shows a continuous decrease. However, this result confirms the Power-law model and Bingham plastic model results [3].

From field experience and practices in the North Sea, the Gulf of Mexico, and the Gulf of Suez. The previous recommendations as to mud type may be developed. A mud with inverted rheology characteristics may be chosen as would a mud such as an "inverted-oil-emulsion" for directional and horizontal drilling. Therefore, inverted-oil-emulsion mud is one possible answer to field-development drilling for central platforms due to the following reasons, namely:

(a) Reducing the excessive pipe rotating torque and drag.
(b) It has good carrying capacity, i.e. an adequate hole cleaning.
(c) Drills a gauge hole.
(d) Most cost effective in terms of the mud cost and drilling days [10].
(e) Prevention of differential-pressure sticking whilst drilling potentially under-pressured permeable pay zone sections. In turn, the permeability damage of pay zone section during drilling can be avoided.

Accordingly, it seems that the inverted-oil-emulsion mud is very useful during drilling pay zone sections or deep drilling operations of directional wells or horizontal drilling section in thin pay zones.

6. VERIFICATION OF REGRESSION APPROACH FOR " μ_e "

The developed regression approach for predicting the effective annular viscosity of drilling fluids given by Equ. (18), where V_m is given by the following equation:

$$V_m = \frac{Q_m}{A_s} \qquad (23)$$

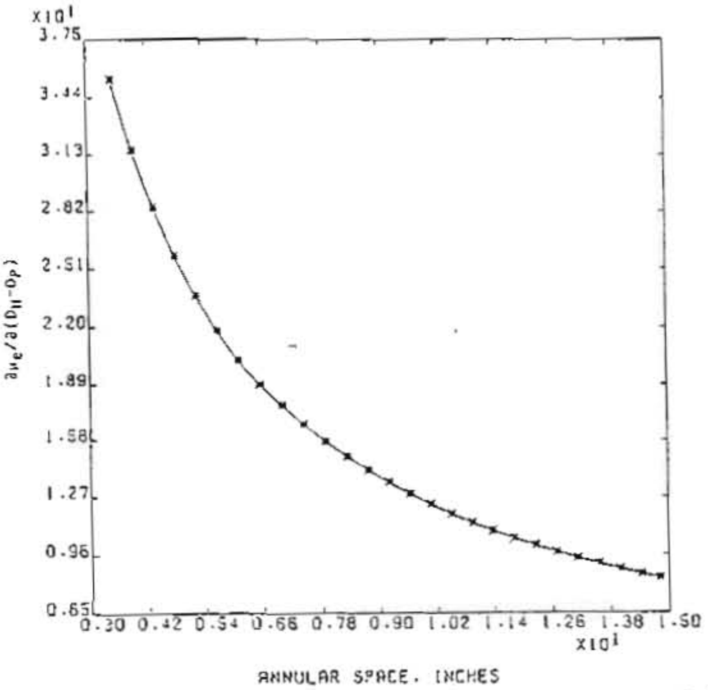

Fig. 3: $\partial\mu_e/\partial(D_H\text{-}D_P)$ Versus Annular Space.

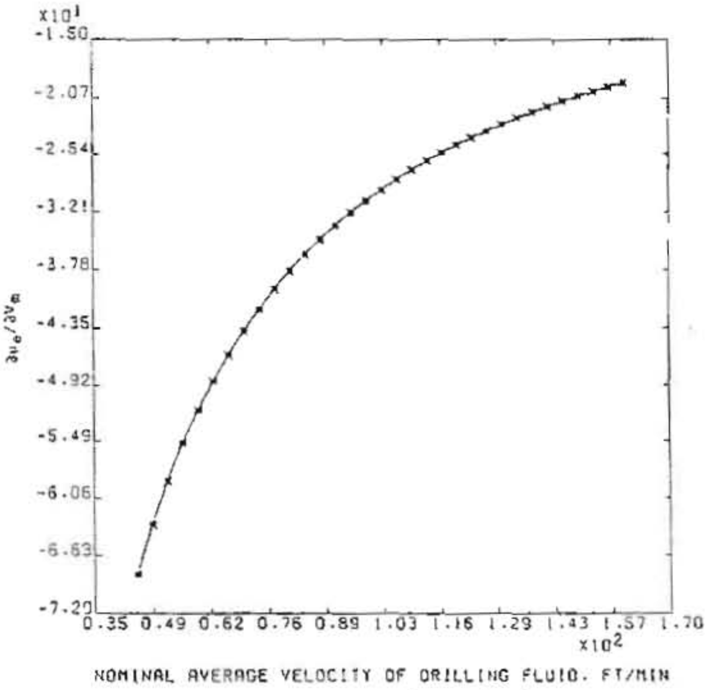

Fig. 4: $\partial\mu_e/\partial Vm$ Versus Nominal Average Velocity of Drilling Fluid.

Thus, Equ. (18) was verified using field data from the North Sea drilling region [10] that shown in Table (1). Also, an additional verification was performed for Equ. (18) utilizing field data from field study 2-wells A, B and C in the North Sea [11] which are given in Table (2). The predictions of Equ. (18) using field data from Tables 1 & 2 are given in Tables 3 & 4, respectively.

Table 1 : The field data from Well X-North Sea Area [10] for the verification of Regression Approach " μ_e ", Case I

No.	Variable Set	Well X					
	Abbreviation	Sec. 17.5"					Sec. 12.25"
1	MD, RKBft	695	1512	1654	3414	6261	11759
2	D_H-D_p, in.	12.5	12.5	12.5	12.5	12.5	7.25
3	Q_m, gpm	954	958	958	958	958	599
4	μ_p, cps	8	9	14	14	8	26
5	Y_p, lb/100 ft^2	40	16	20	49	40	24
6	V_m, ft/min	83.2	83.5	83.5	83.5	83.5	117.5

Table 2 . The field data from Wells A,B,C in North Sea Area [11] for the verification of Regression Approach " μ_e ", Case 2

No.	Well	μ_p, cps	Y_p, lb/100 ft^2	Q_m, gpm	D_H, inches	D_p, inches
1	A	15	13	900	17.5	5
2		34	18	900	17.5	5
3		18	13	600	12.25	5
4		28	18	600	12.25	5
5		18	13	400	8.5	5
6		29	19	400	8.5	5
1	B	16	18	1100	17.5	5
2		30	30	1100	17.5	5
3		19	18	650	12.25	5
4		30	34	650	12.25	5
5		30	26	485	8.5	5
6		35	34	485	8.5	5
1	C	20	20	450	17.5	5
2		30	33	550	17.5	5
3		30	25	550	12.25	5
4		45	30	550	12.25	5

Table 3 : Comparison between Predictions of Regression Approach of Effective Annular Viscosity of Drilling Fluid and Predictions of Bingham Plastic Model of Effective Annular Viscosity of that Fluid, Case 1.

No.	μ_e (pr), cps	μ_e (B), cps	Accuracy, %	Deviation Errors = 100-accuracy = %
	Predictions of Regression Approach of Effective Mud Annular Viscosity	Predictions of Bingham Model of Effective Mud Annular Viscosity[3]	$\frac{\mu_e (pr)}{\mu_e (B)}$ x100	
1	1735	1811	95.8	+ 4.2
2	691	727	95	+ 5.0
3	864	912	94.7	+ 5.3
4	2116	2214	95.6	+ 4.4
5	1727	1804	95.8	+ 4.2
6	560	597	93.8	+ 6.2

Table 4 : Comparison between Predictions of Regression Approach of Effective Annular Viscosity of Drilling Fluid and Predictions of Bingham Plastic Model of Effective Annular Viscosity of that Fluid, Case 2.

Well	No.	μ_e (pr), cps	μ_e (B), cps	Accuracy, %	Deviation Errors = 100-accuracy = %
		Predictions of Regression Approach of Effective Mud Annular Viscosity	Predictions of Bingham Model of Effective Mud Annular Viscosity[3]	$\frac{\mu_e (pr)}{\mu_e (B)}$ x100	
A	1	598	636	94.0	+ 6.00
	2	827	894	92.5	+ 7.50
	3	231	258	89.5	+ 10.5
	4	320	361	88.7	. 7
	5	633	838	75.5	+ 24.5
	6	924	1251	73.9	+ 26.1
B	1	677	720	94.0	+ 6.00
	2	1128	1203	93.8	+ 6.20
	3	269	326	90.6	+ 9.40
	4	558	610	91.5	+ 8.50
	5	104	139	75.4	+ 24.6
	6	136	177	77.2	+ 22.8
C	1	1839	1931	95.2	+ 4.800
	2	2482	2610	95.1	+ 4.900
	3	485	534	90.8	+ 9.20
	4	582	650	89.5	+ 10.5

In the field, the Bingham plastic model is used to predict values for the effective annular viscosity of drilling fluid, this model is given as follows:

$$\mu_e = \mu_p + 300 \left[\frac{Y_p (D_H - D_p)}{V_m} \right] \qquad (24)$$

where:

$$V_m = \frac{Q_m}{\frac{\pi}{4} (D_H^2 - D_p^2)}$$

Details of Equ. (24) is given in reference [3]. Applying the Bingham plastic model that is Equ. (24) and using field data from Tables 1 & 2, the predictions of Bingham's model for the effective annular viscosity of the drilling fluid were obtained as shown in Tables 3 & 4. Table 3 shows the accuracy of prediction, using Equ. (18), the effective annular viscosity of the drilling fluid with respect to the Bingham's model predictions model lie within the range of 93.8 % to 95.8% with deviation error of (+4.2%) to (+6.2%). The average accuracy of the predictions of the equation for the effective annular viscosity of drilling fluid in the North Sea drilling region is 95% with a deviation error of (+5.0%).

Also, from Table (4), The accuracy of the predictions with Equ. (18) for the effective annular viscosity of drilling fluid using field data from Table (2) ranges from 73.9% to 95.2% with deviation errors from (+4.8%) to (26.1%). However, the average accuracy of the predictions is 82% of a deviation error of (+18.0%) based on V_m the estimated nominal average velocity of the drilling fluid. There are two average predicted accuracies using Equ. (18) for the effective annular viscosity of the drilling fluid at two different conditions, namely, using V_m values from equation estimates values of V_m and using estimated values of V_m obtained from mud pump flow rate. The two accuracies are 95.0% as shown in Table (3) and 82.0% as shown in Table (4). Finally, Equ. (18) or regression approach is called as the "regression model" i.e. similar to Bingham plastic model, Polynomial model, and Power-law model to predict the values for the effective annular viscosity of drilling fluid.

7. APPLICATION OF REGRESSION APPROACH OF " μ_e " IN OIL-WELL DRILLING INDUSTRY

The most important application of this approach in oil-well drilling industry is to evaluate the lifting capacity of drilling fluid, i.e. drilled-cuttings transport or annular hole cleaning. This effective annular viscosity of drilling fluid which can be calculated by the regression approach is related directly to the rate of drilled-cuttings will slip through the drilling fluid in the annulus of well-bore.

$$V_s = \frac{4973\, d_c^2 (\rho_c - \rho_m)}{\mu_e} \qquad (25)$$

Also, the net upward rise velocity of drilled-cuttings, V_c ,can be calculated as follows:

$$V_c = V_m - V_s \qquad (26)$$

Slip velocity of drilled-cuttings can be analyzed into the axial and radial components during drilling the deviated or inclined well-bore section from the kick-off point (KOP) to the end of pay zone in directional wells. these components are:

Axial component of slip velocity = $V_{sa} = V_s \cos\theta$

Radial component of slip velocity = $V_{sr} = V_s \sin\theta$

Thus,

$$V_{c\ min} = V_{m\ min} - V_{sa} \qquad (27)$$

Where $V_{c\ min}$ is the minimum net upward rise or transport velocity of drilled-cuttings in the eccentric annulus of directional well during drilling. The net upward rise or transport velocity of drilled-cuttings may affect the concentration of annular drilled-cuttings.

8. CONCLUSIONS

The following are the conclusions which are drawn from this research:

1. The physical situation of predicting the effective viscosity of drilling fluid in the annulus of well-bore during drilling, is a complicated engineering problem but can be described by simple equation linking the dependent and independent parameters which are related and affect the real system.
2. Regression, non-linear, multi-dimensional, dimensionless, effective annular mud viscosity approach for oil well drilling applications was developed.
3. This approach was evaluated parametrically to measure the effect on the approach of changing one of the independent parameters with respect to the dependent parameter, that is referred to as sensitivity analysis or parametric study. From this parametric study, it was found that the effective annular viscosity of drilling fluid increases as the following parameters increase:
   i. The yield point of the drilling fluid.
   ii. The plastic viscosity of the drilling fluid.
   iii. The annular space between the drill-pipe and the wall of the hole.

   Also, the effective annular viscosity of drilling fluid decreases as the designed nominal average velocity of the drilling fluid increases.
4. The regression approach is a practical technique to accurately predict the effective viscosity of drilling fluid in the annulus of well-bore during rotary drilling process. These predicted values of the effective mud viscosity are useful for the following parameters that affect the hole cleaning, i.e. drilled-cuttings transport process, that are:
   i. Absolute slip velocity of drilled-cuttings for vertical drilling process of oil wells.
   ii. Axial and radial components of slip velocity of drilled-cuttings for directional and horizontal drilling processes of oil wells, taking into consideration that the axial component of slip velocity in horizontal drilling process is equal to zero.
   iii. The net upward rise or transport velocity of drilled-cuttings affects the value of annular concentration drilled-cuttings.

5. The regression approach was verified and compared with existing models such as Bingham plastic model. The approach's predictions for the effective annular viscosity of drilling fluid were close with the results obtained from the Bingham plastic model in most cases. Thus, it is considered this regression approach as the fourth model similar to Power-law model, Bingham plastic model and Polynomial model. Accordingly, it was found that the regression approach's predictions were more realistic and reasonable which show a significant virtue that could be used by the oil well drilling industry.

9. RECOMMENDATIONS

The following are the recommendations for the future work:

1. It is recommended that the developed regression approach should be tested practically in actual rotary drilling operations to determine the precision of its predictions under operational drilling conditions.
2. It is recommended that the developed regression approach's predictions should be compared with the results obtained from the Power-law model and the Polynomial model to determine its predictions under operational drilling conditions.
3. This approach can be used for the design of adequate hole cleaning program during rotary drilling process of oil wells, so as to obtain the higher drilling rate at the lowest drilling cost. In other words, this developed regression approach offers real scope for applying drilling optimization technique. In turn, optimum drilling operations design could be obtained for adequate hole cleaning, in order to increase the drilling rate at the lowest possible drilling cost. Therefore, there are several aspects which could be considered, to develop the present approach to take part for drilling optimization applications, these operating conditions or aspects including:
    - i. Mud properties in terms of rheological properties which are:
        1. Plastic viscosity.
        2. Yield point or ratio of yield point to plastic viscosity.
    - ii. Mud hydraulics in terms of nominal average velocity of drilling fluid in the annulus which is supported by the designed mud flow rate.
4. It is recommended that a mud with "inverted rheology" characteristics (higher ratio of yield point to plastic viscosity of that mud) such as inverted-oil-emulsion mud should be used for drilling pay zones in vertical oil wells, and it is also used during drilling directional and horizontal oil wells. Thus, this approach could be used to plan the mud program for inverted rheology characteristics.

10. NOMENCLATURE

A_a = cross sectional area, in^2.
A, B, C, E, m = fluid model constants.
c = unknown constant added by dimensional analysis technique.
Cos = cosine Sin = sine
D_H = hole diameter, inches.
D_P = outside diameter of drill-pipe, inches.
Equ. = Equation.
d_c = equivalent drilled-cutting size or diameter, inches.
f = mathematical function.

M, L, T = dimensions of mass, length and time, respectively.

Ln = natural logarithm.

N = Power-law model exponent.

n = unknown exponent added by dimensional analysis technique.

Y = shear rate, sec^{-1}.

Y_p = yield point of drilling fluid, $lbs/100\ ft^2$.

θ = hole deviation angle, degrees.

μ_e = effective mud annular viscosity, cps.

μ_p = plastic viscosity of drilling fluid, cps.

ρ_c = drilled-cuttings density, ppg.

ρ_m = mud or drilling fluid density, ppg.

ϕ = mathematical function.

V_m = nominal average velocity of drilling fluid in the annulus, ft/min.

V_s = slip velocity of drilled-cuttings in the annulus, ft/min.

V_{sa} = axial component of slip velocity of drilled-cuttings in deviated well, ft/min.

V_{sr} = radial component of slip velocity of drilled-cuttings in deviated well, ft/min.

V_c = net upward rise or transport velocity of drilled-cuttings, ft/min.

$V_{c\ min}$ = minimum net upward rise or transport velocity of drilled-cuttings, ft/min.

$V_{m\ min}$ = minimum velocity of drilling fluid in eccentric annulus of deviated well, ft/min.

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