Menofia University Faculty of Engineering Shebien El-kom First Semester Examination Academic Year : 2017-2018	Minoufia University	Department : Prod. Eng. Year : 1st Subject : Eng. Mathematics Time Allowed : 3 hours Date : 31 / 12/ 2017

Allowed Tables and Charts : None — الامتحان في صفحتين

Answer all the following questions: [100 Marks]

Question 1 [50 Marks]

(A) Find the general solution of the differential equations [10 Marks]

(i) $\dfrac{dy}{dx} = \dfrac{4x+6y+5}{2x+3y+4}$ (ii) $(2xy - \sec^2 x)dx + (x^2 + 2y)dy = 0$

(B) Find the solution of the initial value problem

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} = 1, \quad x = 0, \quad \frac{dx}{dt} = 0 \quad when \ t = 0$$ [5 Marks]

(C) *i)* Solve the differential equation $(x^2D^2 - xD + 2)y = x\ln x$ [5 Marks]

ii) Calculate the volume of the body bounded by the surfaces:

$$z = 4 - x, \qquad x + y = 2, \qquad x = y = z = 0$$ [5 Marks]

(D) *(i)* Solve the following system of simultaneous ordinary differential equations. $\dfrac{d^2x}{dt^2} = y$ and $\dfrac{d^2y}{dt^2} = x$ [5 Marks]

(ii) Solve the following ODEs [10 Marks]

1. $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = e^x$ 2. $(D^2 + 9)y = \cos 2x + \sin 2x$

(E) Solve the differential equation $\dfrac{d^2x}{dt^2} - 4\dfrac{dx}{dt} + 4x = te^t$ **using Laplace transform** with initial conditions: $x(0) = 0$ and $x'(0) = 0$. [10 Marks]

Question 2 [50 Marks]

(A) *i)* Find the interval of convergence of the series $S_n = \sum_{n=1}^{\infty} \frac{(2x)^n}{n}$. [5 Marks]

ii) Calculate the double integral $\iint_D f(x, y)dxdy$ for $f(x, y) = 3 - x^2 - y^3$

and D is bounded by $0 \le x \le 1, \quad 0 \le y \le x$. [5 Marks]

(B) Find the inverse Laplace transform of the functions

(i) $F(s) = \ln \frac{s+1}{s-1}$ *(ii)* $F(s) = \frac{1}{(s)(s-2)(s-4)(s-6)}$ [10 Marks]

(C) Find Laplace transform of the following functions

(i) $f(t) = \frac{1 - \cos t}{t}$ *(ii)* $f(t) = \begin{cases} 0 & t < \frac{2\pi}{3} \\ \cos(t - \frac{2\pi}{3}) & t > \frac{2\pi}{3} \end{cases}$ [10 Marks]

(D) Test the convergence of the following series:

(i) $S_n = \sum_{n=1}^{\infty} \frac{2n-1}{2^n}$ *(ii)* $S_n = \sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$ [5 Marks]

(E) A periodic function $f(x)$ with period 2π is defined as follows:

$$f(x) = x \qquad -\pi \le x \le \pi$$

i) Plot the function.

ii) Find the corresponding Fourier series. [10 Marks]

(F) Evaluate the area bounded by the straight lines $y = x, \quad y=0,$

$x = 1$ and $x = 4$. [5 Marks]

With our best wishes

Dr. Mohammady Bassiouni and

Associate Prof. Dr. Islam M. Eldesoky