



Answer the following questions

First Calculus

Question 1(25 marks)

(a). (6 marks) Find y' of the following functions

i) $y = e^{\tan^{-1} 2x} - \frac{\sin x}{\sinh 2x} + x^3 \ln(1 - 2x)$

ii) $y = \operatorname{sech}^{-1} 2t$ and $x = \sec^{-1} 2t$

(b). (10 marks) i) if $y = \tanh^{-1} x$ prove that $y' = \frac{1}{1-x^2}$.

ii) Find y' using two different methods for the implicit function:

$$y^2 \sinh 2x - x^2 \cosh 2y - 3e^{xy} - 4y^2 + 2x^3 - 7 = 0$$

(c) (9 marks) find the following limits:

i) $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 2x + 2}{x^2 + 1} \right)^{\frac{x^2}{x+1}}$

ii) $\lim_{x \rightarrow 0} (\cot x)^{\frac{1}{\ln x}}$

iii) $\lim_{x \rightarrow 0} (x^3 \ln x)$

Question 2(25 marks)

(a). (10 marks) i) if $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ prove that $(1-x^2)y' = xy + 1$

ii) if $\tan y = \sinh x$ prove that

(1) $x = \ln(\sec y + \tan y)$

(2) $\frac{dx}{dy} = \sec y$ and $\frac{dy}{dx} = \operatorname{sech} x$

(b). (10 marks) i) if $y = e^{ax} \cos(bx + c)$ prove that

$$y_n = (a^2 + b^2)^{n/2} e^{ax} \cos(bx + c + n \tan^{-1} \frac{b}{a})$$

ii) if $y = e^{\sin^{-1} x}$ prove that

$$(1-x^2)y_{n+2} - x(2n+1)y_{n+1} - (n^2+1)y_n = 0$$

(c) (5 marks) if $z = e^{\frac{y}{x}}$, $x = \tanh^2 t$, and $y = \tan^2 t$ find $\frac{dz}{dt}$

Second Algebra

Question 3(35 marks)

(a) (7 marks) Resolve into partial fraction

$$\frac{4x^3 - 28x^2 - 4x - 20}{(x^2 - 2x - 3)(x^2 + 2)}$$

(b) (8 marks) Using Binomial Theorem find the summation of the following series باستخدام نظرية ذات الحدين اوجد مجموع المتسلسلة الآتية

$$\frac{5}{8} + \frac{5.8}{8.12} + \frac{5.8.11}{8.12.16} + \frac{5.8.11.14}{8.12.16.20} + \dots$$

(c) (10 marks) ينتج احد المصانع نوعين من المنتجات ويستخدم في انتاج كل نوع ثلاث آلات كالتالي:

الحد الاقصى للالات	المنتج الثاني	المنتج الاول	الالات
70	1	2	الالة الاولى
40	1	1	الالة الثانية
90	3	1	الالة الثالثة
	70\$	40\$	الربح (Profit)

اوجد حجم الانتاج من كلا النوعين لتحقيق اكبر ربح ممكن .

(d) (10 marks) Find the quotient and remainder of

$$f(x) = x^3 - 12x^2 + 44x - 48 \div (x + 2) \text{ and then write } f(x) \text{ in terms of } (x - 3)$$

Question 4(15 marks)

(a) (5 marks) Verify Cayley Hamilton theorem for matrix A , then find the

inverse of matrix A. where
$$A = \begin{bmatrix} 5 & -2 \\ 7 & 1 \end{bmatrix}$$

(b) (5 marks) For the linear system

$$x + y - z = 1$$

$$x + k y + 3z = 2$$

$$2x + 3y + k z = 3$$

Find the value of k such that the system has

- Unique solution
- Infinite number of solution
- No solution

(c) (5 marks) If the linear system in Q4-b is homogeneous then discuss the solution

With our best wishes

This exam measures the following ILOs

Question Number	Q1-a	Q2-a	Q3-a	Q4-a	Q1-b	Q2-b	Q3-b	Q4-b	Q1-c	Q2-b	Q3-c	Q3-d	Q4-c
Skills	a2-2	a4-1	a5-1	a5-2	b2-1	b5-2	b5-1	b4-2	c4-2	c5-1	c7-2	c5-2	c4-1
	Knowledge & Understanding Skills				Intellectual Skills				Professional Skills				