MINIMUM COST DESIGN OF IRRIGATION CANAL SECTIONS

تصميم قطاعات قنوات الرى بأقل تكلفة إنشاء

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خلاصة:
يتطلب تصميم قطاع القناة ذو التكلفة المنخفضة تخفيض تكاليف الإنشاء للمتر الطولى من القناة. والهدف من هذا البحث هو الوصول إلى التصميم الأمثل لتحقيق تخفيض قيمة دالة الهدف اللاخطية.هذا ويمكن التعبير عن دالة الهدف بدلالة تكاليف الحفر والتبطين للمتر الطولى من القناة. وقد استخدمت معادلةماننج كمعادلة للقيد حيث يجب تحقيقها مبدئيا. تم استخدام طريقة لاجرانج للمضروب للحصول على معادلات التصميم لكلا من قنوات الرى ذات الأشكال الشبه منحرفة والمستطيلة والمثلثة. ويمكن الحصول على الأبعاد المثلى لهذه القنوات باستخدام مجموعة من المنحنيات أخذا فى الاعتبار المسافة بين سطح الأرض ومنسوب المياه. هذه المنحنيات تم رسمها باستخدام النتائج المحسوبة من برنامج حاسب آلى تم إعداده بلغة "فورتران" لحل معادلات التصميم. كما رسمت مجموعة من المنحنيات لتصميم قنوات الرى منخفضة التكلفة بأخذ المسافة بين سطح الأرض ومنسوب المياه فى الاعتبار. وتفيد منحنيات التصميم لقنوات الرى منخفضة التكاليف فى اختيار الأبعاد المثلى لقنوات الرى التى تحقق اقل تكلفة إنشاء. وقد تم عرض مثال تصميمى مع تحليل لحساسية الحل لتوضيح سهولة وإمكانية الاستخدام العملى لهذه الطريقة.

ABSTRACT

Design of minimum cost canal section requires minimization of construction costs per unit length of the canal. The aim is to minimize a nonlinear objective function subject to a nonlinear equality constraint. The objective function has been expressed as the cost per unit length of the canal for excavation and lining. Manning's equation was used as an equality constraint. Using the method of Lagrange multipliers, the necessary equations for the design of minimum cost irrigation canal of trapezoidal, rectangular, triangular shapes can be obtained. The optimal dimensions for the previous canal sections without freeboard could be obtained from a set of design charts. These charts were plotted based on the results obtained using a Fortran computer programs to solve the minimum cost design equations. Also, the effect of freeboard has been taken into consideration and a set of charts was plotted for the design of minimum cost irrigation canals which provided with freeboard. The minimum cost design charts are useful in selecting the optimal canal dimensions guarantying the minimum cost of construction. A design example with sensitivity analysis was presented to indicate the simplicity and practicability of the proposed method.

INTRODUCTION

Irrigation canals in an irrigation system are like veins in a body. They are used to convey, distribute, and apply water to the land. A canal in the irrigation system may be a rigid boundary canal (lined canal) or a mobile boundary canal (unlined canal). In most cases, the purpose of canal lining is to prevent erosion and seepage losses. Also, the smooth surface of lining reduces the friction forces, which enables the canal to be laid on a small slope ensuring a high level at the point of delivery.

The factors to be considered in the design of uniform flow in rigid boundary canals are the kind of nonerodible materials forming the canal surface, the maximum permissible velocity that will not cause erosion of the canal surface, the minimum permissible velocity, to avoid sedimentation, the longitudinal bottom slope of the canal which generally governed by the topography and the purpose of the canal, the freeboard of the canal to prevent waves or fluctuations in water surface from overflowing the sides, and the efficiency of canal section, which indicates how much the section is hydraulically and/or economically efficient [1,2,3].

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The selection of canal dimensions which provide either the greatest hydraulic efficiency or the least construction cost are the objectives of canal geometry optimization. These two objectives usually produce different canals dimensions. The best hydraulic section has the maximum flow rate and minimum perimeter for a given area but not necessarily the most economical canal section. Several investigators have combined canal cost and practicability requirements into an objective function with certain constraints to obtain the optimal canal section [4,5,6,7,9,10,11].

Mainly, the optimum economy of an irrigation project is achieved by minimizing the cost of canals construction. The design of minimum construction cost irrigation canals includes the minimization of the sum of the excavation and lining costs which vary with canal depth subject to uniform flow condition in the canal. This problem represents the minimization of a nonlinear objective function subject to a nonlinear equality constraint, which is difficult to be solved analytically. Using the method of Lagrange multipliers [3,5,7,8], the necessary equations for the design of minimum cost irrigation canal of trapezoidal, rectangular, triangular shapes can be obtained for both the canals with freeboard and the canals without freeboard.

The purpose of this paper is to provide a simple mathematical technique to achieve minimum cost design charts to facilitate the selection of the optimal canal dimensions of trapezoidal, rectangular, and triangular shapes with or without freeboard, guarantying the minimum cost of construction. A design example with sensitivity analysis was presented to demonstrate the simplicity and practicability of the present technique.

COST FUNCTION

The objective function consists of the construction cost per unit length of the canal for excavation and lining. Considering the excavation cost for the flow section, the excavation cost can be written as [5,11]:

$$C_e = c_e A + c_i A\bar{y} \tag{1}$$

in which: c_e = cost per unit volume of excavation at ground level; c_i = increase in the unit excavation cost per unit depth; A = flow area; and $\bar{y}$ = depth of centroid of area from the free water surface.

Considering the cost of canal lining per unit surface area c_l , the cost of lining C_l can be written as:

$$C_l = c_l P \tag{2}$$

where P = wetted perimeter of cross section.

Taking into account the depth of freeboard F, figure (1), equations (1) and (2) may be rewritten as:

$$C_{ef} = c_e A_t + c_i A_t \bar{y}_t \tag{3}$$

$$C_{lf} = c_l P_t \tag{4}$$

where A_t = total cross section area; P_t = total perimeter of cross section and $\bar{y}_t$ = depth of centroid of area from the ground level.

The total cost of construction per unit length C can be obtained by adding the excavation cost and the lining cost as:

$$C = C_e + C_l = c_e A + c_i A\bar{y} + c_l P = c_e A + c_i E + c_l P \tag{5}$$

in which

$$E = A\bar{y} \tag{6}$$

Considering the depth of freeboard, The total cost of construction per unit length C_f can be obtained by adding the excavation cost and the lining cost as:

$$C_f = C_{ef} + C_{lf} = c_c A_t + c_l A_t \bar{y}_t + c_l P_t = c_c A_t + c_l E_t + c_l P_t \tag{7}$$

$$E_t = A_t \bar{y}_t \tag{8}$$

EQUALITY CONSTRAINT FUNCTION

The capacity of a uniformly flowing open channel can be determined, in SI units, by Manning's equation:

$$Q = \frac{1}{n} A R^{2/3} S^{1/2} \tag{9}$$

in which Q = flow rate; n = Manning's roughness coefficient; R = hydraulic radius = A/P; S = channel bed slope. The design flow rate Q, the Manning's roughness coefficient n of the construetion material, and the bed slope of the channel based on topography data are generally given. The design parameters that the engineer must determine are A and R, which are functions of the geometric elements of the cross section and the normal depth y. Since a canal is designed to sustain uniform flow, the equality constraint function can be written as:

$$\phi = A R^{2/3} - \frac{Qn}{S^{1/2}} = \frac{A^{5/3}}{P^{2/3}} - \frac{Qn}{S^{1/2}} = 0 \tag{10}$$

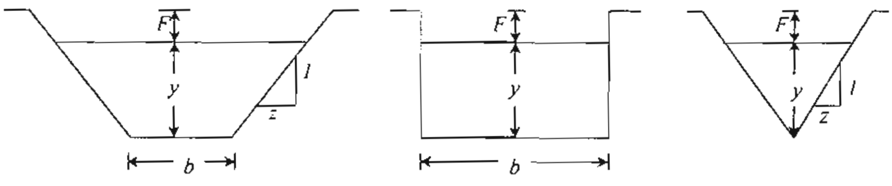

Fig.1. Canal Sections: *(a)* Trapezoidal Section; *(b)* Rectangular Section; and *(c)* Triangular Section

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For canals of trapezoidal section C and ϕ are functions of water depth y, bed width b, and side slope z. In the case of rectangular section canals, C and ϕ are functions of water depth y, and bed width b, while in the case of triangular section canals C and ϕ are functions of water depth y, and side slope z.

Trapezoidal Section Case

By applying the Lagrange method of undetermined multipliers for trapezoidal canal with freeboard, figure (1-a), the minimum value of the function $C_f(y, b, z)$ can be evaluated from the following four equations:

$$\frac{\partial C_f}{\partial y} + \lambda \frac{\partial \phi}{\partial y} = 0 \tag{11}$$

$$\frac{\partial C_f}{\partial b} + \lambda \frac{\partial \phi}{\partial b} = 0 \tag{12}$$

$$\frac{\partial C_f}{\partial z} + \lambda \frac{\partial \phi}{\partial z} = 0 \tag{13}$$

and $\phi(y, b, z) = 0$ (14)

Elimination of λ between equations (11), (12), and (13) results in:

$$\frac{\partial C_f}{\partial y}\frac{\partial \phi}{\partial b} - \frac{\partial C_f}{\partial b}\frac{\partial \phi}{\partial y} = 0 \tag{15}$$

$$\frac{\partial C_f}{\partial y}\frac{\partial \phi}{\partial z} - \frac{\partial C_f}{\partial z}\frac{\partial \phi}{\partial y} = 0 \tag{16}$$

in which

$$\frac{\partial C_f}{\partial y} = c_e\frac{\partial A_t}{\partial y} + c_t\frac{\partial E_t}{\partial y} + c_l\frac{\partial P_t}{\partial y} \tag{17}$$

$$\frac{\partial C_f}{\partial b} = c_e\frac{\partial A_t}{\partial b} + c_t\frac{\partial E_t}{\partial b} + c_l\frac{\partial P_t}{\partial b} \tag{18}$$

$$\frac{\partial C_f}{\partial z} = c_e\frac{\partial A_t}{\partial z} + c_t\frac{\partial E_t}{\partial z} + c_l\frac{\partial P_t}{\partial z} \tag{19}$$

$$\frac{\partial \phi}{\partial y} = \frac{5}{3}\frac{A^{2/3}}{P^{2/3}}\frac{\partial A}{\partial y} - \frac{2}{3}\frac{A^{5/3}}{P^{5/3}}\frac{\partial P}{\partial y} \tag{20}$$

$$\frac{\partial \phi}{\partial b} = \frac{5}{3}\frac{A^{2/3}}{P^{2/3}}\frac{\partial A}{\partial b} - \frac{2}{3}\frac{A^{5/3}}{P^{5/3}}\frac{\partial P}{\partial b} \tag{21}$$

$$\frac{\partial \phi}{\partial z} = \frac{5}{3}\frac{A^{2/3}}{P^{2/3}}\frac{\partial A}{\partial z} - \frac{2}{3}\frac{A^{5/3}}{P^{5/3}}\frac{\partial P}{\partial z} \tag{22}$$

Substituting equations (17), (18), (20), and (21) into equation (15) yields:

$$\left(\frac{\partial A_t}{\partial y} + k_e\frac{\partial E_t}{\partial y} + k_l\frac{\partial P_t}{\partial y}\right)\left(\frac{5}{3}\frac{A^{2/3}}{P^{2/3}}\frac{\partial A}{\partial b} - \frac{2}{3}\frac{A^{5/3}}{P^{5/3}}\frac{\partial P}{\partial b}\right) -$$
$$\left(\frac{\partial A_t}{\partial b} + k_e\frac{\partial E_t}{\partial b} + k_l\frac{\partial P_t}{\partial b}\right)\left(\frac{5}{3}\frac{A^{2/3}}{P^{2/3}}\frac{\partial A}{\partial y} - \frac{2}{3}\frac{A^{5/3}}{P^{5/3}}\frac{\partial P}{\partial y}\right) = 0 \tag{23}$$

in which $k_e = c_t / c_e$, and $k_l = c_l / c_e$.

Substituting equations (17), (19), (20), and (22) into equation (16) yields:

$$\left(\frac{\partial A_t}{\partial y} + k_e\frac{\partial E_t}{\partial y} + k_l\frac{\partial P_t}{\partial y}\right)\left(\frac{5}{3}\frac{A^{2/3}}{P^{2/3}}\frac{\partial A}{\partial z} - \frac{2}{3}\frac{A^{5/3}}{P^{5/3}}\frac{\partial P}{\partial z}\right) -$$
$$\left(\frac{\partial A_t}{\partial z} + k_e\frac{\partial E_t}{\partial z} + k_l\frac{\partial P_t}{\partial z}\right)\left(\frac{5}{3}\frac{A^{2/3}}{P^{2/3}}\frac{\partial A}{\partial y} - \frac{2}{3}\frac{A^{5/3}}{P^{5/3}}\frac{\partial P}{\partial y}\right) = 0 \tag{24}$$

where: $A_t = b(y+F) + z(y+F)^2$, $P_t = b + 2(y+F)\sqrt{1+z^2}$,

$$E_t = \frac{b(y+F)^2}{2} + \frac{z(y+F)^3}{3}, \quad \frac{\partial A_t}{\partial y} = b + 2z(y+F), \quad \frac{\partial A_t}{\partial b} = y + F,$$

$$\frac{\partial A_t}{\partial z} = (y+F)^2, \quad \frac{\partial P_t}{\partial y} = 2\sqrt{1+z^2}, \quad \frac{\partial P_t}{\partial b} = 1, \quad \frac{\partial P_t}{\partial z} = \frac{2z(y+F)}{\sqrt{1+z^2}},$$

$$\frac{\partial E_t}{\partial y} = b(y+F) + z(y+F)^2, \quad \frac{\partial E_t}{\partial b} = \frac{(y+F)^2}{2}, \text{ and } \frac{\partial E_t}{\partial z} = \frac{(y+F)^3}{3} \tag{25}$$

The optimal values of y, b, z can be obtained by solving equations (14), (15), and (16) simultaneously. Practically, the side slope z of the canal section is determined based on other factors than the economic design. These factors involve the angle of repose of soil, the building material, and the method of construction [2,7,9]. In this case, the side slope z is considered constant, $\frac{\partial C_f}{\partial z} = \frac{\partial \phi}{\partial z} = 0$, and the optimal values of y, b can be obtained by solving equations (14), and (15) simultaneously.

When $F = 0$, the optimal values of y, b, z can be obtained for trapezoidal canal without considering freeboard by solving equation (14) simultaneously with the following two equations:

$$\frac{\partial C}{\partial y}\frac{\partial \phi}{\partial b} - \frac{\partial C}{\partial b}\frac{\partial \phi}{\partial y} = 0 \qquad (26)$$

$$\frac{\partial C}{\partial y}\frac{\partial \phi}{\partial z} - \frac{\partial C}{\partial z}\frac{\partial \phi}{\partial y} = 0 \qquad (27)$$

in which

$$\frac{\partial C}{\partial y} = c_e \frac{\partial A}{\partial y} + c_t \frac{\partial E}{\partial y} + c_l \frac{\partial P}{\partial y} \qquad (28)$$

$$\frac{\partial C}{\partial b} = c_e \frac{\partial A}{\partial b} + c_t \frac{\partial E}{\partial b} + c_l \frac{\partial P}{\partial b} \qquad (29)$$

$$\frac{\partial C}{\partial z} = c_e \frac{\partial A}{\partial z} + c_t \frac{\partial E}{\partial z} + c_l \frac{\partial P}{\partial z} \qquad (30)$$

Substituting equations (20), (21), (26), and (27) into equation (24) yields:

$$\left(\frac{\partial A}{\partial y} + k_e \frac{\partial E}{\partial y} + k_l \frac{\partial P}{\partial y}\right)\left(\frac{5}{3}\frac{A^{2/3}}{P^{2/3}}\frac{\partial A}{\partial b} - \frac{2}{3}\frac{A^{5/3}}{P^{5/3}}\frac{\partial P}{\partial b}\right) -$$

$$\left(\frac{\partial A}{\partial b} + k_e \frac{\partial E}{\partial b} + k_l \frac{\partial P}{\partial b}\right)\left(\frac{5}{3}\frac{A^{2/3}}{P^{2/3}}\frac{\partial A}{\partial y} - \frac{2}{3}\frac{A^{5/3}}{P^{5/3}}\frac{\partial P}{\partial y}\right) = 0 \qquad (31)$$

Substituting equations (20), (22), (26), and (28) into equation (25) yields:

$$\left(\frac{\partial A}{\partial y} + k_e \frac{\partial E}{\partial y} + k_l \frac{\partial P}{\partial y}\right)\left(\frac{5}{3}\frac{A^{2/3}}{P^{2/3}}\frac{\partial A}{\partial z} - \frac{2}{3}\frac{A^{5/3}}{P^{5/3}}\frac{\partial P}{\partial z}\right) -$$

$$\left(\frac{\partial A}{\partial z} + k_e \frac{\partial E}{\partial z} + k_l \frac{\partial P}{\partial z}\right)\left(\frac{5}{3}\frac{A^{2/3}}{P^{2/3}}\frac{\partial A}{\partial y} - \frac{2}{3}\frac{A^{5/3}}{P^{5/3}}\frac{\partial P}{\partial y}\right) = 0 \qquad (32)$$

where: $A = by + zy^2$, $P = b + 2y\sqrt{1+z^2}$, $E = \frac{by^2}{2} + \frac{zy^3}{3}$, $\frac{\partial A}{\partial y} = b + 2zy$, $\frac{\partial A}{\partial b} = y$,

$\frac{\partial A}{\partial z} = y^2$, $\frac{\partial P}{\partial y} = 2\sqrt{1+z^2}$, $\frac{\partial P}{\partial b} = 1$, $\frac{\partial P}{\partial z} = \frac{2zy}{\sqrt{1+z^2}}$, $\frac{\partial E}{\partial y} = by + zy^2$,

$\frac{\partial E}{\partial b} = \frac{y^2}{2}$, and $\frac{\partial E}{\partial z} = \frac{y^3}{3}$ (33)

Rectangular Section Case

When $z = 0$, the solution of equations (14), and (23) simultaneously gives the optimal values of y, and b for rectangular canal including freeboard considerations. Referring to Fig. (1-b), equations (25) can be simplified as follows:

$A_t = b(y+F)$, $P_t = b + 2(y+F)$, $E_t = \frac{b(y+F)^2}{2}$, $\frac{\partial A_t}{\partial y} = b$,

$\frac{\partial A_t}{\partial b} = y + F$, $\frac{\partial P_t}{\partial y} = 2$, $\frac{\partial P_t}{\partial b} = 1$, $\frac{\partial E_t}{\partial y} = b(y+F)$, and $\frac{\partial E_t}{\partial b} = \frac{(y+F)^2}{2}$ (34)

For rectangular canal section without considering freeboard, $z = 0$ and $F = 0$, the optimal values of y. and b can be obtained by solving equations (14), and (31) simultaneously.

Triangular Section Case

When $b = 0$, the solution of equations (14), and (24) simultaneously gives the optimal values of y, and z for triangular canal including freeboard considerations. Referring to Fig. (1-c), equations (25) can be simplified as follows:

$$A_t = z(y+F)^2,\ P_t = 2(y+F)\sqrt{1+z^2},\ E_t = \frac{z(y+F)^3}{3},\ \frac{\partial A_t}{\partial y} = 2z(y+F),$$

$$\frac{\partial A_t}{\partial z} = (y+F)^2,\ \frac{\partial P_t}{\partial y} = 2\sqrt{1+z^2},\ \frac{\partial P_t}{\partial z} = \frac{2z(y+F)}{\sqrt{1+z^2}},\ \frac{\partial E_t}{\partial y} = z(y+F)^2,$$

$$\text{and } \frac{\partial E_t}{\partial z} = \frac{(y+F)^3}{3} \qquad (35)$$

- For triangular canal section without considering freeboard, $b = 0$ and $F = 0$, the optimal values of y, and z can be obtained by solving equations (14), and (32) simultaneously.

GRAPHICAL SOLUTION

A graphical solution of the minimum cost design equations for a trapezoidal canal section with a specified value of side slopes z, allows a quick and illustrative determination of minimum cost dimensions. Based on Newton Raphson method, Fortran computer programs was prepared to solve these equations. The values of side slopes z equal to *1*, *1.5*, *2*, $1/\sqrt{3}$, and *0* (rectangular section) was used separately in the computations. Considering freeboard, equations (14) and (23) were solved separately and a separate graph was drawn for each value of the side slope z, as shown from figure (2) to figure (6). Also, equations (14) and (31) were solved separately and a separate graph for canal without freeboard was drawn for each value of the side slope z, as shown from figure (7) to figure (11). For a triangular canal section with freeboard, equations (14) and (24) was solved separately and the computed results was drawn in figure (12). equations (14) and (32) were solved separately for a triangular canal section without freeboard and the computed results was drawn in figure (13).

There are two sets of curves in all the illustrated figures. Each curved line in the first set of curves represent one value of the section factor ($AR^{2/3}$) which is in turn equal to $Qn/S^{1/2}$. The second set of curves for different values of k_e and k_l , represents the solution of equation (23) in case of trapezoidal channel and rectangular channel with freeboard, as shown from figure (2) to figure (6). From figure (7) to figure (11), the second set of curves represents the solution of equation (31) in case of trapezoidal channel and rectangular channel without freeboard. In figure (12) the second set of curves for different values of k_e and k_l represents the solution of equation (24) for triangular channel with freeboard, while the second set of curves in figure (13) represents the solution of equation (32) for triangular channel without freeboard.

For known values of Q, n, S, z, k_e and k_l, the section factor ($Qn/S^{1/2}$) is computed. In the case of both trapezoidal and rectangular channel, the minimum cost design water depth and bed width of the channel is determined directly at the point where the section factor line crosses the k_e and k_l cost ratio line. The minimum cost design water depth and side slope of

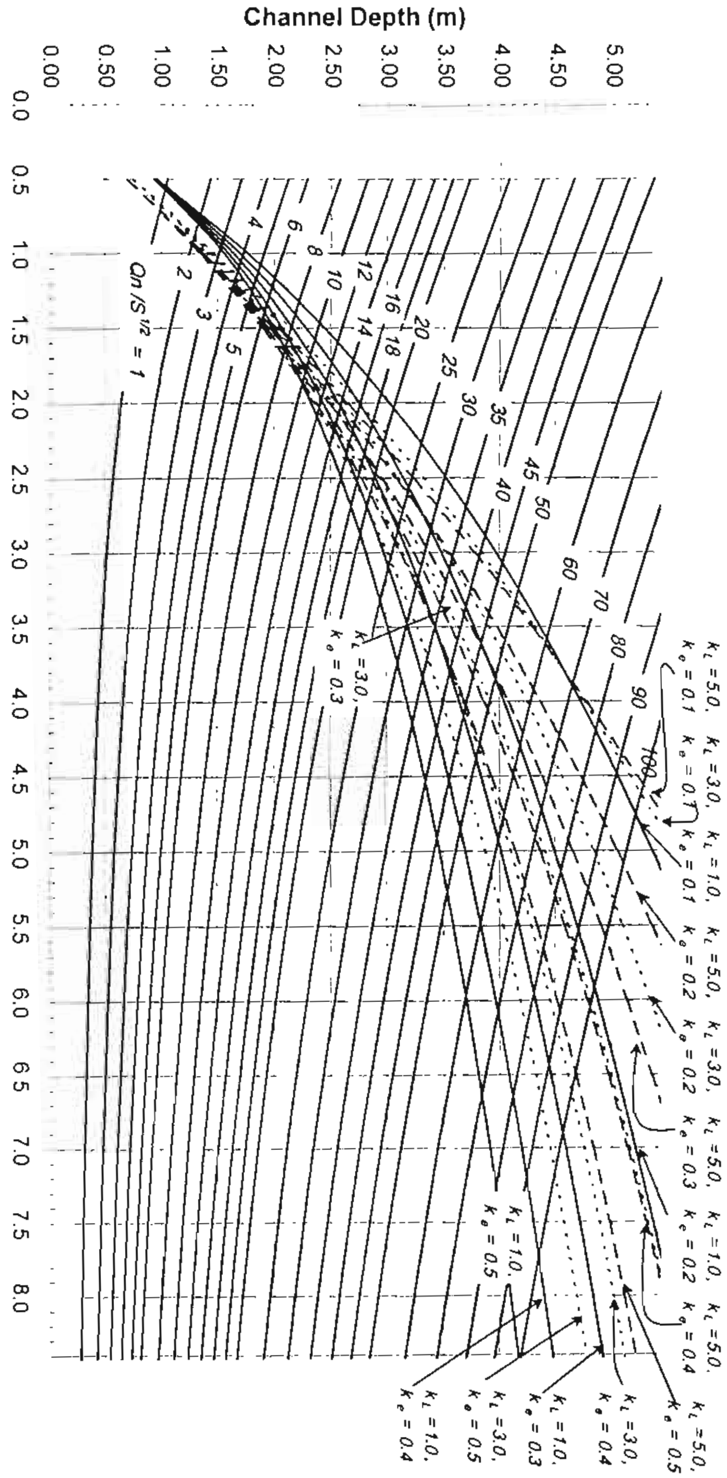

Fig. 2. Minimum Cost Design Curves for Lined Trapezoidal Channel with Freeboard = *1 m* and Side Slopes *z* = *1.0*.

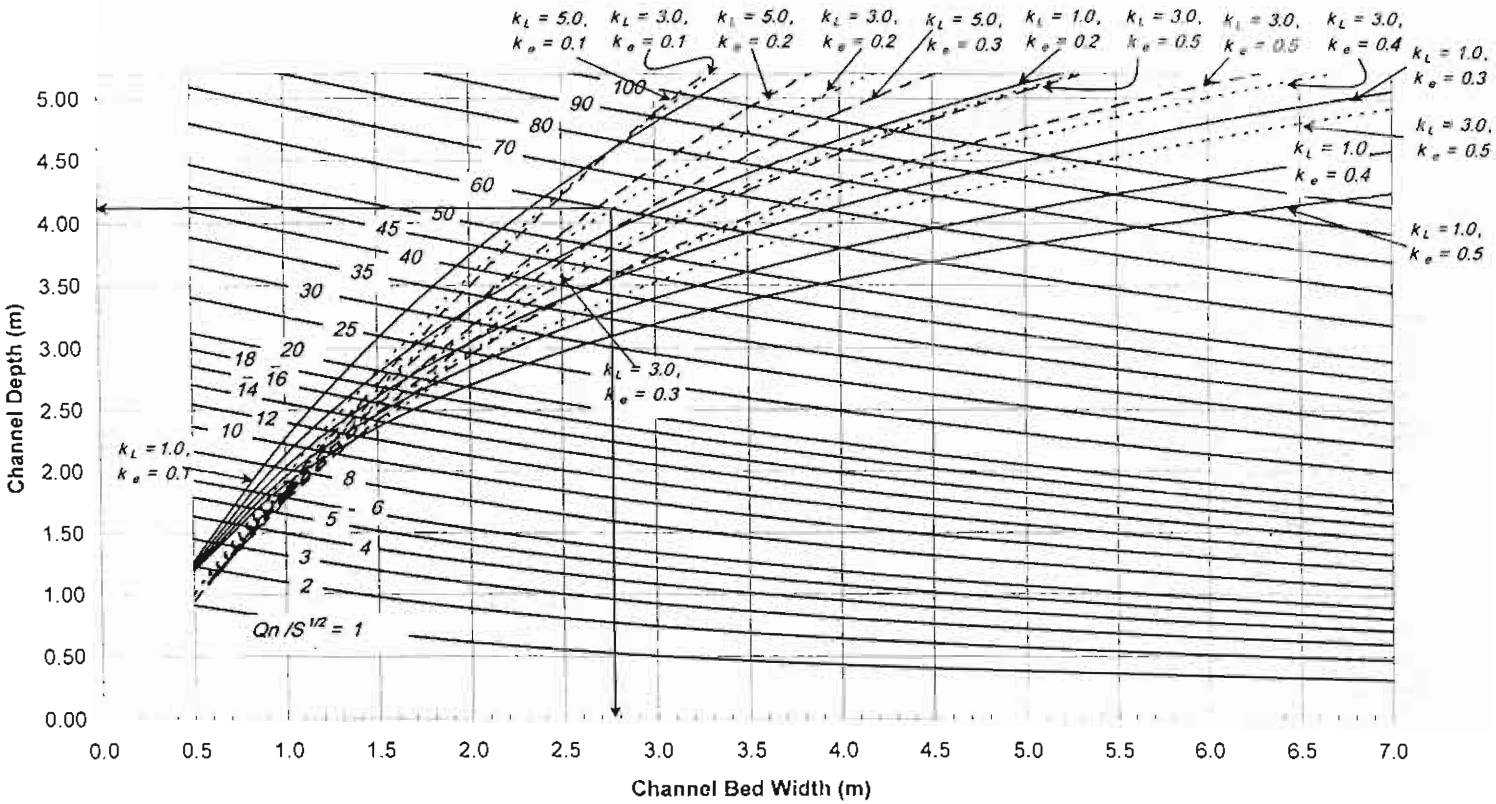

Fig.3. Minimum Cost Design Curves for Lined Trapezoidal Channel with Freeboard = *1 m* and Side Slopes *z = 1.5*.

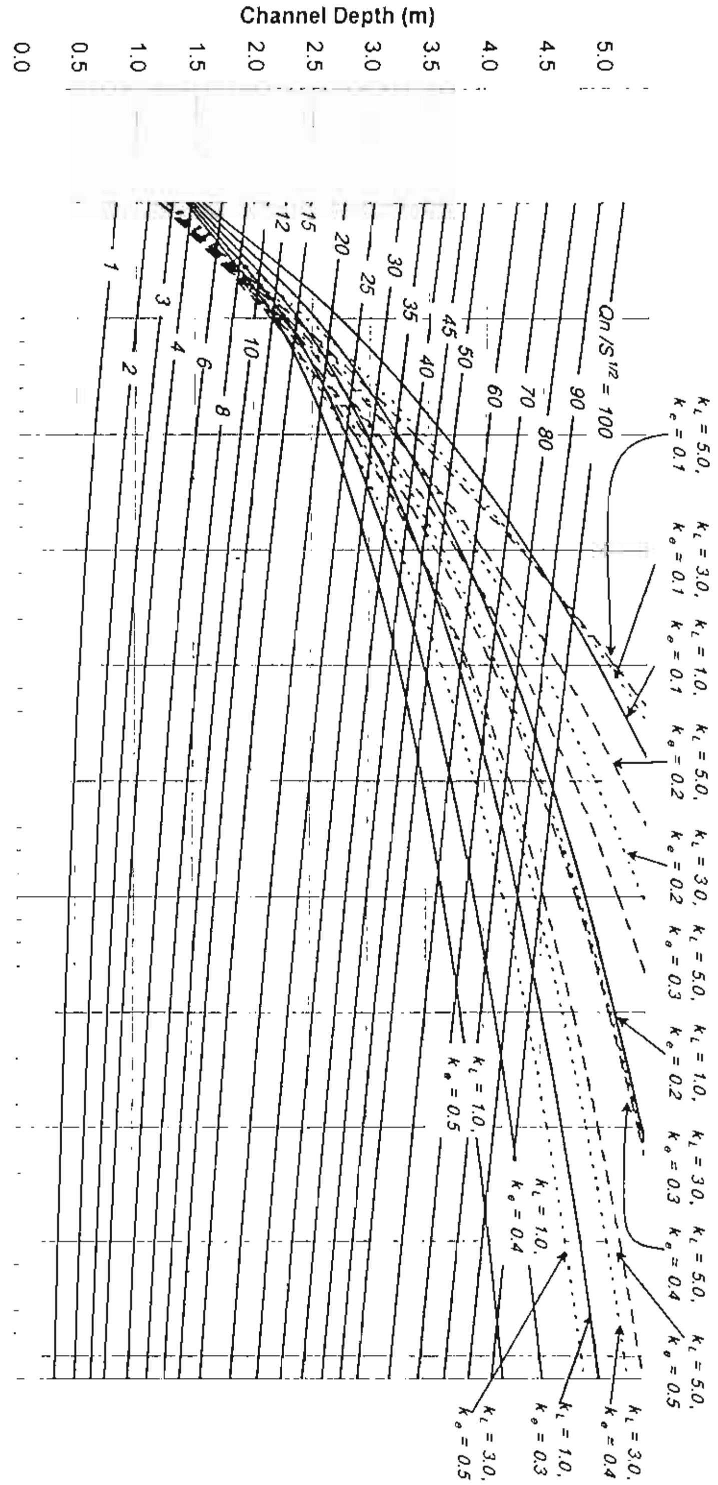

Fig.4. Minimum Cost Design Curves for Lined Trapezoidal Channel with Freeboard = *1m* and Side Slopes *z* = *2*.

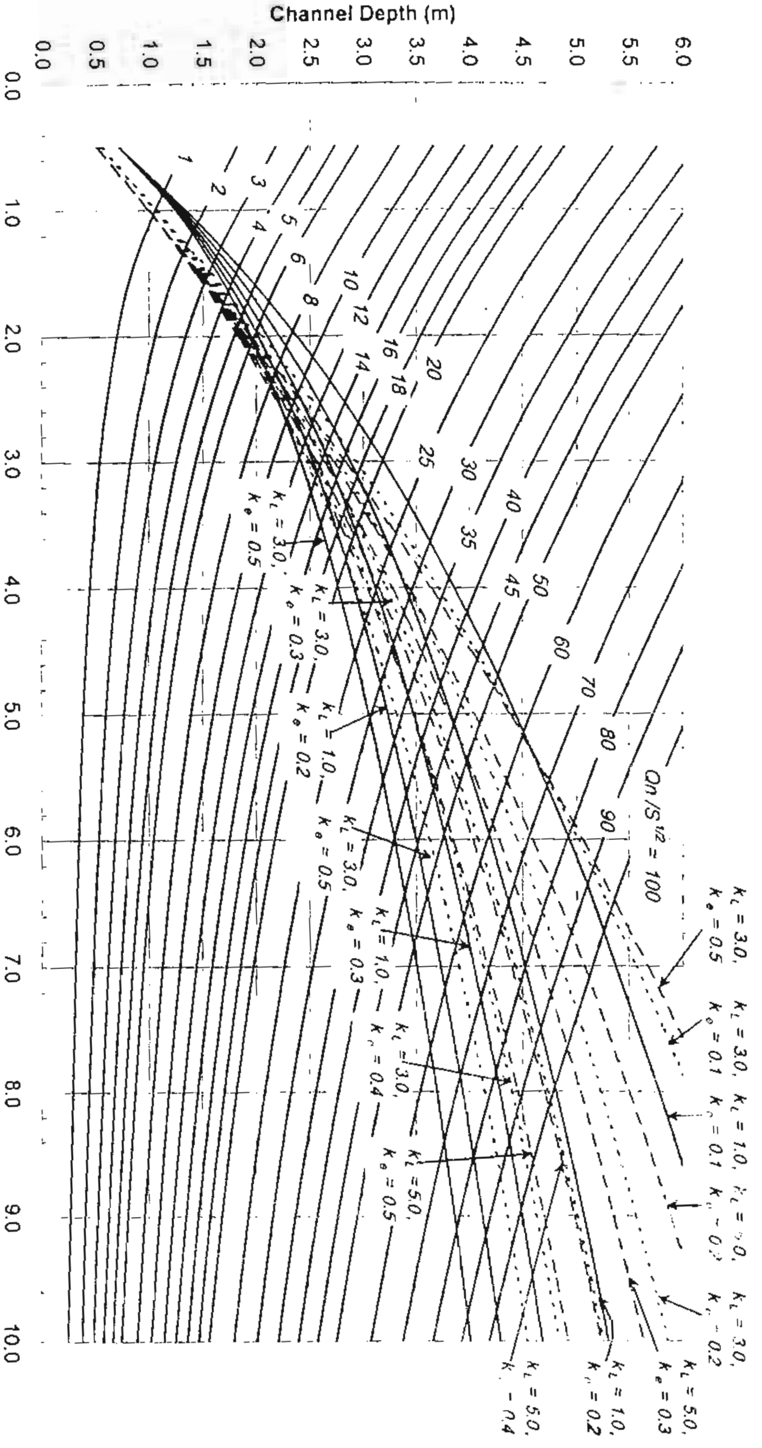

Fig.5. Minimum Cost Design Curves for Lined Trapezoidal Channel with Freeboard = *1m* and Side Slopes *z* = *1/(3)*$^{1/2}$.

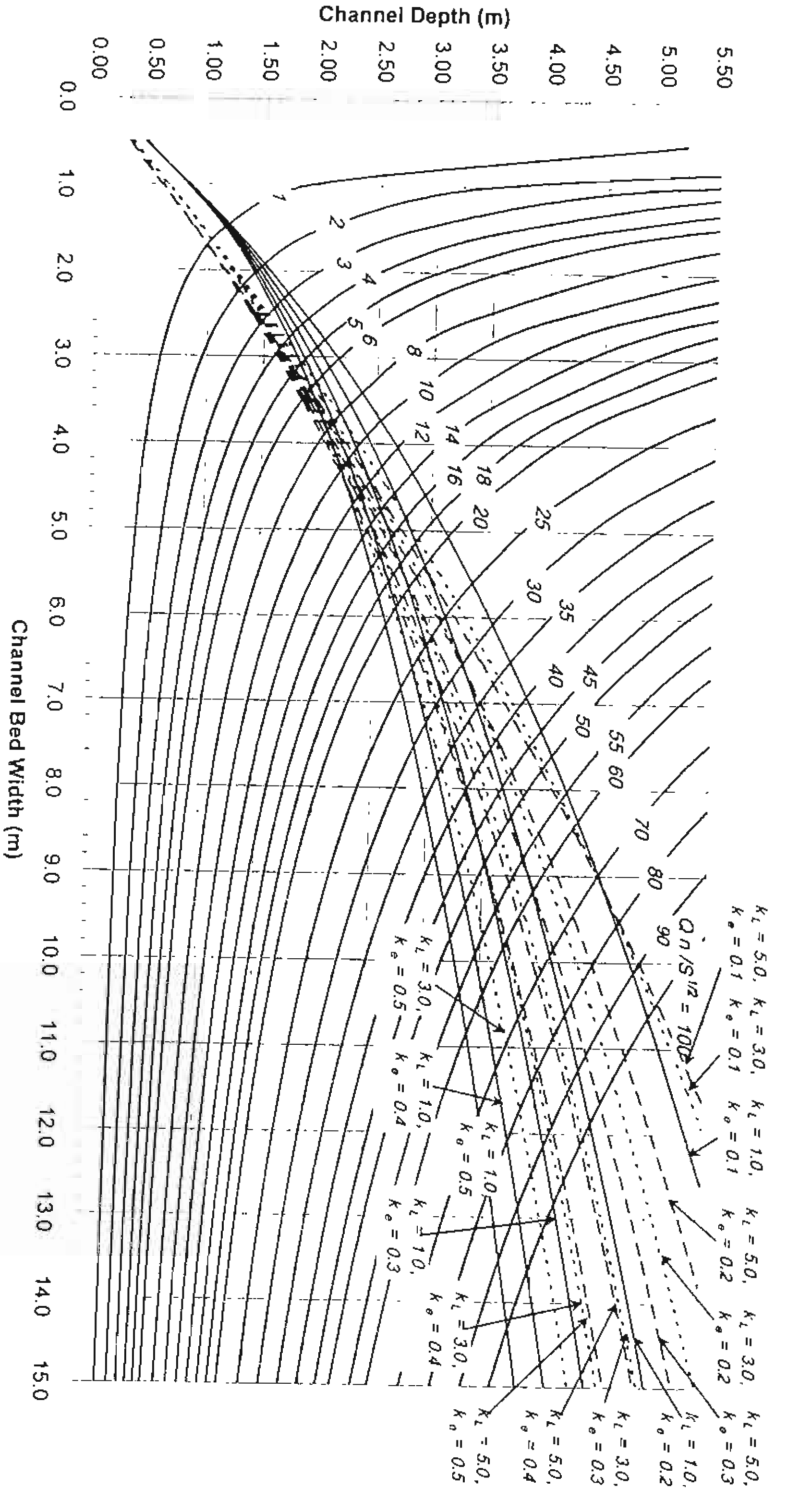

Fig.6. Minimum Cost Design Curves for Lined Rectangular Channel with Freeboard = 1 *m*.

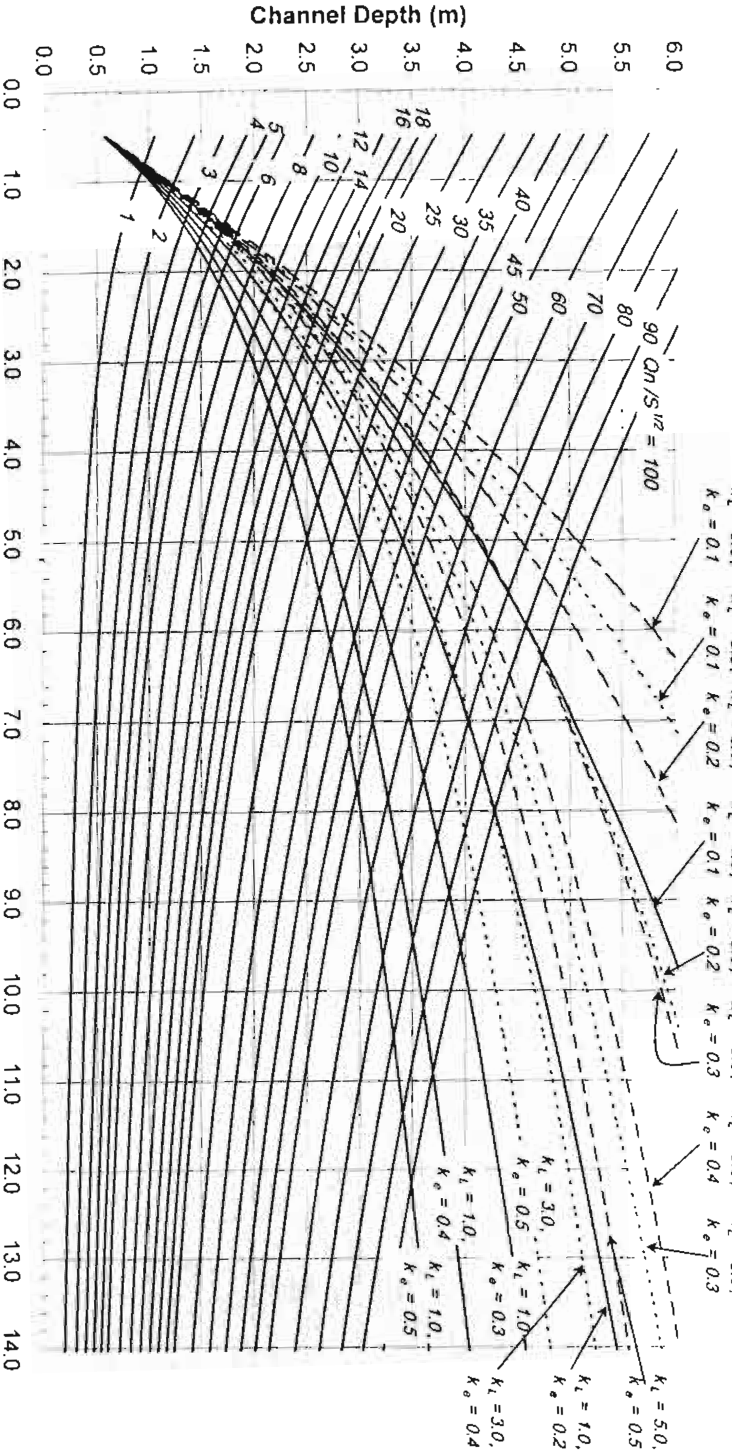

Fig. 7. Minimum Cost Design Curves for Lined Trapezoidal Channel without Freeboard and with Side Slopes *z* = *1.0*.

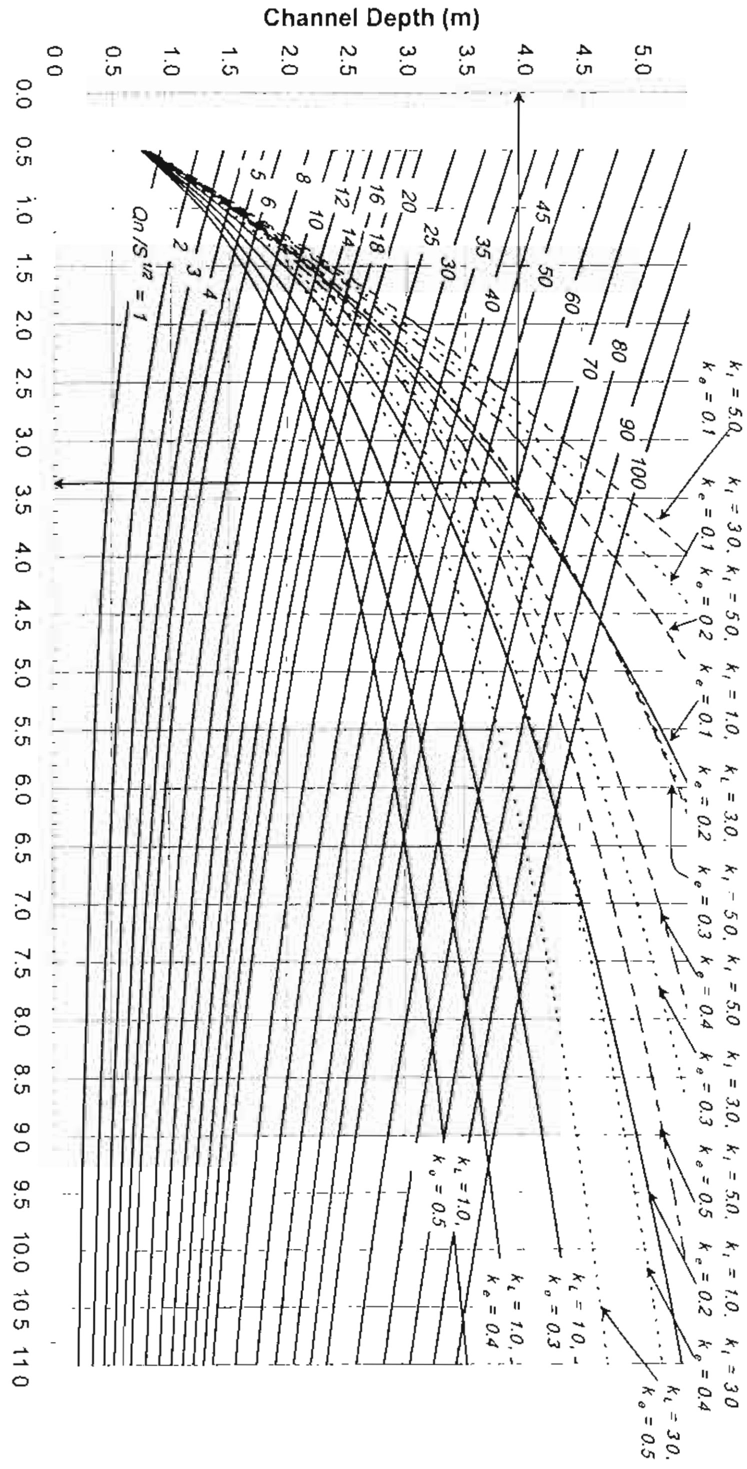

Fig. 8. Minimum Cost Design Curves for Lined Trapezoidal Channel without Freeboard and with Side Slopes $z = 1.5$.

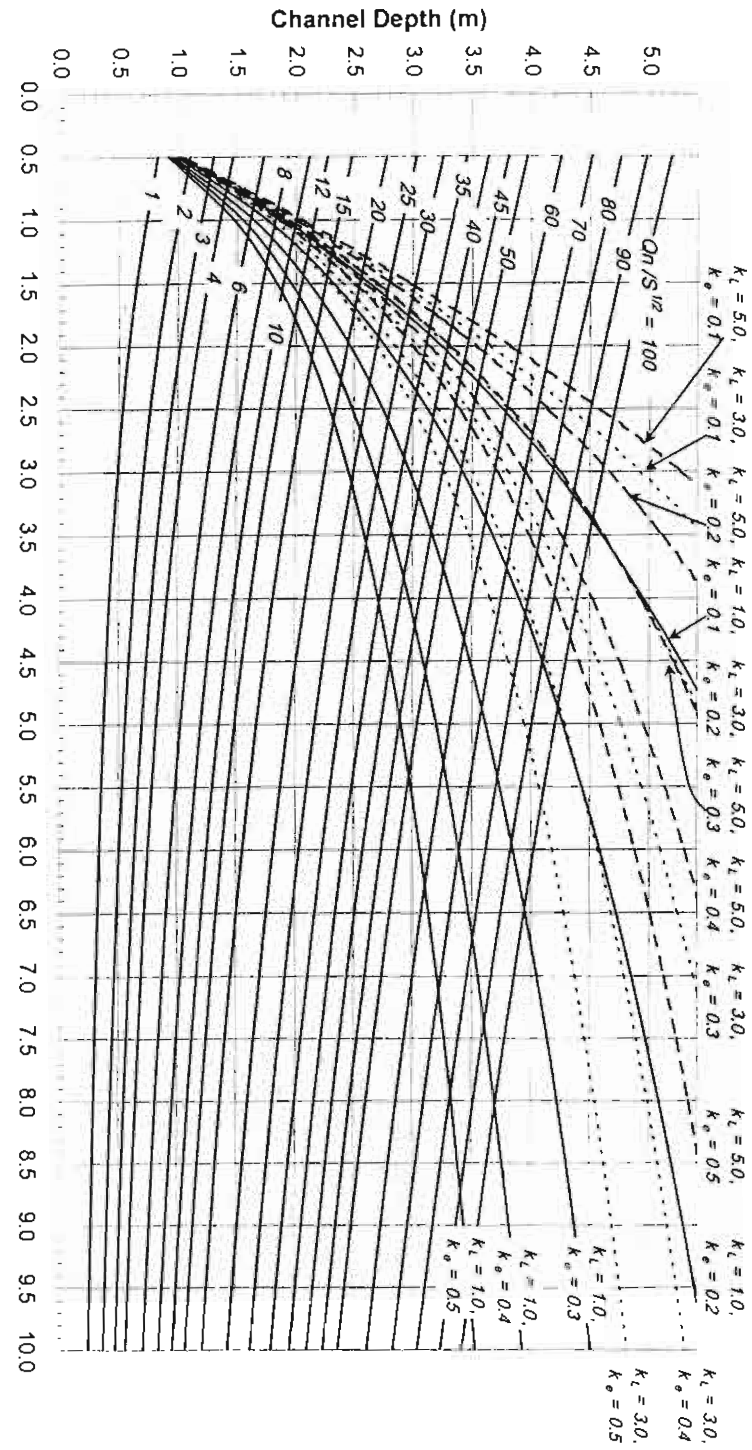

Fig.9. Minimum Cost Design Curves for Lined Trapezoidal Channel without Freeboard and with Side Slopes *z* = *2.0*.

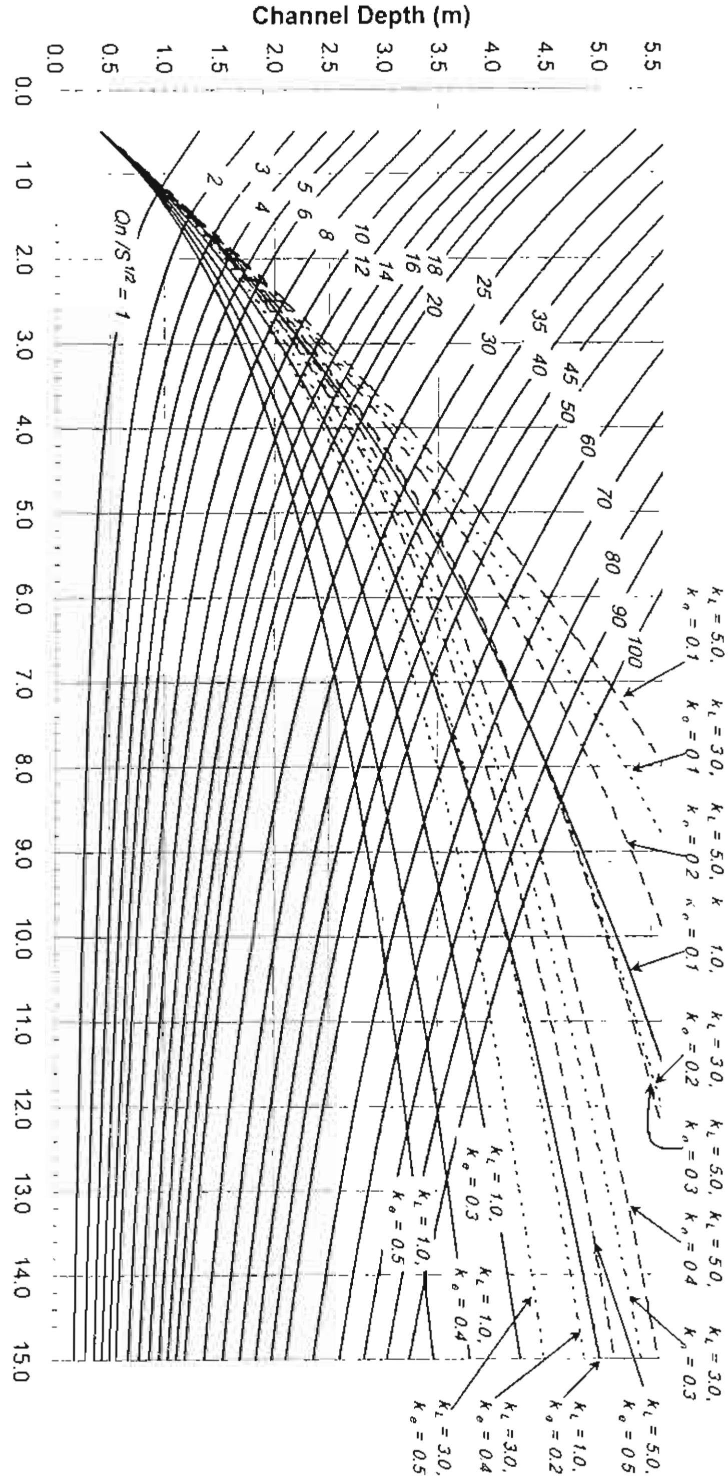

Fig.10. Minimum Cost Design Curves for Lined Trapezoidal Channel without Freeboard and with Side Slopes $z = 1/(3)^{1/2}$.

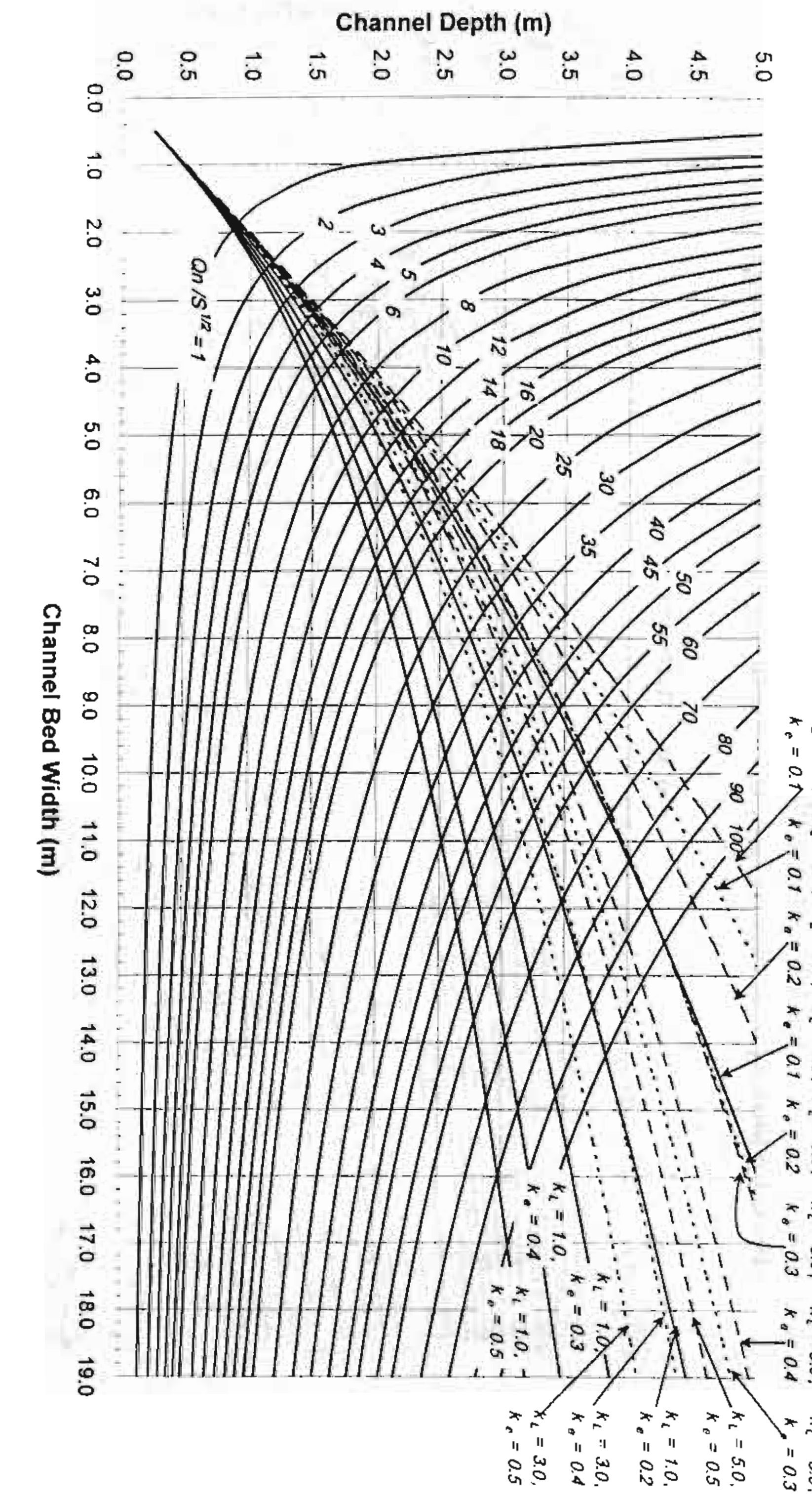

Fig. 11. Minimum Cost Design Curves for Lined Rectangular Channel without Freeboard.

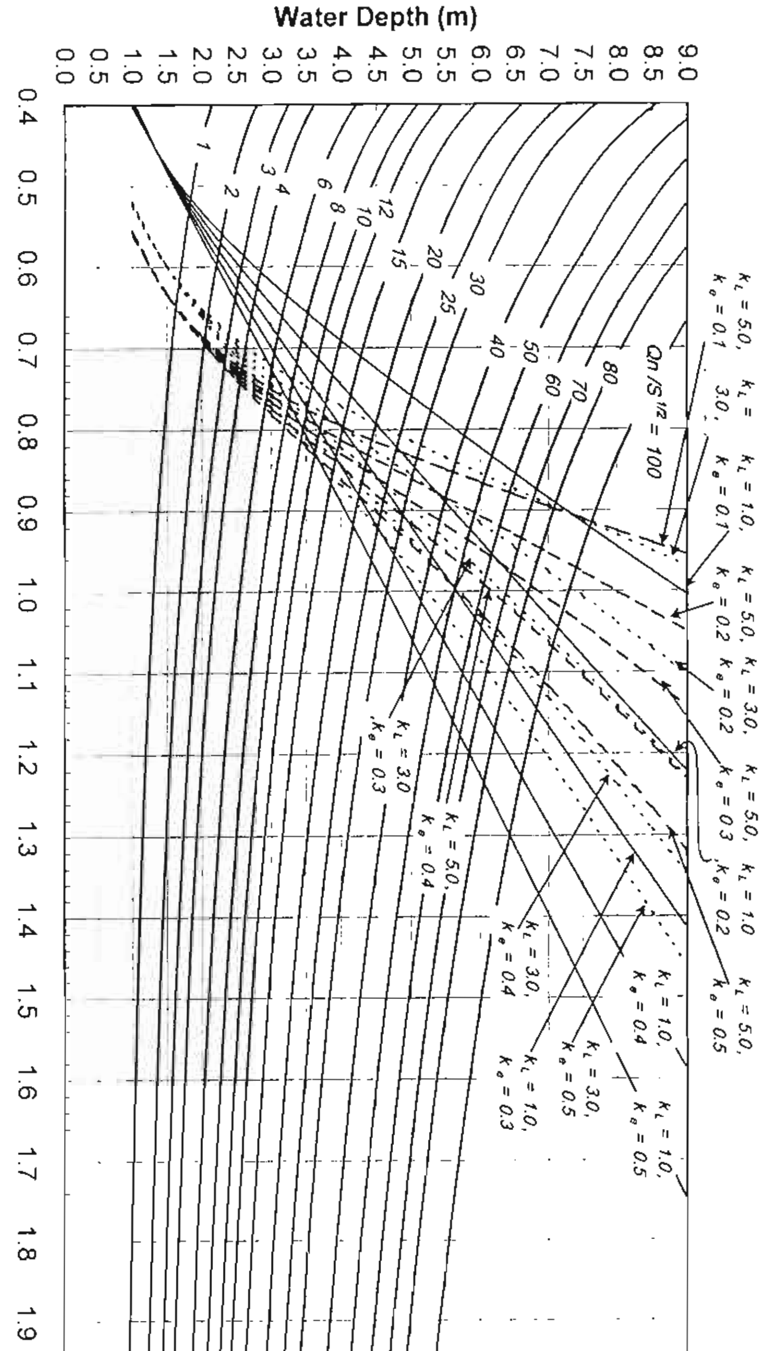

Fig.12. Minimum Cost Design Curves for Lined Triangular Channel with Freeboard = *1 m*.

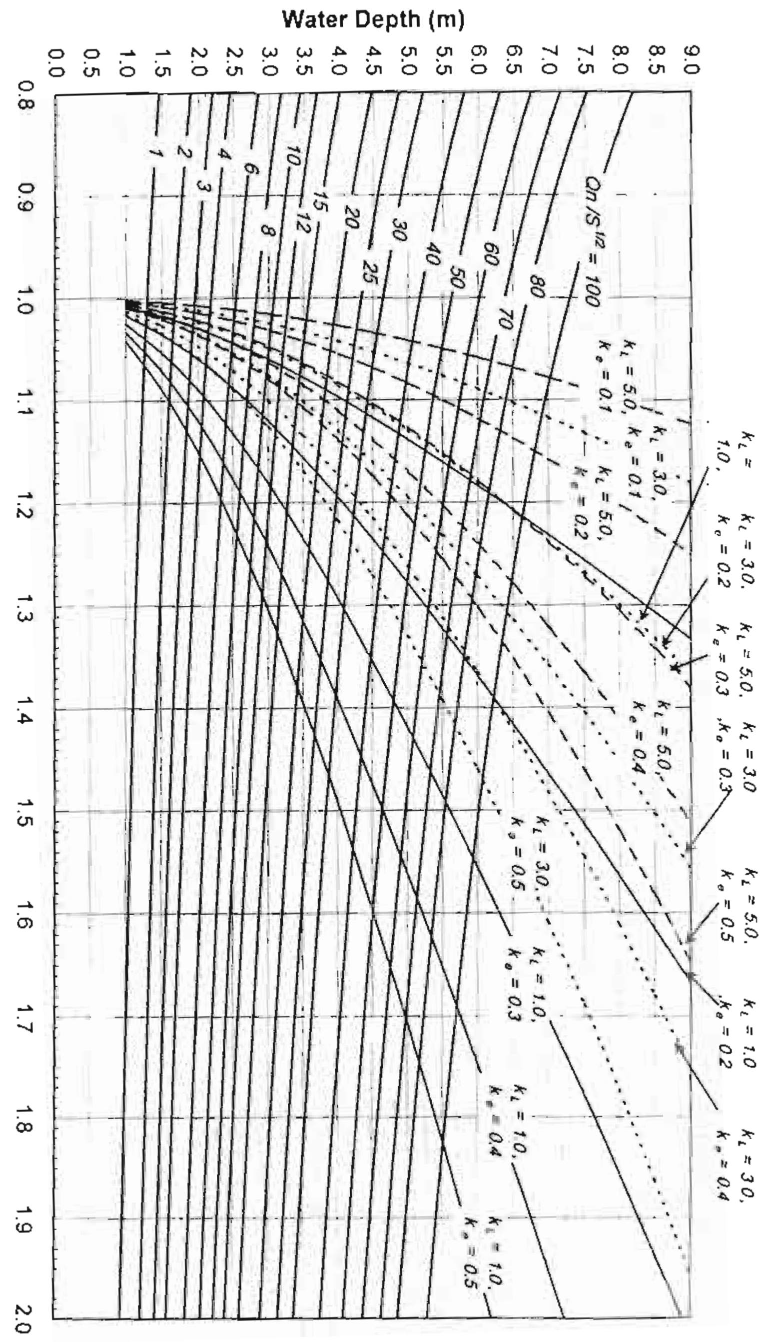

Fig.13. Minimum Cost Design Curves for Lined Triangular Channel without Freeboard.

the triangular channel is also determined at the point where the section factor line crosses the k_e and k_l cost ratio line. The cost of construction could be obtained by substituting the minimum cost design dimensions into equation (5), in the case of neglecting the freeboard. In the case of considering the freeboard, the cost of construction could be obtained by substituting the minimum cost design dimensions into equation (7).

The minimum cost design dimensions could be obtained for other values of $Qn/S^{1/2}$ greater than that presented in the given figures by using the appropriate equations to draw the section factor line and the k_e and k_l cost ratio line. The point of intersection of these two lines determines the minimum cost design dimensions.

Velocity Limits

The average flow velocity of the designed section can be obtained by dividing the given discharge value by the computed section area. This velocity should be greater than the nonsilting velocity but less than the maximum permissible velocity. The maximum permissible velocity depends on the type of lining material [11]. If the average velocity is greater than the maximum permissible velocity, a superior lining material should be used. otherwise, the method of permissible velocity should be used.

DESIGN EXAMPLE

Design a concrete lined trapezoidal canal section with $n = 0.012$ and side slopes $z = 1.5$ to carry a discharge of $50\ m^3/sec$ on a longitudinal bed slope of 0.0001. The canal passes through a stratum of ordinary soil for which $c_e = 10$ *Egyptian pound/m²/m'*, $c_i = 2$ *Egyptian pound/m³/m'*, $c_l = 30$ *Egyptian pound/m/m'*.

Solution

The section factor $Qn/S^{1/2} = 60$ is computed using the given values. The values of $k_e = 0.2\ m^{-1}$ and $k_l = 3.0\ m$ is computed, where $k_e = c_i / c_e$ and $k_l = c_l / c_e$. The minimum cost dimensions, bottom width $= 2.77\ m$ and water depth $= 4.11\ m$ can be obtained from figure (3) for trapezoidal channel with side slopes $z = 1.5$ and depth of freeboard $= 1\ m$. The exact solution of bottom width $= 2.771265\ m$ and water depth $= 4.11\ m$ can be obtained by solving equation (14) simultaneously with equation (23). For trapezoidal channel with side slope $z = 1.5$ without considering depth of freeboard the minimum cost dimensions, bottom width $= 3.38\ m$ and water depth $= 3.95\ m$, can be obtained from figure (8). The exact solution of bottom width $= 3.379963m$ and water depth $= 4.11\ m$ can be obtained by solving equation (14) simultaneously with equation (31).

The total cost of construction of only the flow area obtained from equation (5) is *1011.22 pounds/m'* in the case of freeboard consideration and *1010.56 pounds/m'* in the channel without freeboard. By adding freeboard depth of *1m* in both the two cases, the total cost of construction of total canal section obtained from equation (7) is *1374.82 pounds/m'* in the case of freeboard consideration and *1375.89 pounds/m'* in the case of the channel without freeboard consideration. These results mean that, in case of considering flow area only, the design charts for cross section without free board gave the least cost of construction, while the design charts with free board gave the least cost of construction in case of considering the total cross section area.

The average flow velocity of the designed section is equal to *1.361 m/sec* in the case of freeboard consideration and equal to *1.3603 m/sec* in the case of design without freeboard consideration. These values, which must be within the permissible values, can be obtained by dividing the given discharge value by the computed section area.

Sensitivity of Optimal Design

The water depth of flow were calculated using equation (14) for bed width ranging from *0.5* to *40* and side slope ranging from *0* to *2*. The total construction cost was obtained using equation (5). The variation of construction cost with bed width and side slopes *z* is shown in figure (14). In this figure the least minimum construction cost is *949.72 pounds/m'* for trapezoidal canal with side slope *0.6* and bed width *6.4* m. It is obvious that the construction cost values for the different side slopes is less sensitive to the change in the channel bed width around the point of minimum construction cost. The minimum construction cost of *1016.79 pounds/m'* for rectangular section (*z* = *0*)occurs at channel bed width equal to *10.0 m*. Also, for the design data, figure (14) shows that the rectangular section is more economical for bed width greater than *14*, and the trapezoidal section the most economical section otherwise.

The sensitivity of the minimum construction cost to the variations in the side slopes is shown in figure (15). Also from this figure it is obvious that the minimum construction cost occurs at side slopes *0.6*. The construction cost is less sensitive to the variation in side slopes values between *0.5* and *0.8*.

This design example indicates that the construction cost values are not sensitive to the variations of cross sectional dimensions as long as they do not deviate far from the optimum dimensions.

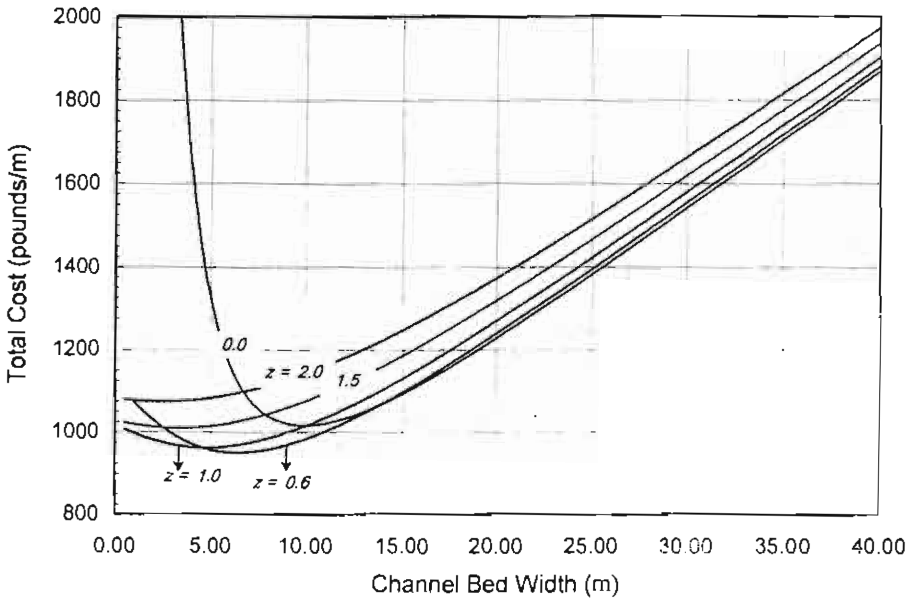

Fig. 14. Senstivity Analysis of Optimal Design

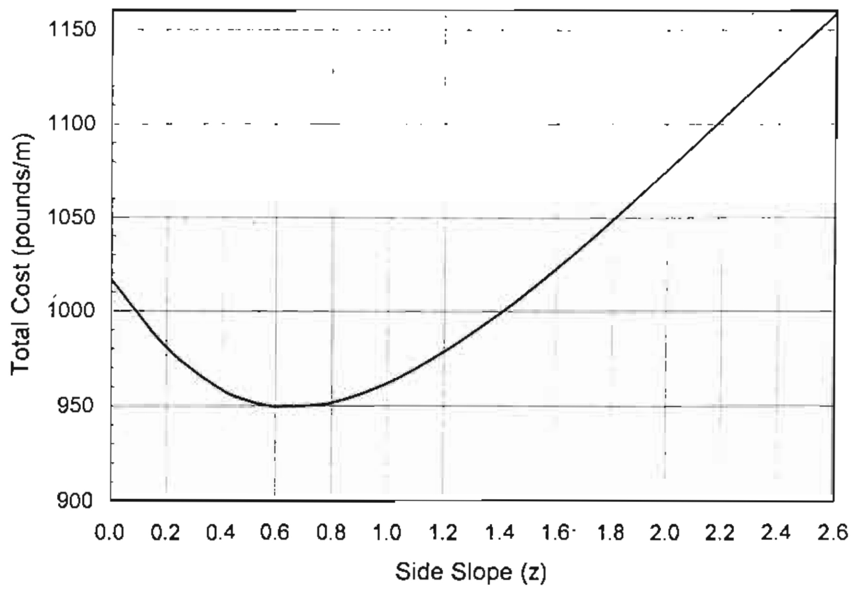

Fig. 15. Minimum Total Cost Versus Side Slopes for The Design Example

SUMMARY AND CONCLUSION

Mathematical technique and graphical solutions were presented for the minimum cost design of trapezoidal, rectangular and triangular irrigation canals. The objective nonlinear function has been expressed as the cost per unit length of the canal for excavation and lining. Manning's equation was used as a nonlinear equality constraint. The method of Lagrange multipliers was applied to the objective function and the equality constraint function to get the required equation for the minimum cost design.

Fortran computer programs were prepared to solve the design equations separately. Using the results of computer program, minimum cost design charts of trapezoidal, rectangular and triangular canal sections were plotted considering both the case of including the freeboard depth and the case of neglecting the freeboard depth in the computations. The minimum cost design charts are considered useful in direct selection of the optimal canal dimensions.

A design example with sensitivity analysis was provided to demonstrate the simplicity and practicability of the present method. The sensitivity analysis indicated that small deviations from optimal dimensions did not cause a significant increase in costs.

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NOTATION

The following symbols are used in this paper:

A = flow area;
A_t = total excavation area including freeboard ;
b = bed width of canal;
C = the total cost of construction per unit length of canal;
C_e = cost of excavation per unit length of canal;
C_{ef} = cost of excavation per unit length of canal including freeboard;
C_l = the cost of lining per unit length of canal;
C_{lf} = the cost of lining per unit length of canal including freeboard;
c_e = cost per unit volume of excavation at ground level;
c_i = increase in the unit excavation cost per unit depth;
c_l = the cost of canal lining per unit surfaee area;
E = $A\bar{y}$;
E_t = $A_t\bar{y}_t$;
F = depth of freeboard;
k_e = c_i / c_e;
k_l = c_l / c_e.
n = Manning's roughness coefficient;
Q = flow rate;
P = wetted perimeter;
P_t = total perimeter of cross section including freeboard;
R = mean hydraulic radius;
S = ehannel bed slope;
y = depth of flow;
$\bar{y}$ = depth of centroid of area from the free water surface;
$\bar{y}_t$ = depth of centroid of area from the ground level;
z = side slope of canal;
ϕ = equality constraint funetion; and
λ = Lagrange multiplier.