

COMPUTER AIDED DESIGN OF VIBRATION
ISOLATORS REGARDING SOIL INTERACTION (VID)

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ABSTRACT

The main objective of this paper is to help designers of vibration isolators for a more exact solution of isolation problem taking into account soil interaction. A computer aided design technique using the machinery and soil information is introduced.

KEYWORDS

Vibration, isolation, transmissibility, response, spectrum.

INTRODUCTION

In the classical design procedure of vibration isolators soil effects are neglected, assuming a perfectly stiff soil [1,2]. In fact, soil stiffness changes from one place to another. So, soil interaction is expected to have varying effects on the isolation procedure. Both machine and foundation responses, change by this interaction at various degrees according to the soil parameters. For this reason, a more exact solution for machinery isolation was introduced.

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THE SUGGESTED MATHEMATICAL MODEL :

Fig. (1) show the suggested mathematical model, for more realistic approach to machinery isolation problem.

Development of Equations of Motion :

The equations of motion which describe the behavior of the mathematical model Fig. (1) "natural frequencies, mode shapes, transmissibility, and vibration response" are developed using the method of dynamic equilibrium equations [3] taking into consideration the following assumptions :

Isolator stiffness is linear, soil stiffness is linear, rigid inertia block, infinite stiffness of concrete block, the excitation is due to a known unbalance weight, the motion of the machine and foundation is restricted to the vertical and horizontal directions only, and no vibration transmitted to the foundation from the neighborhood.

The condition of equilibrium of a mass at any instant of time under the influence of forces and reactions is considered. In order to account for dynamic equilibrium, the mass inertia force is included.

For the vertical excitation of the system (the machine, inertia block, and soil mechanism), the dynamic equilibrium equations are derived as follows :

$$m_1 \ddot{z}_1 + K_{v1}(z_1 - z_2) + C_{v1}(\dot{z}_1 - \dot{z}_2) = F_z(t) \quad (1)$$

$$m_2 \ddot{z}_2 + C_{v2} \dot{z}_2 + C_{v1}(\dot{z}_2 - \dot{z}_1) + K_{v1}(z_2 - z_1) + K_{v2} z_2 = 0 \quad (2)$$

Substitute $F_z e^{i\omega t}$ for $F_z(t)$, $Z_1 e^{i(\omega t - \phi_1)}$ for z_1 , and $Z_2 e^{i(\omega t - \phi_2)}$ for z_2 .

Rearranging the terms of equations (1) and (2) and dividing all through by $e^{i\omega t}$, the equations of motion become

$$(K_{v1} - m_1 \omega^2 + i\omega C_{v1}) Z_1 e^{-i\phi_1} - (K_{v1} + i\omega C_{v1}) Z_2 e^{-i\phi_2} = F \quad (3)$$

$$(K_{v2} + K_{v1} - m_2 \omega^2 + i\omega C_{v2} + i\omega C_{v1}) Z_2 e^{-i\phi_2} - (K_{v1} + i\omega C_{v1}) Z_1 e^{-i\phi_1} = 0 \quad (4)$$

, The damping ratio term is ignored while calculating the natural frequencies and modes of the system. However, the

influence of damping is considered in response calculations.

The phase angles between the exciting force and the response z_1, z_2 are given by :

$$\phi_{1v} = \tan^{-1} [\omega(A_2 \cdot A_6 - A_1 \cdot A_7)/A_9] \quad (5)$$

$$\phi_{2v} = \tan^{-1} [\omega(K_{v1} \cdot A_6 - C_{v1} \cdot A_7)/A_{10}] \quad (6)$$

The undamped natural frequencies of the system are

$$\omega_{1v} = Q_1^{\frac{1}{2}} \quad (7)$$

$$f_{n1v} = \omega_{1v}/2\pi \quad (8)$$

$$\omega_{2v} = Q_2^{\frac{1}{2}} \quad (9)$$

$$\text{and } f_{n2v} = \omega_{2v}/2\pi \quad (10)$$

where :

$$Q_1 = [B_v + (B_v^2 - 4 A_{11} C_z)^{\frac{1}{2}}]/2 A_{11} \quad (11)$$

$$Q_2 = [B_v - (B_v^2 - 4 A_{11} C_z)^{\frac{1}{2}}]/2 A_{11} \quad (12)$$

The steady state response of the system can be given as :

$$z_1 = F. / \sqrt{(\omega^2 \cdot (A_2 \cdot A_6 - A_1 \cdot A_7)^2 + (A_2 \cdot A_7 + \omega^2 \cdot A_1 \cdot A_6)^2)} / A_8 \quad (13)$$

$$z_2 = F. / \sqrt{((K_{v1} \cdot A_7 + \omega^2 \cdot C_{v1} \cdot A_2)^2 + \omega^2 (K_{v1} \cdot A_6 - C_{v1} \cdot A_7)^2)} / A_8 \quad (14)$$

The response of both base/bed system are

$$vZ_1 = \omega Z_1 \quad (15)$$

$$vZ_2 = \omega Z_2 \quad (16)$$

$$\text{and the absolute transmissibility is } T = (vZ_2/vZ_1)100 \quad (17)$$

The characteristics of this model in the horizontal direction are similar to the above model except that all the parameters are related to the horizontal axis.

Method of Analysis :

The application of computer programs in obtaining the performance of vibration isolators in terms of transmissibility spectra, and also the response of both base/bed system at different frequencies are calculated for both vertical and

horizontal mode. In addition, these programs have sufficient built-in flexibility in input/output operation so that design and performance of isolators can be performed in single run. A flow chart of a typical application is shown in Fig. (2).

Table (1) represents a specimen of the results obtained from this program.

DISCUSSIONS OF RESULTS :

Figs.(3) and (4) represent the response of both base/bed system for two cases of soils, a stiff soil Fig.(3) and an elastic soil Fig. (4). From these figures it is clearly noticed that in case of rigid foundation the base response has a higher value of vibration displacement, with only one mode of vibration, while the elastic foundation reflects lower values for base vibration amplitude with more modes. A contradictory effect is noticed for the bed, with lower values of vibration amplitudes at all modes in case of stiff soil.

This means that erecting a machine foundation on an elastic soil will decrease the isolation efficiency and help vibration transmission to other machinery. This fact should be taken into consideration for designing a good isolator.

This conclusion can be clearly demonstrated from the results taken from VID program representing transmissibility spectra and shown in Fig.(5) for the vertical direction and Fig.(6) in the horizontal direction.

SUGGESTED DESIGN PROCEDURE FOR ISOLATORS :

For more accurate design of isolators that takes into consideration the soil interactions, the following procedure is suggested :

a) Data required :

Soil information

- 1] Shear modulus of soil G .
- 2] Poisson's ratio of soil ν .
- 3] Soil unit weight γ .

Foundation information :

- 1] Concrete unit weight γ_c .
- 2] Area of foundation A_f .
- 3] Foundation height H_f .

These should be stated using the recommendation given below and used for calculating the foundation mass.

Machine information :

- 1] Machine weight W_1 .
- 2] Machine dimensions.
- 3] Forcing frequencies of machine f .
- 4] Machine center of gravity co-ordinates.

RECOMMENDATIONS REGARDING THE FOUNDATION DIMENSIONS & MASS :

The following items apply to block-type foundation resting on soil [4,5] :

- 1] A rigid block-type foundation resting on soil should have a mass of two to three times the mass of the supported machine for centrifugal machines. However, when the machine is reciprocating, the mass of the foundation should be three to five times the mass of the machine.
- 2] The top of the block is usually kept 30 cm above the finished floor or pavement elevation to prevent damage from surface water runoff.
- 3] The vertical thickness of the block should not be less than 60 cm., or as dictated by the length of anchor bolts used. The vertical thickness may also be governed by the other dimensions of the block in order that the foundation be considered rigid. The thickness is seldom less than one fifth of the least dimension or one tenth the largest dimension.
- 4] The foundation should be wide to increase damping in the rocking mode. The width should be at least 1 to 1.5 times the vertical distance from the base to the machine centerline.
- 5] Once the thickness and width have been selected, the length is determined according to (1) above, provided that sufficient plan area is available to support the machine plus 30 cm

clearance from the edge of the machine base to the edge of the block for maintenance purposes.

- 6] The length and width of the foundation are adjusted so that the center of gravity of the machine plus equipment coincides with center of gravity of the foundation. The combined center of gravity should coincide with the center of resistances of the soil.
- 7] For large reciprocating machines, it may be desirable to increase the embedded depth in soil such that 50% to 80% of the depth is embedded. This will increase the lateral restraint and the damping ratios for all modes of vibration.
- 8] Should the dynamic analysis predict resonance with the acting frequency, the mass of the foundation is increased or decreased so that, generally, the modified structure is overtuned or undertuned for reciprocating and centrifugal machines, respectively.

b] Design steps :

With this program together with the above design data in hand, the following steps are requested to be followed :

- 1] Given the machine mass (m_1) together with the calculated values of (m_2) and K_{v2} of the foundation, the main program VID will calculate the transmissibility spectra for different isolator to soil stiffness ratio K_{v1}/K_{v2} .
- 2] Traces of family of K_{v1}/K_{v2} ratio's as transmissibility spectra can be obtained from the same program Fig. (7).
- 3] With the forcing frequency of the machine in question on the abscissa and the required transmissibility on the ordinate of the above traces, their intersection will determine the required value of the stiffness ratio K_{v1}/K_{v2} as shown in Fig. (7).
- 4] Since the value of K_{v2} "soil stiffness" is already known, the required isolator stiffness that fulfills the proposed isolation efficiency is hence determined.
- 5] To check the resonant frequencies of the designed system the traces shown in Fig. (8) can be obtained from the same program for the soil stiffness in question. These traces

represent the natural frequencies of the major modes as function of stiffness ratio's K_{v1}/K_{v2} .

Given the value of K_{v2} and the designed ratio K_{v1}/K_{v2} , the frequencies of the major modes can be obtained.

CONCLUSIONS :

As far as the results of the theoretical work reported here in, are concerned, the following conclusions are drawn :

- 1] Soil interaction plays a great role in the vibration isolation process, and should be taken very seriously during the design stage.
- 2] A computer aided design procedure for vibration isolation taking into consideration the soil interaction has been created as a part of this work.
- 3] Stiff soil is much more favourable in vibration isolation than elastic soils, piles, are thus recommended for elastic soils whenever it is economically feasible.

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NOMENCLATURE :

A_{00}	$= m_2 \omega^2.$
A_0	$= m_1 \omega^2.$
A_1	$= C_{v1} + C_{v2}.$
A_2	$= K_{v1} + K_{v2} - A_{00}.$
A_3	$= K_{v1} \cdot (K_{v2} - A_{00}).$
A_4	$= C_{v1} \cdot (K_{v2} - A_{00}).$
A_5	$= C_{v1} \cdot C_{v2} \cdot \omega^2.$
A_6	$= A_4 + (C_{v2} \cdot K_{v1}) - (A_0 \cdot A_1).$
A_7	$= A_3 - (A_0 \cdot A_2) - A_5.$
A_8	$= [(A_7)^2 + (\omega A_6)^2]^2.$
A_9	$= A_2 \cdot A_7 + \omega^2 \cdot A_1 \cdot A_6.$
A_{10}	$= K_{v1} \cdot A_7 + \omega^2 \cdot C_{v1} \cdot A_6.$
B	= Mount width
C_{v1}	$= 2 \cdot \xi_1 / \sqrt{K_{v1} \cdot m_1}.$
C_{v2}	$= 2 \cdot \xi_2 / \sqrt{K_{v2} \cdot m_2}.$
D	= Mount diameter.
F	$= m e \omega^2.$
H	= Mount height.
L	= Mount length.
ξ_1, ξ_2	= Isolator and soil damping factor.
ω	$= 2\pi f.$

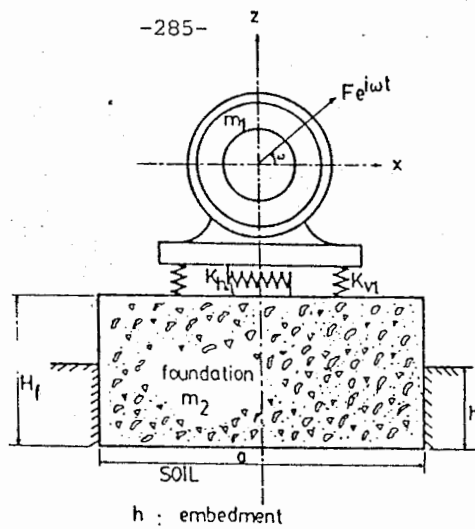


Fig. (3-7) Machine supported on concrete foundation and vibration isolated from foundation.

- m_1 Equivalent machine mass.
- m_2 Equivalent foundation mass.
- k_{v1} Equivalent isolator stiffness.
- k_{v2} Equivalent soil stiffness.
- c_{v1} Isolator damping coefficient.
- c_{v2} Soil damping coefficient.

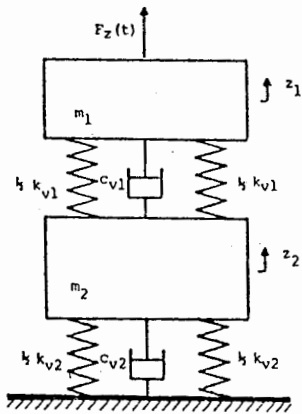


Fig. (3-7a) VERTICAL EXCITATION.

- m_1 Equivalent machine mass.
- m_2 Equivalent foundation mass.
- k_{h1} Equivalent isolator stiffness.
- k_{h2} Equivalent soil stiffness.
- c_{h1} Isolator damping coefficient.
- c_{h2} Soil damping coefficient.

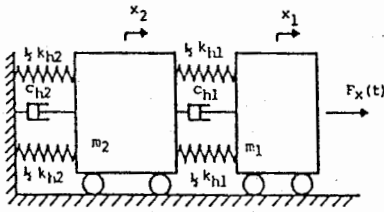


Fig. (3-7b) HORIZONTAL EXCITATION.

Fig. (1) MATHEMATICAL MODEL.

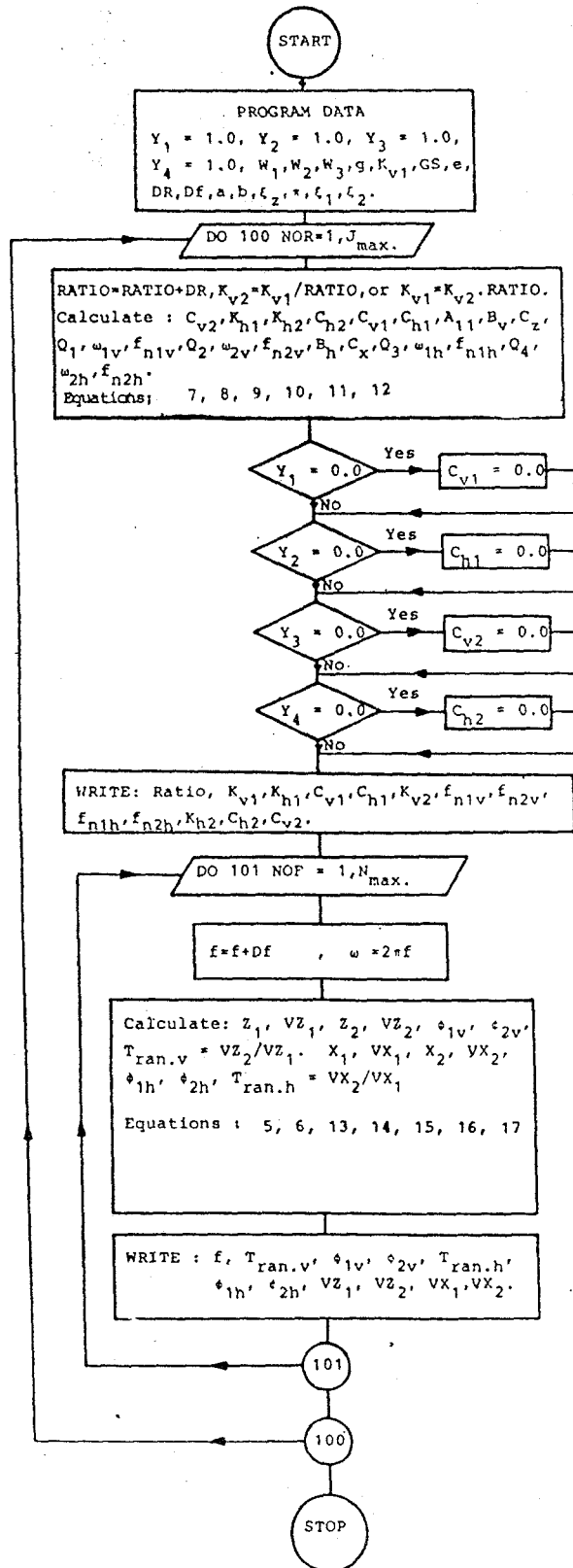


Fig. (2) Flow chart(VID PROGRAM) for designing isolator or performance of isolator considering soil interaction effect.

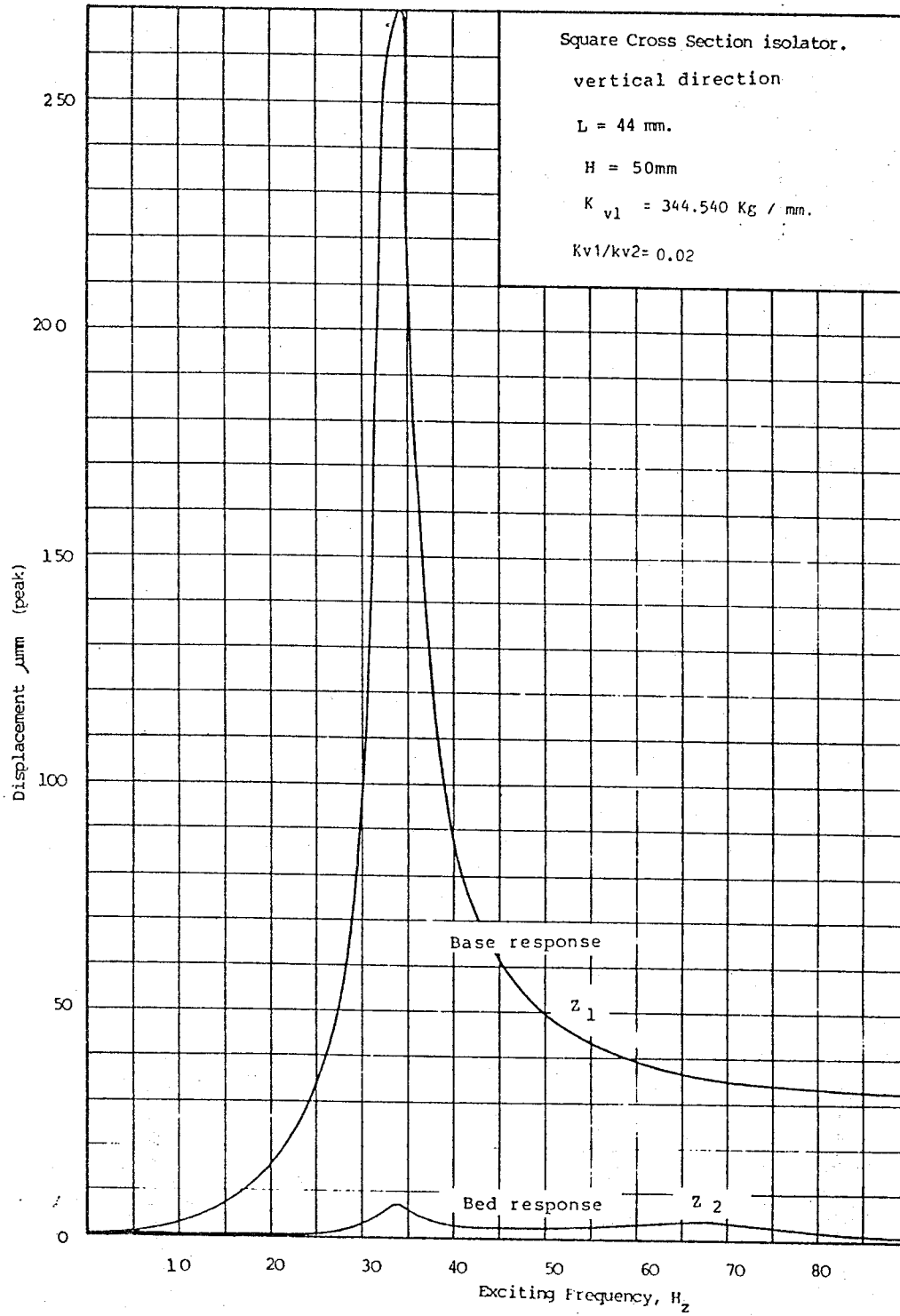


Fig. (3) Motion response for the case of rigid soil.

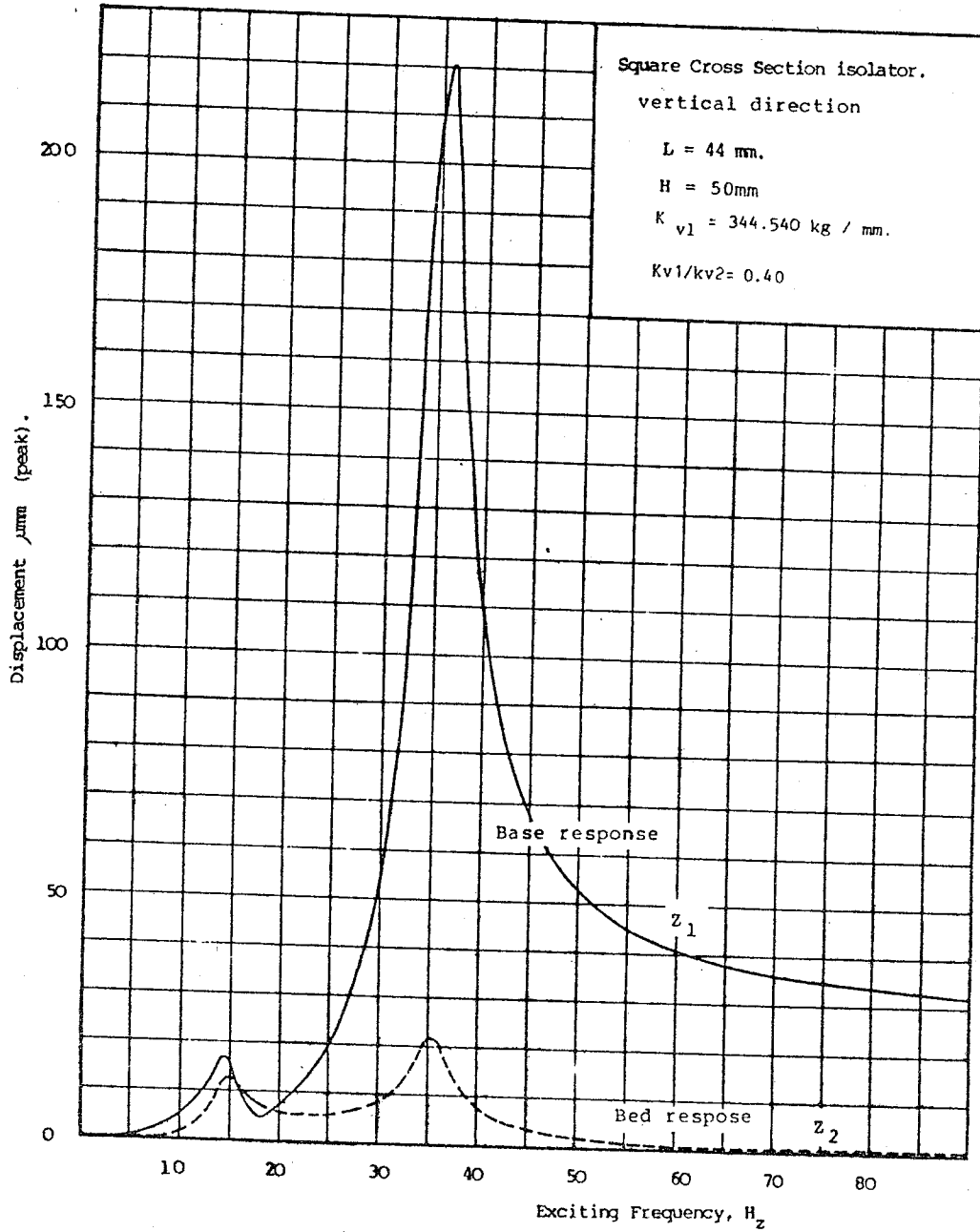


Fig. (4) Motion response for the case of elastic soil.

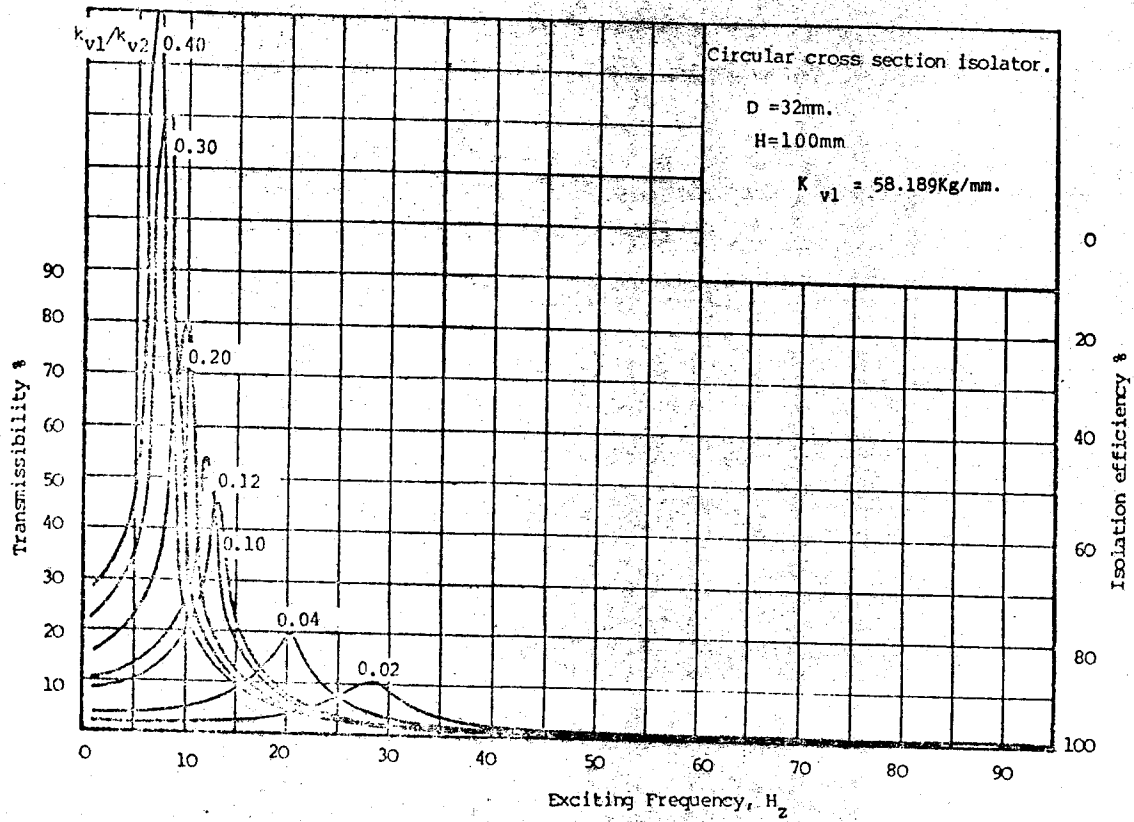


Fig. (5) Soil interaction effect in the vertical direction.

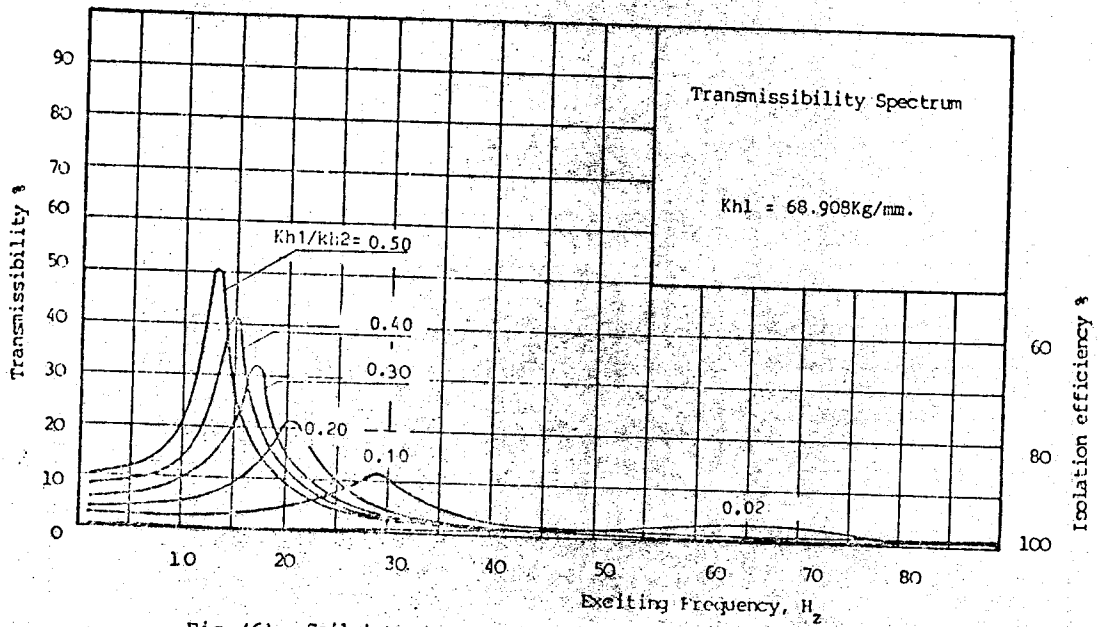


Fig. (6) Soil interaction effect in the horizontal direction.

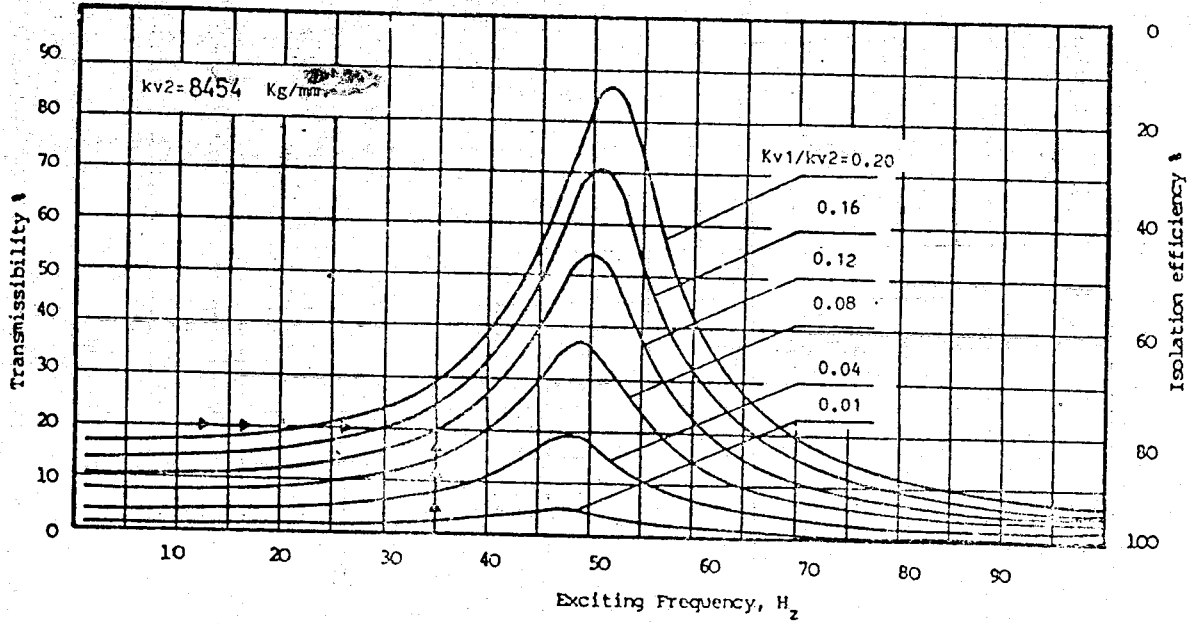


Fig. (7) Design curves considering soil interaction effects.

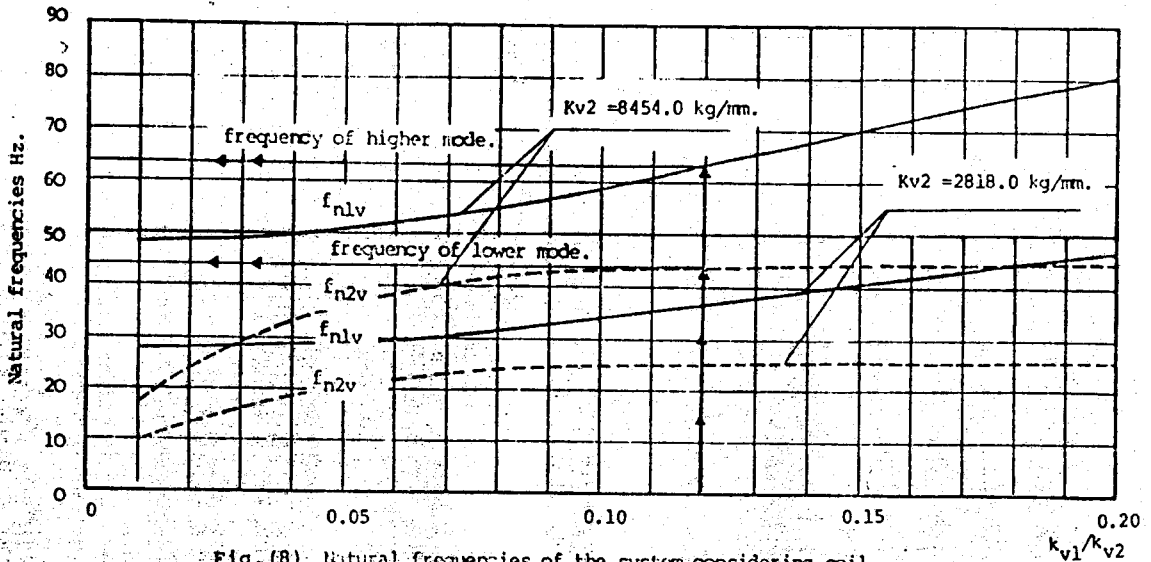


Fig. (8) Natural frequencies of the system considering soil interaction effect (CHECK CURVES).

Table (1) Specimen of the results obtained from (VID) program.

CV = 0.140000 DZ CM = 0.50000000 LK CM = 0.15000000 M CV = 0.156437E 01
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Table with 12 columns: I, YEAR, PH2V, PH2W, PH2X, PH2Y, PH2Z, V1, V2, V3, V4, V5, VZ. The table contains a grid of numerical data points for various specimens.