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Kinetic Energy and Momentum Coefficients for Egyptian Irrigation Canals

معاملى طاقة وكمية الحركة فى قنوات الرى المصرية

Zidan, A. R. [a]; Abdalla, M. G. [b]; Khalaf S. [c]; and Saqr, A. M. [d]

[a] *Prof., Irrigation & Hydraulics Dept., Fac. of Engrg., EL-Mansoura Univ.*

[b] *Assistant Prof., Irrigation & Hydraulics Dept., Fac. of Engrg., EL-Mansoura Univ.*

[c] *Lecturer, Irrigation & Hydraulics Dept., Fac. of Engrg., EL-Mansoura Univ.*

[d] *Demonstrator, Irrigation & Hydraulics Dept., Fac. of Engrg., EL-Mansoura Univ.*

الملخص العربي:- يَهدِف البحثُ إلى دِراسة مُعاملي تصحيح طاقة و كمية الحركة (α و β) فى بعض قنوات الرى المصرية. تم اختيار ست حالات دراسية مختلفة من هذه القنوات ؛ تتدرج من الريّاحات إلى القنوات التوزيعية بواقع عشرة قطاعات متتالية لكل حالة دراسية بتصرفات تتراوح من 489.7 م3 /ث إلى 0.4 م3 /ث طبقا للقنوات المختارة. تم الحصول على خصائص القطاعات المائية وكذلك خصائص السريان المار خلال القطاعات الخاصة بكل ترعة من الترع تحت الدراسة. وقد أُستخدمت الطريقة الحسابية لدقتها فى حساب قيم المعاملين عن طريق إعداد برنامج اكسل لتسهيل عملية الحسابات. وُجِد أنّ القيم المحسوبة للمُعاملين تتطابق تقريبا مع القيم المعطاه فى جدول **Chow** بقيم متوسطة تساوى **1.30**و **1.10** والتى تستخدم فى الأغراض العملية لمُعاملي تصحيح طاقة وكمية الحركة على الترتيب ، كما أن القيم المتوسطة للمعاملين تزيد بزيادة درجة المجرى المائى ؛ ولكن القيم العظمى والصغرى للمعاملين قد لا تعتمد بشكل كبيرعلى درجة المجرى المائى. أُستنتجت علاقة تربط المعاملين لكل قناة ؛ وكذلك علاقة عامة لكل حالات الدراسة. تم إيجاد معادلات ؛ و رسمت علاقات بيانية مقارنة تجمع بين المعاملين من جهة ؛ ونسبة علاقة السرعة القصوى بالسرعة المتوسطة (ϵ) ؛ ومعاملماننج للخشونة (n) ؛ وبعض خصائص القطاع ؛ والتصرفات المختلفة من جهة أخرى. أَوضحت الدراسةُ أن قيم المعاملين لا تتأثر بشكل ملحوظ بخصائص القطاع ؛ وإنما بشكل توزيع السرعات ؛ وأن رقم **Froude** له تأثير كبيرعلى قيم المعاملين ، أما رقم **Reynolds** فإن تأثيره قد يكون غير ملموس.

KEYWORDS
Energy coefficient, Momentum coefficient, Egyptian irrigation canals, Manning's coefficient, Froude number, Reynolds number

Abstract: - **The main objective of this research work is to investigate the energy coefficient (α) and momentum coefficient (β) for some Egyptian irrigation canals. Six cases of study from them having different degrees, starting from rayyahs to distributer canals with 10 successive cross sections for each reach, were carefully selected with discharge values ranged from 489.7 m3/s to 0.4 m3/s. The different properties and flow characteristics of each cross section were obtained. The arithmetic method was used in the computation process of α and β values due to its accuracy using a prepared excel program to facilitate the calculations. It was found that the computed values of α and β approximately match with Chow's table with average values of about 1.30 and 1.11 respectively, for canals under study, which could be used for field applications. It was also found that the average values of α and β for each canal increased with the increase of the canal degree but the maximum and the minimum values may be independent of the canal degree. General relationships relating α and β for each canal and for all canals under study have been derived. Moreover, correlation relationships for both α and β with a ratio correlating maximum and mean velocities (ϵ), Manning's roughness coefficient (n), and different values of discharge and channel characteristics were performed. The cross section properties had intangible effect on the values of both α and β but these values were strongly dependent on the velocity distribution shape. Froude number had a pronounced effect on the value of α and β but Reynolds number could have a negligible effect.**

1. INTRODUCTION

Open channels are considered as one of the main infrastructures which are in continuous application in water conveyance activities such as irrigation and drainage engineering, hydropower, and water supply and sanitation from day to day (Alonso et al., 2009; Venkateshwarlu, 2012). Moreover, a continuous investigation of open channels should be carried out to upgrade their design. Furthermore, the design process of open channel is considered as a complicated process because it requires more skill, experience, knowledge, and comprehensive education.

Energy and momentum coefficients are involved in many hydraulic equations namely law of conversation of energy and law of conversation of momentum which affect open channels design (Chaudhry, 2008; Luo, 2012) Also, many hydraulic problems can be solved using energy and momentum coefficients, for example: determining the water surface profiles in many computer models such as HEC-RAS (Al-Khatib, 2013).

Velocity distribution is not uniformly distributed over a channel cross section. The velocity is maximum at

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the middle of the cross section at a distance approximately varies between 0.05 and 0.25 of the water depth measured from the water surface and decreases gradually until it reaches zero at the channel boundary due to different channel characteristics such as cross section irregularities, channel alignment, obstructions, and boundary roughness (Akan, 2006; Field et al., 2010).

Due to the variation in velocity along channel cross section, the values of velocity head and the momentum flux are greater than the values computed by using the average velocity, so these values ought to be corrected using the energy and momentum coefficients (Chen, 1990; Field et al., 2000).

2. LITERATURE REVIEW

The energy coefficient, also called Coriolis coefficient (Chanson, 2004), equals the actual energy based on the actual velocity divided by the calculated energy using mean velocity of cross section (Subramanya, 1996). It can be calculated using the following equation based on velocity distribution measurements (Sturm, 2010; Wali, 2013):

$$\alpha = \frac{\int v^3 dA}{V^3 A} = \frac{\sum v^3 dA}{V^3 A} \qquad (1)$$

Where:

α : energy coefficient;
V : mean velocity of cross section;
v : velocity of an elementary area of cross section;
A : total water area of cross section; and
dA : elementary water area of cross section.

Similarly, the momentum coefficient, also called Boussinesq coefficient, equals the actual momentum based on the actual velocity divided by the calculated momentum based on the mean velocity of cross section. It can be calculated using the following equation:

$$\beta = \frac{\int v^2 dA}{V^2 A} = \frac{\sum v^2 dA}{V^2 A} \qquad (2)$$

Where:

β : momentum coefficient.

2.1 Methods for Estimating α and β

The values of α and β can be calculated by different methods which can be summarized as follows (Hulsing et al., 1966; Thandaveswara, 2012):

2.1.1 Arithmetic method

In this method, the energy and momentum coefficients are calculated by using Eqs. (1) and (2) respectively, based on cross section incremental areas and their corresponding mean velocities by using point velocities at different depths. The more point velocities and incremental areas are; the more accuracy of computation is. Therefore, this method saves time but it needs more data.

2.1.2 Graphical method

Also, Eqs. (1) and (2) are used in the calculation process but in this method, the area between each two successive isovels and the mean velocity between these isovels are used. Unfortunately, the isovels drawing method, the magnitude of the areas between each two consecutive isovels, and the point velocity measurements greatly affect the calculation process. Hence, this method is costly and extremely slow.

2.1.3 Grid method

Similarly, the calculation process is mainly based on Eqs. (1) and (2) but the area of flow is divided into small elementary areas like the grid shape, Fig. (1), and each elementary area and its corresponding measured point velocity at its center is used in the calculation. The precision of this method increases with the increase of both the grid elements and the number of point velocities measured, so this method is of great cost and requires more time.

2.1.4 Empirical method

As it has been cited by Li and Hager (1991), based on the known logarithmic velocity distribution, Rehbock gave the following empirical relationships for calculating α and β in 1922:

$$\alpha = 1 + 3\epsilon^2 - 2\epsilon^3 \qquad (3)$$
$$\beta = 1 + \epsilon^2 \qquad (4)$$

For linear velocity distribution, he gave the following relationships for the computation of α and β:

$$\alpha = 1 + \epsilon^2 \qquad (5)$$
$$\beta = 1 + \epsilon^2/3 \qquad (6)$$
$$\epsilon = \frac{V_{max}}{V} - 1 \qquad (7)$$

Where:

ϵ : ratio correlating cross section maximum and mean velocities; and
V_{max} : maximum cross sectional velocity.

Mohanty et al. (2013) suggested the following relationships on the basis of experimental data assuming logarithmic velocity distribution for smooth trapezoidal main channel flanked with two smooth wide symmetrical flood plains:

$$\alpha = 1 - 0.4\,\epsilon^2 + 2.36\,\epsilon^3 \qquad (8)$$
$$\beta = 1 + 0.54\,\epsilon^2 \qquad (9)$$

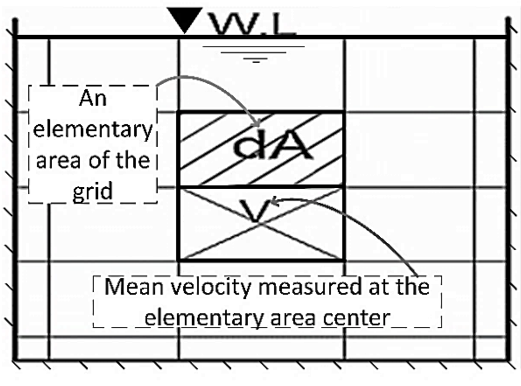

Fig. (1): Grid method for computing α and β, (Thandaveswara, 2012).

2.1.5 Table method

Many researchers have given tables for predicting α and β values based on field and experimental data such as that given by Chow (1959), Table (1).

Table (1): Values of α and β for different channels, (Chow, 1959)

Channels	α			Β		
	min	avg	max	min	avg	max
Regular channels	1.10	1.15	1.20	1.03	1.05	1.07
Natural streams	1.15	1.30	1.50	1.05	1.10	1.17
Ice cover rivers	1.20	1.50	2.00	1.07	1.17	1.33
Flooded rivers	1.50	1.75	2.00	1.17	1.25	1.33

2.2 Research Studies Concerning α and β

In 1922, Rehbock derived the following relationship based on experimental data (Li and Hager, 1991) :

$$\beta = \frac{\alpha+2}{3} \quad (10)$$

Hulsing et al. (1966) related the energy coefficient with Manning's roughness coefficient with the following equation:

$$\alpha = 14.8\,n + 0.884 \quad (11)$$

where:

n : Manning's roughness coefficient.

In 1968, Cobb gave the following correlation between α and β for open channels (Seckin et al., 2009):

$$\alpha = 2.66\beta - 1.66 \quad (12)$$

In 1970, Jagannadha investigated the values of α and β along the hydraulic jump based on experimental data for a flume of 0.6 m width and he found that the maximum values of α and β approximately are 14 and 4 respectively (Thandaveswara, 2012).

Al-Khatib and Gocus (1999) suggested that the values of α and β vary from 1.029 to 1.063 and range between 1.005 and 1.034 respectively, for straight compound cross section flumes.

In case of undirectional non-uniform velocities, the following relationship was recommended by Seckin et al. (2009):

$$(\alpha - 1) = 3(\beta - 1) + \frac{1}{A}\int_A \left(\frac{v-V}{V}\right)^3 dA \quad (13)$$

Seckin et al. (2009) stated that the values of energy and momentum coefficients are 1.094 and 1.034 respectively for compound laboratory flumes with main channel lined with sand and smooth flood plains. In addition, the following relationship between α and β was derived as:

$$\alpha = 2.6777\beta - 1.6748 \quad (14)$$

Kamal and Matin (2010) investigated α and β in a symmetrical rectangular cross section with a dredging area and gave an average value of 1.1233 for α and 1.0514 for β for main channel and an average value of 2.6150 for α and 1.4871 for β for dredging area.

As it has been cited by Thandaveswara (2012) , in case of reverse flow, the values of α and β can be calculated by using any method mentioned before but the velocity in the reverse flow region should be taken with a negative sign.

Al-Khatib (2013) investigated α and β in symmetrical compound smooth cross section flumes, and he deduced that α equals 1.1525 while β equals 1.1261 respectively, as an average value. He also derived the following relationship:

$$\alpha = 0.99\beta + 0.0375 \quad (15)$$

Mohanty et al. (2013) derived the following relationship based on experimental data for straight trapezoidal channels with flood plains:

$$\alpha = 2.898\,\beta - 1.894 \quad (16)$$

3. FIELD WORK

3.1 Site Description

Six reaches from different six man-made Egyptian irrigation canals, that work with their full capacity along the irrigation season without any control structures along their length, were selected. For each reach, ten successive cross sections along it were used for obtaining data starting from cross section No. (1) upstream to cross section No. (10) downstream. Table (1) presents the locations of the cross sections for each canal under study. The selected reaches under study are arranged from the biggest degree to smallest degree as follows:

1) ***First reach***: the first reach was selected from El-Tawfiqy rayyah, first order large-sized canal. This rayyah starts after El-Khaireya group barrages and moves to north direction in the east of Delta region.

2) ***Second reach***: the second reach from the main canal named El-Ibrahimeya takes its water just after Assuit control barrage and moves to north direction to feed the west valley of The Nile River with water.

3) ***Third reach***: the third reach was taken from Bahr-Mois canal which is classified as a main canal and it takes its water from El-Tawfiqy rayyah to feed El-Sharqiya Governorate with water.

4) ***Fourth reach***: the fourth reach was from El-Saideya canal and it is considered as a branch canal which takes its water from El-Ismaileya canal to feed some villages in El-Sharqiya Governorate with water.

5) ***Fifth reach***: the fifth reach was selected from the branch canal called Bahr-Tnah canal. It takes its water from the main canal called El-Mansuriya canal to feed the east of Dahakliya governorate with water.

6) ***Sixth reach***: the sixth reach was chosen from Om-Glagel distributer canal. It takes its water from El-Mansuriya canal to supply the west irrigation regions in Dahakliya governorate with water.

Table (2): locations of cross sections of canals under study (distance in km from the canal intake).

C. S. \ Canal	(1)	(2)	(3)	(4)	(5)	(6)
1(upstream)	0.50	6	0.3	1	0.5	0.3
2	4.50	15	2.8	5	3.5	2.3
3	8.00	20	5.0	7	6.5	4.3
4	10.50	26	7.5	9	9.5	6.3
5	13.50	30	10.0	11	12.5	8.3
6	17.00	35	13.0	13	15.5	10.3
7	20.00	41	15.5	15	18.5	12.3
8	24.50	47	18.5	17	21.5	14.3
9	29.50	50	21.5	19	24.5	16.3
10(downstream)	33.75	53	24.0	21	27.5	18.3

3.2 Field Data Collection

The current meter was used for discharge measurement and the tilting level and levelling staff were used for water surface elevations measurements. The cross section was divided into many strips that were bounded by two verticals, Fig. (2). The velocity at each vertical was measured at 0.2, 0.5, and 0.8 of the vertical depth from the water surface for deep zones. For shallow zones less than one meter, the velocity was measured at 0.6 of the vertical depth from the water surface. Then, the average velocity at each vertical for deep zones was calculated as:

$$\mathbf{V_{avg}} = \frac{\mathbf{V_a * 0.2d + \left(\frac{V_a+V_b}{2}\right) * 0.3d + \left(\frac{V_b+V_c}{2}\right) * 0.3d + \left(\frac{V_c}{2}\right) * 0.2d}}{\mathbf{d}} \quad (17)$$

Where:

V_{avg} : vertical average velocity;
V_a : velocity at 0.2 of the total depth;
V_b : velocity at 0.5 of the total depth;
V_c : velocity at 0.8 of the total depth; and
d : total water depth at each vertical.

For shallow zones, the mean velocity was measured from the water surface at 0.6 of the water depth.

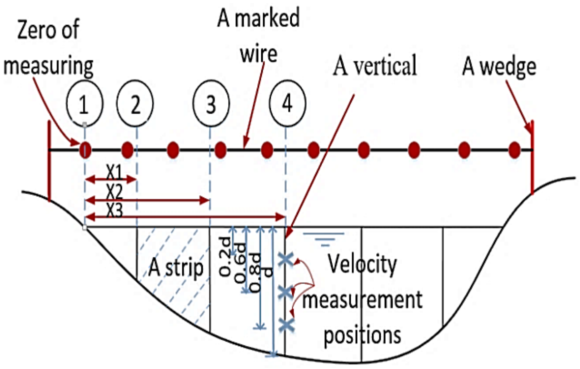

Fig. (2): Velocity measurement process by using the current meter.

4. DATA ANALYSIS

Using the velocity measurements data, the velocity-area method was used to calculate the discharge by using the following equation:

$$Q = \sum_{i=1}^{N} V_i A_i \quad (18)$$

Where:

Q : discharge;
A_i : water area of each strip;
V_i : average velocity of each strip; and
N : number of strips.

Based on the velocity measurements, the geometrical properties of the channel cross section were measured such as top width (T), mean depth (D), wetted perimeter (P), max depth (Y_{max}), and mean width (B) and flow parameters such as Froude number (Fr), Reynolds number (Re), and shear velocity (V*) were calculated from the following
equations:

$$R_e = VR_h/v \quad (19)$$

$$F_r = V/\sqrt{gD} \quad (20)$$

$$V^* = \sqrt{gR_hS_f} \quad (21)$$

Where:

ν : kinematic viscosity; and
S_f : friction slope.

The average water surface slopes for the canals under study were calculated using linear regression analysis for the measured water surface elevations which were measured for each canal for 3 successive days at most to approximately achieve steady state condition and to cover the fluctuation in water surface elevations.

Manning's roughness coefficient values were determined using Manning's roughness equation:

$$V = \frac{1}{n} R_h^{2/3} \sqrt{S_f} \quad (22)$$

Manning's equation in this form is used for steady uniform flow. In order to apply Manning's roughness coefficient for steady varied flows, the term named friction slope in Manning's roughness coefficient is modified to reflect the boundary friction losses by the following equations (Dalrymple and Benson, 1989):

$$S_f = \frac{\Delta h + (\Delta h_V/2)}{L} \quad \text{(for positive } \Delta h_V) \quad (23)$$

$$S_f = \frac{\Delta h + \Delta h_V}{L} \quad \text{(for negative } \Delta h_V) \quad (24)$$

Where:

Δh : water surface elevation difference;
Δh_V : upstream velocity head minus downstream velocity head; and
L : length of the reach.

As a result, the term Δh_V is always very small and may be neglected so the friction slope is approximately equal to the water slope (S_w).

5. RESULTS AND DISCUSSION

In order to calculate the values of α and β, the arithmetic method was used for its precision than the other methods as mentioned before. An excel program was prepared to calculate the values of α and β for all the 60 cross sections of canals under study, sample is given in Table (3). The results of data measurements and analysis are shown in tables (4) and (5).

It is marked from tables (4) and (5) that ($\alpha > \beta > 1$) as mentioned by Al-Khatib (2013). In addition, referring to the computed values of α and β for the six canals, the maximum, minimum, and average values of α and β for each canal and for all canals under study are listed in table (6). It is clear from the table that the maximum value of α from canals under study is 1.48, the minimum value is 1.10, and the average value is 1.30. It is also evident from the table that the maximum value of β for canals under study is 1.18, the minimum value is 1.04, and the average value is 1.11. In conclusion, all the previous mentioned values for α and β approximately match with the values given by Chow (1957).

Moreover, it is obvious from table (6) that the average value of α or β mainly depends on the degree of the canal. The maximum average values of α and β are 1.39 and 1.15 respectively, for the first order large-sized canal called El-Tawfiqy rayyah No. (1). Furthermore, the average values of α and β decrease with the decrease of the canal degree until the minimum average value are 1.19 and 1.07 receptively for Om-Glagel canal No. (6), the smallest degree of canals under study. In contrast, canal No. (2), the main canal, has discharge and dimension values larger than that for canal No. (1) but it has average values of α and β smaller than that for canal No. (1). This could be explained as the average values of α and β increase with the increase of the canal degree and may not increase with the increase of the discharge.

Fig. (3) shows poor correlation between both α and β; and Q. It is also apparent from figs. (4), (5), and (6) that there is no correlation between both α and β; and mean velocity (V), and maximum velocity (V_{max}) while this correlation is significant for the shear velocity (V^*) as it is related to the friction along the cross section which affect the velocity distribution and consequently, the values of α and β.

The maximum values of α and β are given in table (6) for canals No. (3) and (4) not canal No. (1). Also, the minimum values of α and β are recorded for canals No. (4) and (6). As a result, there may be no relationship between the maximum and minimum values of α and β; and the canal degree.

Distance from left bank: (m)	Vertical depth: (m)	Width of each strip :<wi>: (m)	Average water depth of each strip:<di> (m)	Area of each strip <Ai> = wi*di :(m2)	Vertical average velocity <Vavg>: (m/s)	Average velocity for each strip <Vi>: (m/s)	Discharge for each strip <Q> = Ai *Vi :(m3/s)	Calculate: (Ai*Vi2)	Calculate :(Ai*Vi3)
0.00	0.00	8.00	1.85	14.80	0.00	0.13	1.92	0.25	0.03
8.00	3.70	8.00	3.90	31.20	0.26	0.57	17.93	10.30	5.92
16.00	4.10	8.00	4.40	35.20	0.89	0.98	34.67	34.14	33.63
24.00	4.70	8.00	5.05	40.40	1.08	1.21	48.89	59.17	71.61
32.00	5.40	8.00	5.70	45.60	1.34	1.32	60.20	79.48	104.94
40.00	6.00	8.00	5.75	46.00	1.30	1.22	55.90	67.93	82.54
48.00	5.50	8.00	4.85	38.80	1.13	0.99	38.22	37.65	37.08
56.00	4.20	8.00	3.85	30.80	0.84	0.56	17.40	9.83	5.55
64.00	3.50	8.00	1.75	14.00	0.29	0.14	2.03	0.29	0.04
72.00	0.00				0.00				
∑				296.80			277.16	299.04	341.35
					V			β	α
					0.933815			1.155	1.412

Table (3): An excel program for calculating α and β values using the arithmetic method (sample of a cross section calculations).

Table (4): Data measurements and analysis for canals No. (1), (2) and (3)

Canal	C. S.	Km.	Cross section properties							S_w	Q (m^3/s)	V (m/s)	V_{max} (m/s)	ϵ	V^* (m/s)	R_e	F_r	n	α	β
			A (m^2)	T (m)	D (m)	P (m)	R (m)	Y_{max} (m)	B (m)											
El-Tawfiqy rayyah No. (1)	1	0.5	296.8	72.0	4.12	73.78	4.02	6.00	49.47	0.000086	277.2	0.93	1.69	0.81	0.366	3756376.8	0.147	0.0251	1.412	1.155
	2	4.5	277.6	70.0	3.97	71.54	3.88	5.80	47.86		252.9	0.91	1.63	0.79	0.353	3535495.3	0.146	0.0251	1.397	1.149
	3	8.0	271.4	69.0	3.93	70.71	3.84	5.70	47.61		238.1	0.88	1.64	0.87	0.349	3367170.1	0.141	0.0259	1.457	1.169
	4	10.5	259.1	67.0	3.87	68.56	3.78	5.50	47.11		227.1	0.88	1.63	0.86	0.344	3312452.9	0.142	0.0257	1.422	1.161
	5	13.5	246.3	64.0	3.85	65.79	3.74	5.30	46.47		214.2	0.87	1.61	0.85	0.341	3255851.1	0.142	0.0257	1.439	1.160
	6	17.0	227.6	62.0	3.67	63.44	3.59	5.20	43.77		206.1	0.91	1.56	0.72	0.326	3248422.6	0.151	0.0240	1.330	1.126
	7	20.0	219.4	61.0	3.60	62.46	3.51	5.00	43.88		191.6	0.87	1.63	0.87	0.320	3067532.0	0.147	0.0245	1.449	1.159
	8	24.5	214.4	60.0	3.57	61.46	3.49	4.90	43.76		177.5	0.83	1.50	0.81	0.317	2888038.3	0.140	0.0258	1.384	1.142
	9	29.5	190.4	58.0	3.28	59.22	3.22	4.80	39.67		166.3	0.87	1.51	0.73	0.293	2807725.2	0.154	0.0231	1.265	1.099
	10	33.75	181.5	56.0	3.24	57.58	3.15	4.70	38.61		146.6	0.81	1.45	0.80	0.287	2545497.7	0.143	0.0247	1.370	1.133
El-Ibrahimeya canal No. (2)	1	6.0	539.0	93.0	5.80	95.63	5.64	8.00	67.37	0.000058	489.7	0.91	1.64	0.80	0.421	5120513.6	0.121	0.0265	1.396	1.147
	2	15.0	500.5	89.0	5.62	91.75	5.45	7.90	63.35		445.6	0.89	1.59	0.78	0.408	4857036.5	0.120	0.0265	1.389	1.143
	3	20.0	477.8	87.0	5.49	89.71	5.33	7.70	62.05		433.5	0.91	1.57	0.73	0.398	4832708.5	0.124	0.0256	1.336	1.123
	4	26.0	452.7	85.0	5.33	87.48	5.17	7.50	60.36		415.2	0.92	1.56	0.70	0.387	4746651.5	0.127	0.0248	1.298	1.111
	5	30.0	436.0	84.0	5.19	86.14	5.06	7.40	58.92		388.2	0.89	1.54	0.73	0.378	4506521.1	0.125	0.0252	1.324	1.119
	6	35.0	422.1	82.0	5.15	84.41	5.00	7.20	58.62		375.0	0.89	1.52	0.71	0.374	4442774.5	0.125	0.0251	1.320	1.119
	7	41.0	411.3	80.0	5.14	82.51	4.98	7.10	57.93		360.0	0.88	1.53	0.75	0.372	4363625.2	0.123	0.0254	1.332	1.122
	8	47.0	400.2	79.0	5.07	81.95	4.88	6.90	58.00		327.5	0.82	1.51	0.84	0.365	3996036.4	0.116	0.0268	1.434	1.157
	9	50.0	382.4	76.0	5.03	78.52	4.87	6.80	56.24		313.0	0.82	1.50	0.84	0.364	3986531.8	0.117	0.0267	1.430	1.158
	10	53.0	373.6	75.0	4.98	77.57	4.82	6.60	56.61		294.8	0.79	1.51	0.91	0.360	3800043.1	0.113	0.0275	1.473	1.172
Bahr-Mois canal No. (3)	1	0.3	175.1	55.0	3.18	56.80	3.08	4.40	39.80	0.000093	131.3	0.75	1.42	0.90	0.292	2311396.8	0.134	0.0272	1.482	1.177
	2	2.8	159.2	52.0	3.06	53.73	2.96	4.30	37.01		126.9	0.80	1.40	0.76	0.280	2361886.0	0.146	0.0249	1.347	1.128
	3	5.0	148.9	51.0	2.92	52.09	2.86	4.10	36.32		115.1	0.77	1.41	0.82	0.270	2210226.6	0.144	0.0251	1.369	1.136
	4	7.5	137.5	48.0	2.86	49.41	2.78	4.00	34.36		111.4	0.81	1.35	0.67	0.263	2254730.9	0.153	0.0235	1.235	1.088
	5	10.0	128.3	47.0	2.73	48.29	2.66	3.80	33.75		103.1	0.80	1.33	0.66	0.251	2135210.4	0.155	0.0230	1.208	1.078
	6	13.0	118.5	45.0	2.63	46.29	2.56	3.70	32.03		84.7	0.72	1.29	0.80	0.242	1830649.2	0.141	0.0252	1.378	1.135
	7	15.5	109.5	42.0	2.61	43.10	2.54	3.60	30.40		76.8	0.70	1.30	0.85	0.240	1781550.2	0.139	0.0256	1.411	1.150
	8	18.5	96.2	38.0	2.53	39.18	2.46	3.40	28.29		71.1	0.74	1.23	0.66	0.232	1815836.4	0.148	0.0237	1.268	1.098
	9	21.5	91.6	37.0	2.48	38.31	2.39	3.20	28.63		68.4	0.75	1.25	0.67	0.226	1786600.9	0.152	0.0231	1.224	1.084
	10	24.0	85.3	35.0	2.44	36.50	2.34	3.10	27.50		63.2	0.74	1.23	0.66	0.221	1732574.6	0.152	0.0229	1.217	1.084

Table (5): Data measurements and analysis for canals No. (4), (5), and (6).

Canal	C. S.	Km.	Cross section properties							S_w	Q (m^3/s)	V (m/s)	V_{max} (m/s)	ϵ	V^* (m/s)	R_e	F_r	n	α	β
			A (m^2)	T (m)	D (m)	P (m)	R (m)	Y_{max} (m)	B (m)											
El-Saideya canal No. (4)	1	1.0	78.3	33.0	2.37	34.26	2.28	3.0	26.08	0.000111	60.7	0.78	1.32	0.71	0.236	1771505.5	0.161	0.0236	1.267	1.098
	2	5.0	72.2	31.0	2.33	32.45	2.22	3.0	24.07		52.2	0.72	1.37	0.89	0.230	1609759.5	0.151	0.0248	1.480	1.179
	3	7.0	67.9	30.0	2.26	31.29	2.17	2.9	23.40		50.6	0.75	1.31	0.76	0.224	1616136.8	0.158	0.0237	1.288	1.105
	4	9.0	59.6	29.0	2.05	30.03	1.98	2.7	22.06		43.3	0.73	1.12	0.54	0.205	1442362.7	0.162	0.0229	1.121	1.046
	5	11.0	56.4	28.0	2.01	29.01	1.94	2.6	21.69		39.0	0.69	1.20	0.73	0.201	1345598.8	0.156	0.0237	1.301	1.111
	6	13.0	52.7	27.0	1.95	28.02	1.88	2.5	21.06		37.4	0.71	1.09	0.54	0.194	1336080.0	0.163	0.0226	1.102	1.039
	7	15.0	47.9	25.0	1.92	26.14	1.83	2.5	19.16		29.5	0.62	1.10	0.79	0.189	1129441.0	0.142	0.0256	1.347	1.121
	8	17.0	44.9	24.0	1.87	25.22	1.78	2.4	18.71		28.2	0.63	1.12	0.78	0.184	1116275.4	0.146	0.0247	1.355	1.126
	9	19.0	39.0	23.0	1.70	23.90	1.63	2.3	16.96		24.3	0.62	1.05	0.69	0.169	1016220.8	0.153	0.0234	1.213	1.080
	10	21.0	34.8	21.0	1.66	21.95	1.59	2.2	15.82		20.1	0.58	1.05	0.82	0.164	915801.7	0.143	0.0248	1.357	1.128
Bahr-Tnah canal No. (5)	1	0.5	30.7	19.5	1.57	20.41	1.50	2.15	14.26	0.000129	19.4	0.63	1.05	0.66	0.167	951780.9	0.161	0.0235	1.236	1.091
	2	3.5	29.8	19.0	1.57	19.92	1.50	2.10	14.19		19.3	0.65	1.02	0.58	0.167	968862.6	0.165	0.0229	1.140	1.053
	3	6.5	28.5	18.5	1.54	19.49	1.46	2.00	14.23		18.4	0.65	1.03	0.60	0.163	945335.8	0.167	0.0226	1.159	1.063
	4	9.5	27.6	18.0	1.53	19.02	1.45	1.95	14.13		17.5	0.64	1.07	0.68	0.161	921968.9	0.164	0.0228	1.194	1.078
	5	12.5	23.5	17.0	1.38	17.78	1.32	1.85	12.69		12.4	0.53	0.99	0.87	0.147	695682.8	0.143	0.0259	1.391	1.147
	6	15.5	21.4	16.5	1.30	17.19	1.25	1.75	12.25		11.8	0.55	0.95	0.72	0.139	686971.9	0.154	0.0239	1.301	1.108
	7	18.5	19.0	15.5	1.23	16.15	1.18	1.70	11.17		10.3	0.54	0.89	0.64	0.131	636718.5	0.156	0.0234	1.227	1.082
	8	21.5	16.9	14.0	1.20	14.78	1.14	1.65	10.21		8.1	0.48	0.94	0.94	0.127	551391.9	0.141	0.0256	1.374	1.142
	9	24.5	13.8	12.0	1.15	12.73	1.08	1.60	8.59		7.6	0.55	0.87	0.59	0.120	593346.1	0.164	0.0218	1.146	1.056
	10	27.5	12.2	11.5	1.06	12.17	1.00	1.50	8.11		6.1	0.50	0.80	0.60	0.111	500382.3	0.155	0.0227	1.165	1.066
Om-Glagel canal No. (6)	1	0.3	10.4	10.0	1.04	10.76	0.96	1.35	7.67	0.000147	4.9	0.48	0.96	1.02	0.114	458491.7	0.150	0.0248	1.351	1.139
	2	2.3	9.1	9.5	0.95	10.26	0.88	1.20	7.55		4.5	0.50	0.81	0.63	0.105	440865.9	0.163	0.0224	1.155	1.063
	3	4.3	7.6	9.0	0.85	9.54	0.80	1.15	6.62		3.2	0.42	0.87	1.09	0.095	332620.3	0.145	0.0250	1.376	1.150
	4	6.3	6.5	8.5	0.76	8.98	0.72	1.10	5.89		2.7	0.42	0.69	0.63	0.086	305660.0	0.155	0.0230	1.187	1.071
	5	8.3	5.7	8.0	0.71	8.42	0.67	1.05	5.38		2.5	0.45	0.71	0.59	0.080	298478.5	0.169	0.0209	1.100	1.040
	6	10.3	4.6	7.5	0.62	7.83	0.59	0.90	5.13		1.8	0.40	*–	–	0.070	232963.4	0.161	0.0215	1.175	1.068
	7	12.3	3.3	6.0	0.54	6.32	0.51	0.80	4.06		1.2	0.38	*–	–	0.061	195566.4	0.165	0.0205	1.129	1.050
	8	14.3	2.1	5.0	0.41	5.23	0.39	0.75	2.75		0.7	0.34	*–	–	0.047	134059.8	0.169	0.0192	1.089	1.037
	9	16.3	1.7	4.5	0.37	4.72	0.35	0.70	2.39		0.5	0.30	*–	–	0.042	105208.9	0.155	0.0205	1.118	1.055
	10	18.3	1.5	4.0	0.36	4.21	0.34	0.60	2.42		0.4	0.28	*–	–	0.041	95932.6	0.148	0.0214	1.140	1.059

*: Maximum velocity was not measured because the velocity was only measured at 0.6 of the water depth from the water surface (Shallow depths).

Table (6): Maximum, minimum, and average values of α and β for canals under study.

Canal	A			β		
	min	avg	max	min	avg	max
(1)	1.26	1.39	1.46	1.10	1.15	1.17
(2)	1.30	1.37	1.47	1.11	1.14	1.17
(3)	1.21	1.31	1.48	1.08	1.12	1.18
(4)	1.10	1.28	1.48	1.04	1.10	1.18
(5)	1.14	1.23	1.39	1.05	1.09	1.15
(6)	1.10	1.19	1.37	1.04	1.07	1.14
All*	1.10	1.30	1.48	1.04	1.11	1.18

*: For all canals under study.

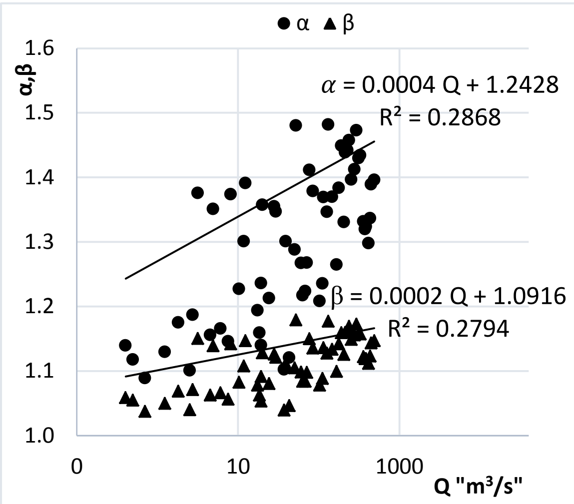

Fig. (3): Correlation between both α and β; and Q for canals under study.

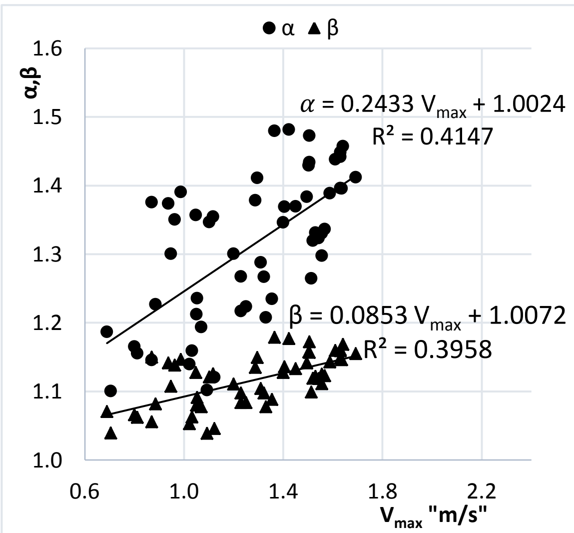

Fig. (4): Correlation between both α and β; and V_{max} for canals under study.

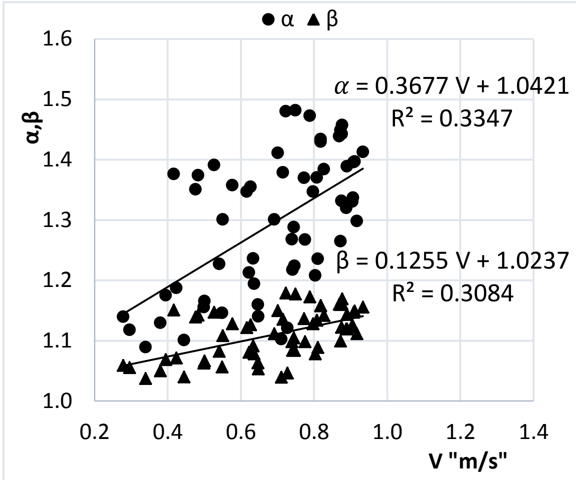

Fig. (5): Correlation between both α and β; and V for canals under study.

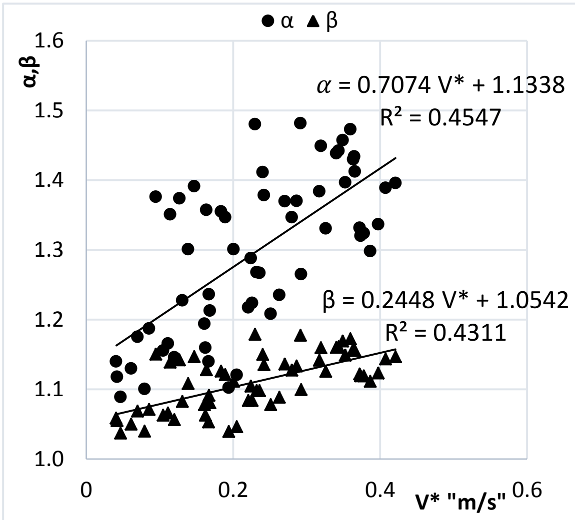

Fig. (6): Correlation between both α and β; and V^* for canals under study.

A regression analysis has been carried out between α and β for all canals under study as demonstrated in figs. (7) through (12). A linear regression relationship between α and β is already recommended as reported by Seckin et al. (2009) because the determination coefficient approximately equals 1 for all figures. It is noticeable from these figures that all data are in a good agreement and the slope of the regression line increases with the increase of the canal degree. The following general relationship for all reaches can be determined from fig. (13) for β varies between 1.037 and 1.079:

$$\alpha = 2.80\,\beta - 1.82 \qquad R^2 = 0.99 \qquad (25)$$

A comparison between Eq. (25) and the mentioned equations in the study is shown in fig. (14). It is clear from the figure that Eq. (25) gives a good agreement with Cobb and Seckin et al. equation because Cobb equation was basically derived for open channels and Skin et al. equation was derived for compound channel with sand main channel which is similar to the study canals boundary material which is silty sand. Al-Khatib, Rehbock, and Mohanty et al. equations deviate from Eq.

(25) as they were derived for different models and conditions.

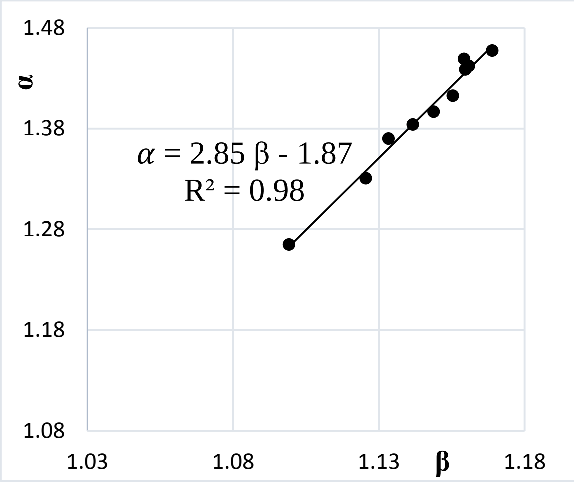

Fig. (7): Relationship between α and β for El-Tawfiqy rayyah.

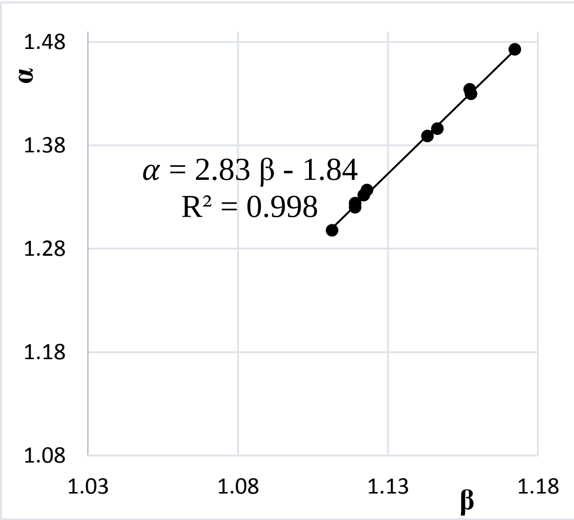

Fig. (8): Relationship between α and β for El-Ibrahimeya canal.

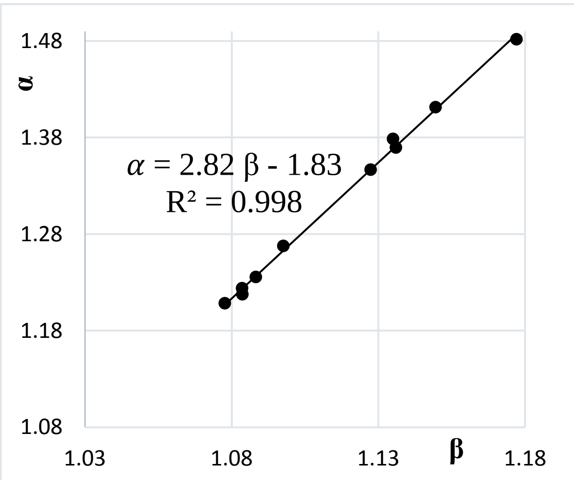

Fig. (9): Relationship between α and β for Bahr-Mois canal.

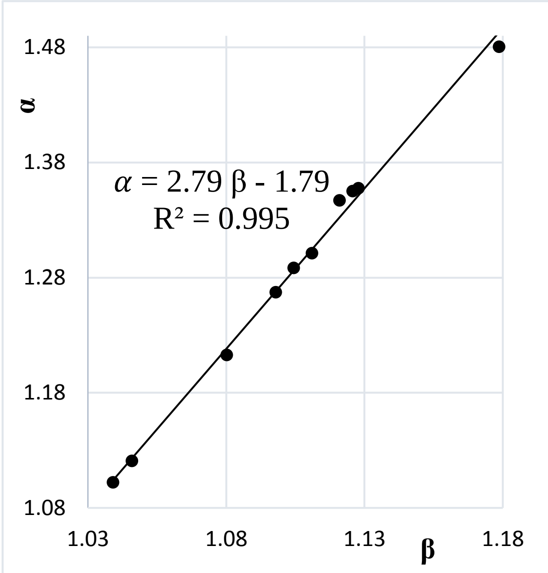

Fig. (10): Relationship between α and β for El-Saideya canal

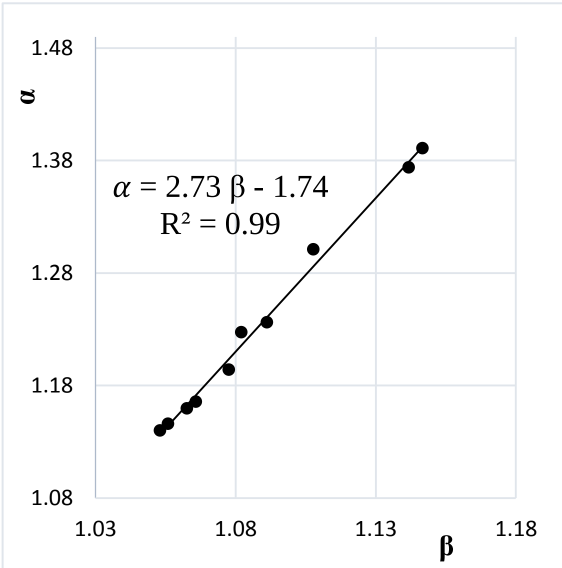

Fig. (11): Relationship between α and β for Bahr-Tnah canal.

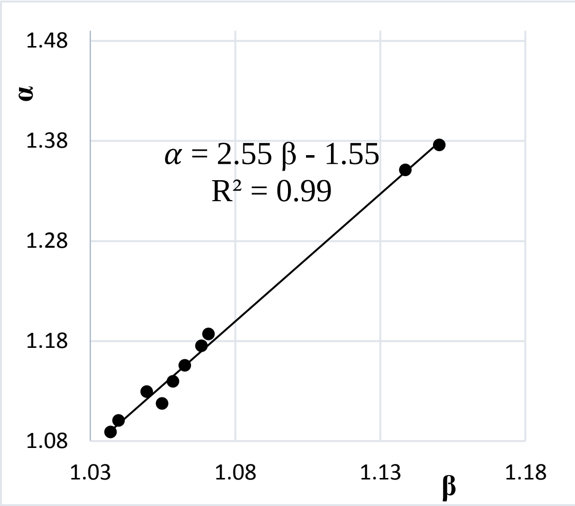

Fig. (12): Relationship between α and β for Om-Glagel canal.

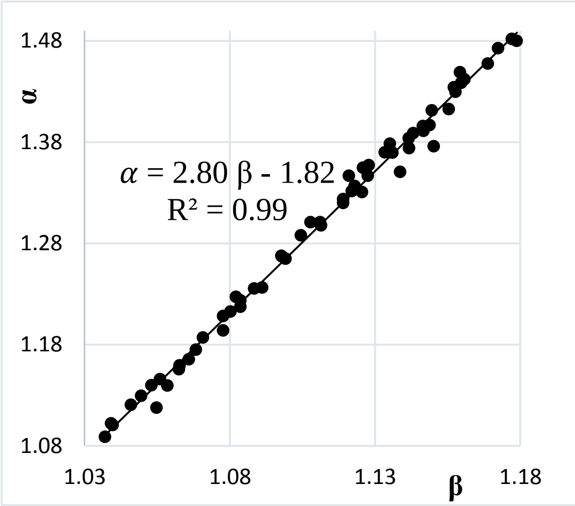

Fig. (13): Relationship between α and β for canals under study

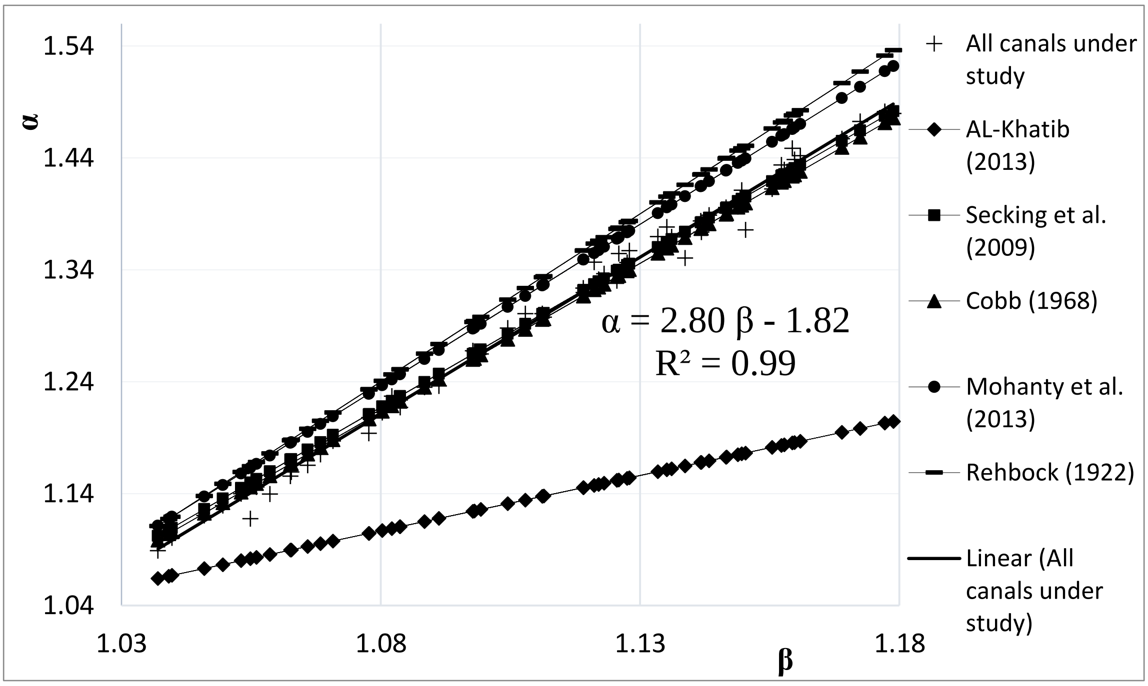

Fig. (14): Comparison between the derived relationship between α and β for all canals under study and that given in the literature.

The relationships between (α and ϵ); and (β and ϵ) for the canals under study are compared with the relationships given by Rehbock and Mohanty et al. equations, Figs. (15) and (16), respectively. The results show that Rehbock and Mohanty et al. equations are not applicable to use for the canals under study because they were derived for different models and conditions. Therefore, the following relationships could be used for ϵ ranges from 0.54 to 1.09:

$$\alpha = -5.65\,\epsilon^3 + 11.45\,\epsilon^2 - 6.55\,\epsilon + 2.19 \qquad R^2 = 0.93 \qquad (26)$$

$$\beta = -0.68\,\epsilon^2 - 1.35\,\epsilon + 0.50 \qquad R^2 = 0.91 \qquad (27)$$

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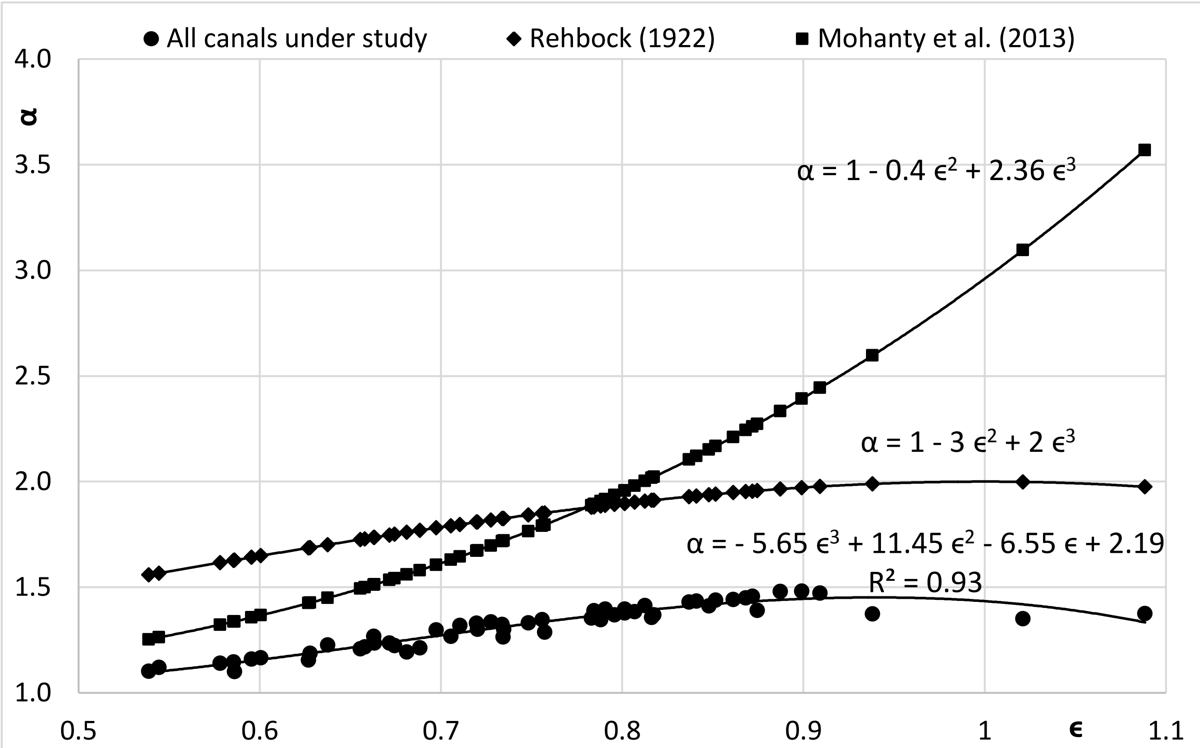

Fig. (15): Comparative relationships between α and ϵ for canals under study and Rehboch, and Mohanty et al. equations.

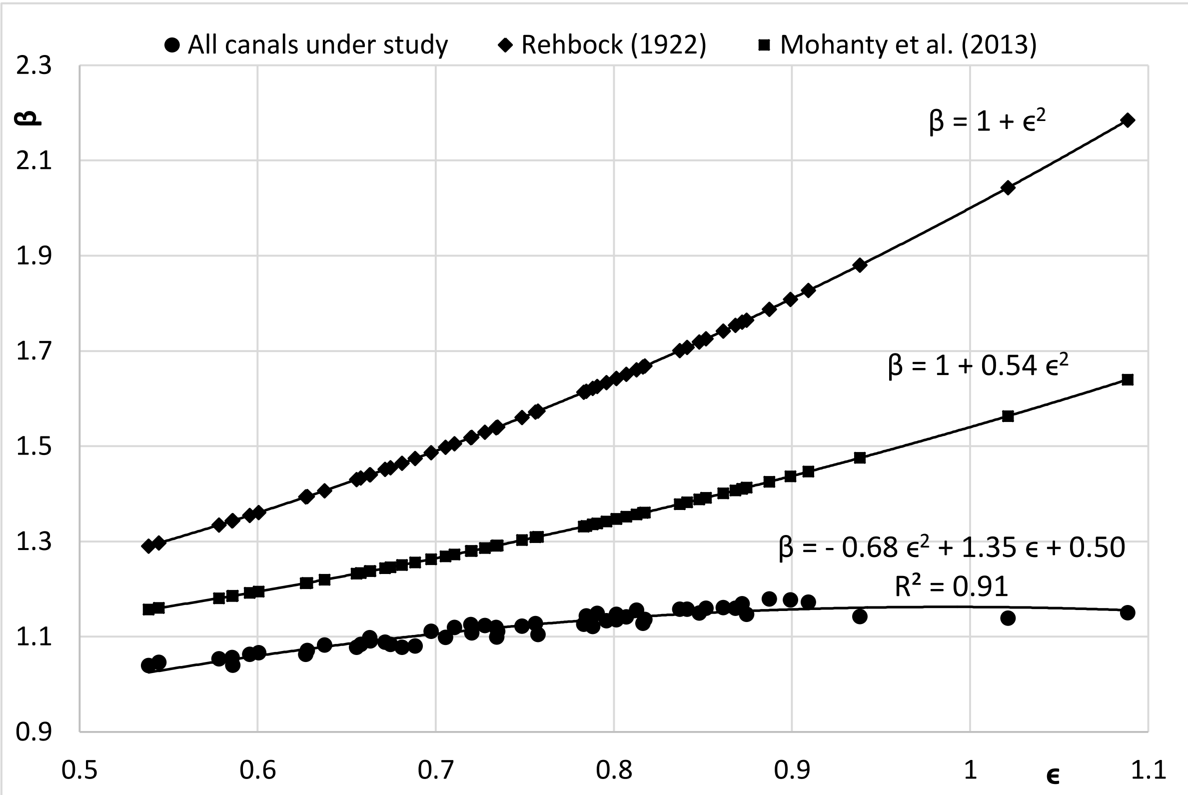

Fig. (16): Comparative relationships between β and ϵ for canals under study and Rehboch, and Mohanty et al. equations

On the other hand, a comparison between the relationship of α and n for the canals under study and Hulsing et al. equation has been carried out, Fig. (17). It is clear from the figure that Hulsing et. al equation is not valid to use for the study canals for the different assumptions and conditions that it was based on, so the following equation is recommended to be used for n varies between 0.0192 and 0.0275:

$$\alpha = 55.60\, n + 0.04 \qquad R^2 = 0.81 \tag{28}$$

Also, second order polynomial relationships are recommended, Fig. (18), and given by:

$$\alpha = -1.8 * 10^6 \, n^3 + 1.3 * 10^5 \, n^2 - 3 * 10^3 \, n + 23.77 \quad R^2 = 0.85 \tag{29}$$

$$\beta = -6.2 * 10^5 \, n^3 + 4.4 * 10^4 \, n^2 - 1 * 10^3 \, n + 8.67 \quad R^2 = 0.82 \tag{30}$$

These relationships could be applied for n ranges from 0.0192 to 0.0275

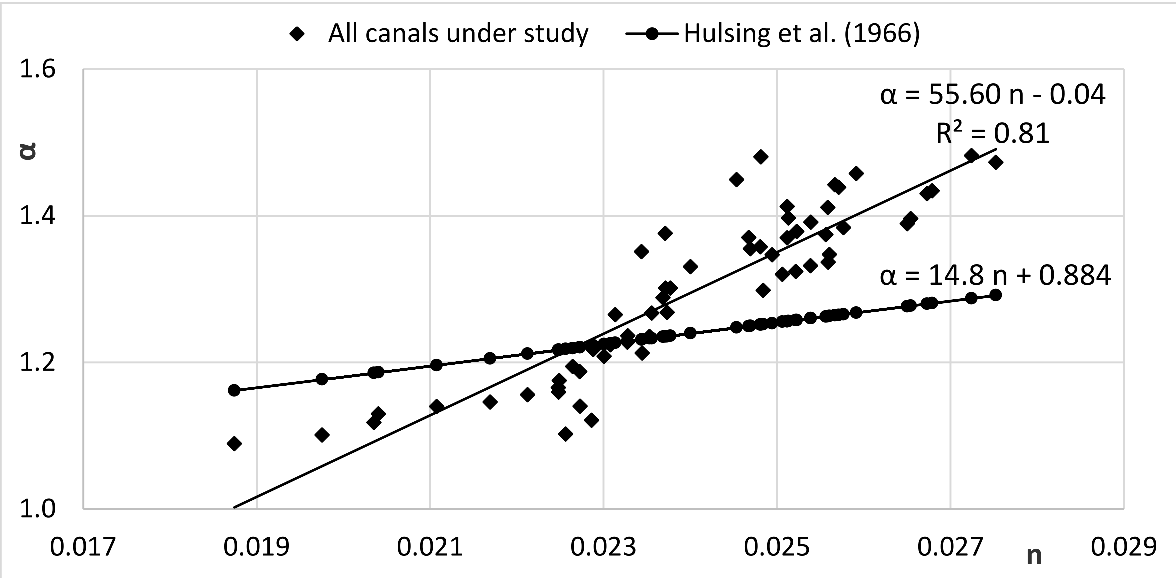

Fig. (17): Comparative relationships between α and n for canals under study and Hulsing et al. equation.

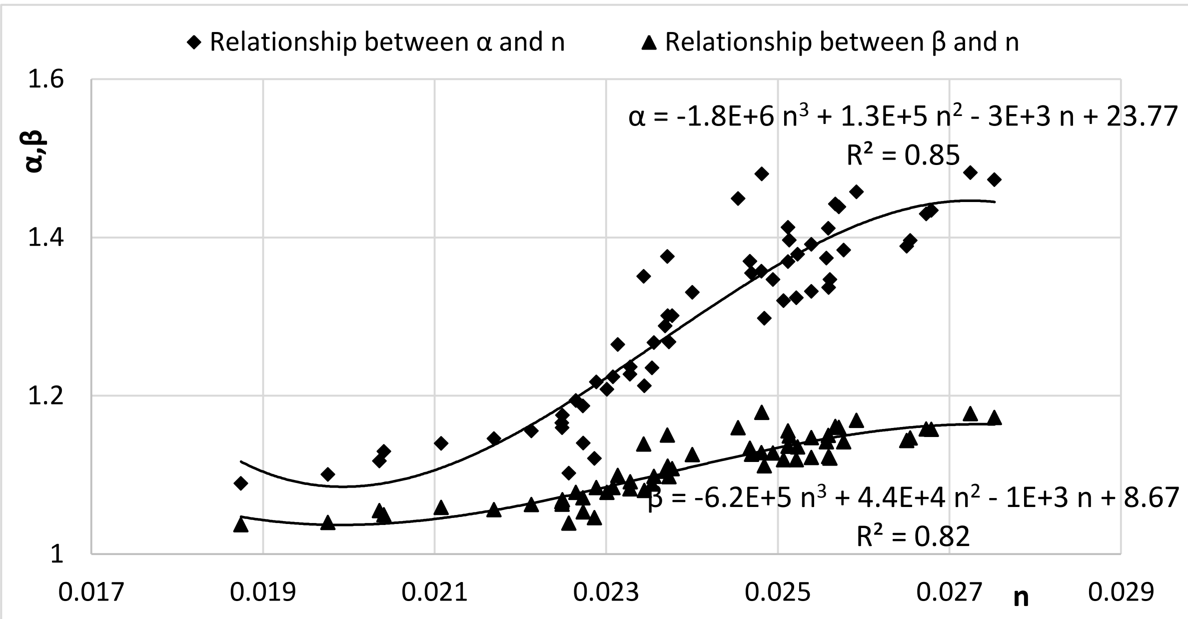

Fig. (18): Relationships between both α and β; and n for canals under study

Correlation relationships between both α and β; and cross sections' properties of canals under study including: A, T, R_h, P, B, D, and Y_{max} are shown in figs. (19) through (25). It is clear from the figures that the values of α and β are not strongly related to the cross section properties and it is logical because the velocity distribution is the main effective of the computation of α and β. In adition, the horizontal dimensions of the cross section namely T, B, and P may be more efective than the vertical dimensions such as R_h, D, and Y_{max} on the values of α and β as large coefficients of determination are given.

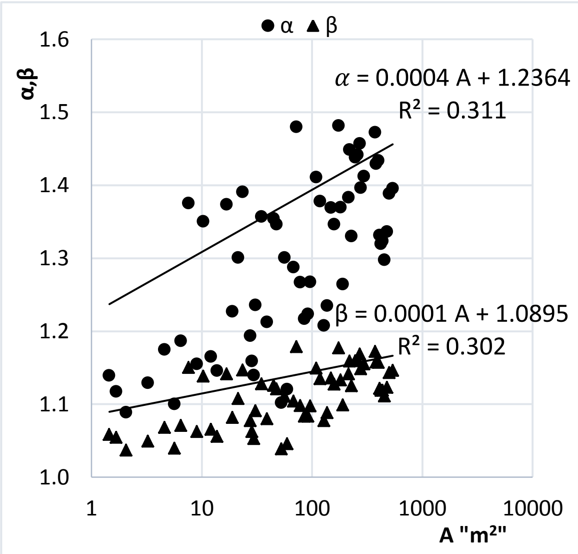

Fig. (19): Correlation between both α and β; and A for canals under study.

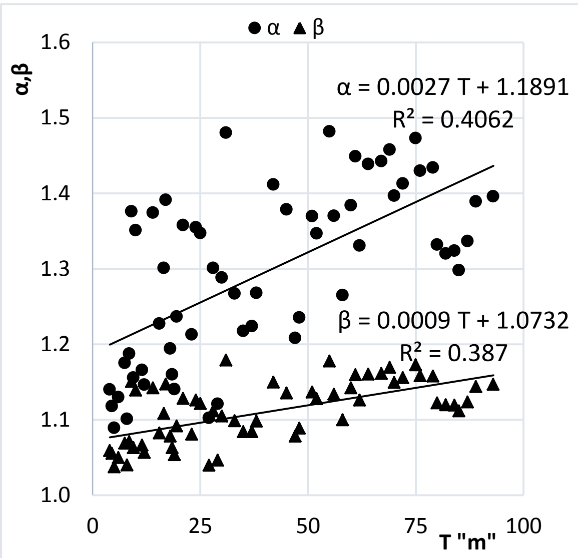

Fig. (20): Correlation between both α and β; and T for canals under study.

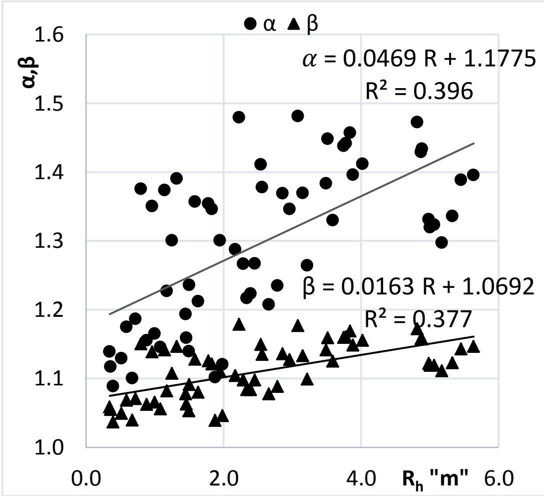

Fig. (21): Correlation between both α and β; and R for canals under study.

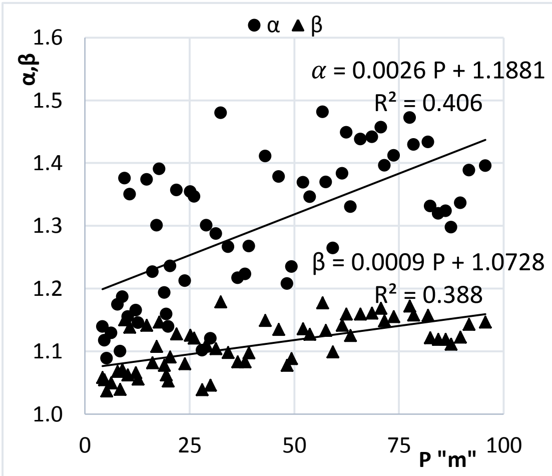

Fig. (22): Correlation between both α and β; and P for canals under study.

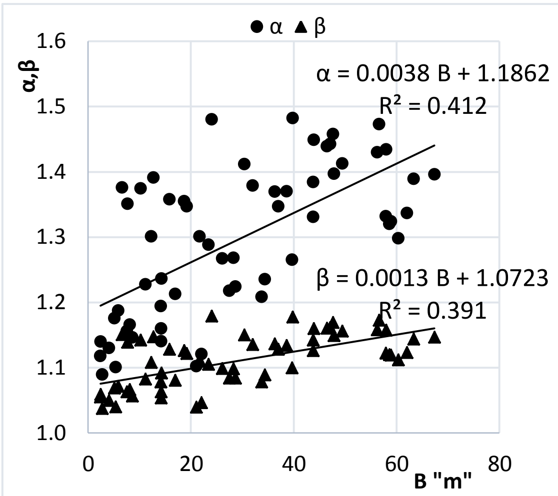

Fig. (23): Correlation between both α and β; and B for canals under study.

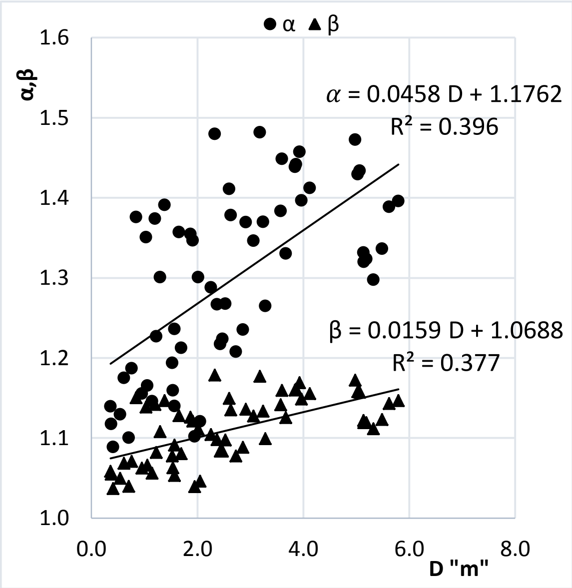

Fig. (24): Correlation between both α and β; and D for canals under study.

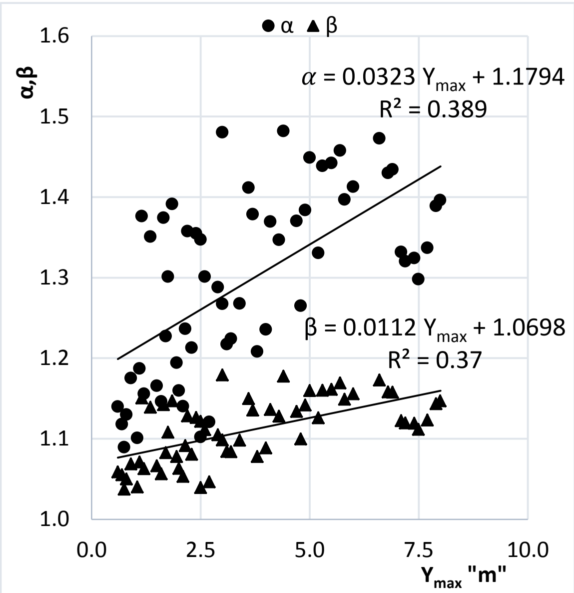

Fig. (25): Correlation between both α and β; and Ymax for canals under study.

Also, relationships between both α and β; and Froude number (F_{r}) and Reynolds (R_e) are demonstrated in fig. (26) and fig. (27) respectively. It is shown from the figures that the second order polynomial regression relationships are recommended to give higher values of coefficient of determination. Moreover, it is obvious from the figures that F_r is more effective than R_e on the computation of the values of α and β and it is rational because the Froude number is dominant in open channel flow due to gravitational force. The relationship between both α and β; and F_r can be considered as follows:

$$\alpha = -181.32\ \mathrm{F_r}^2 + 45.87\ \mathrm{F_r} - 1.49 \qquad \mathrm{R}^2 = 0.66 \tag{34}$$

$$\beta = -65.35\ \mathrm{F_r}^2 + 16.57\ \mathrm{F_r} + 0.10 \qquad \mathrm{R}^2 = 0.66 \tag{35}$$

for F_r ranges between 0.113 and 0.169

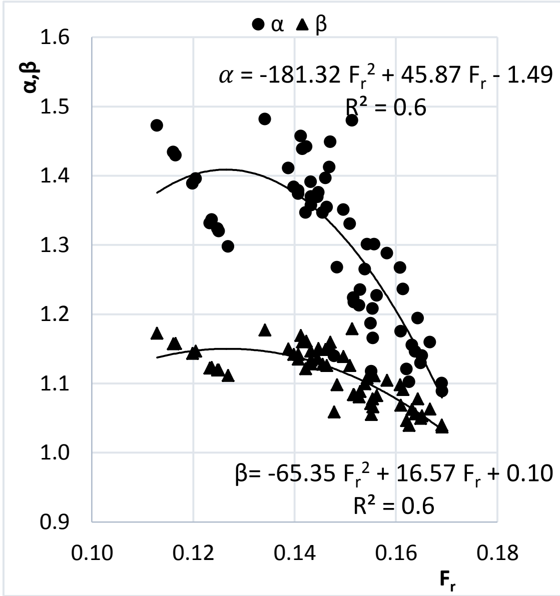

Fig. (26): Correlation between both α and β; and Fr for canals under study.

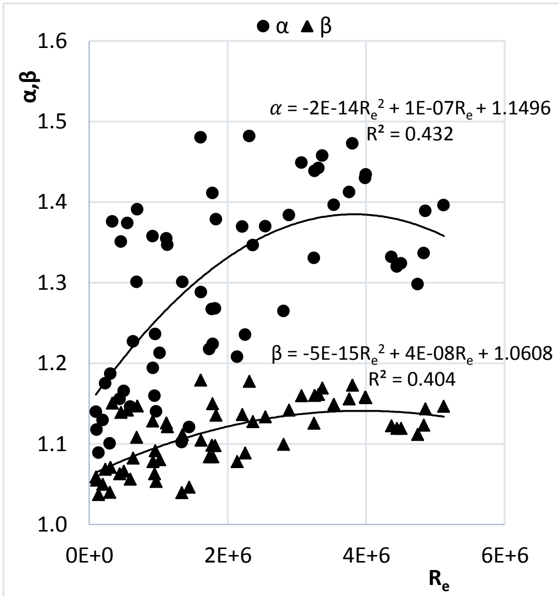

Fig. (27): Correlation between both α and β; and Re for canals under study.

6. CONCLUSIONS

The following conclusions are drawn from this research work:

- The average values of α and β for all canals under study equal 1.30 and 1.11 respectively which match with the table values for natural channels given by Chow (1957) and can be used for practical usages.
- The average values of α and β for every canal under study are given and the average values of α and β increase with the increase of the canal degree while the maximum and minimum values of α and β for all canals under study may not depend on the degree of the canal.
- α and β values are not strongly related to the mean velocity, maximum velocity, shear velocity, and

the discharge as they mainly depend on the shape of velocity distribution.

- The shear velocity slightly affects the values of α and β more than the maximum and mean velocity as it depends on the friction slope which strongly affects the velocity distribution.
- The relationship between α and β is considered as a linear regression relationship and the slope of the line increases with the increase of the channel degree.
- A general relationship between α and β for all reaches of canals under study can be written as:

$$\alpha = 2.80\beta - 1.82$$

- General relationships between both α and β; and ϵ can be considered for canals under study respectively as:

$$\alpha = -5.65\,\epsilon^3 + 11.45\,\epsilon^2 - 6.55\,\epsilon + 2.19$$
$$\beta = -0.68\,\epsilon^2 - 1.35\,\epsilon + 0.50$$

- A general linear regression relationship between α and n can be given for all canals under study as:

$$\alpha = 55.60\,n + 0.03$$

- Second order polynomial relationships are recommended between both α and β; and n:

$$\alpha = -2 * 10^6\,n^3 + 130734\,n^2 - 3010\,n + 23.77$$
$$\beta = -620449\,n^3 + 43966\,n^2 - 1013\,n + 8.67$$

- The values of α and β are not strongly related to the cross section properties but the horizontal properties are slightly more effective on the values of α and β more than the vertical ones.
- Froude number has tangible effect than Reynolds number on the values of α and β and can be related by the following relationships:

$$\alpha = -181.32\,{F_r}^2 + 45.87\,F_r - 1.49$$
$$\beta = -65.35\,{F_r}^2 + 16.57\,F_r + 0.10$$

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NOTATION

The following symbols are used in this paper:

A	total water area of cross section;
$\mathbf{A_i}$	water area of each strip;
B	mean water width;
D	mean water depth;
d	total water depth at each vertical;
dA	elementary water area of cross section;
$\mathbf{d_i}$	average water depth of each strip;
$\mathbf{F_r}$	Froude number;
$\mathbf{\Delta h}$	water surface elevation difference;
$\mathbf{\Delta h_V}$	upstream velocity head minus downstream velocity head;
L	length of the reach;
N	number of strips;
n	Manning's roughness coefficient;
P	wetted perimeter;
Q	discharge;
$\mathbf{Q_i}$	discharge for each strip;
$\mathbf{R_e}$	Reynolds number;
$\mathbf{R_h}$	hydraulic radius;
$\mathbf{S_f}$	friction slope;
$\mathbf{S_w}$	water slope;
T	top width;
V	mean velocity of the cross section;
$\mathbf{V_a}$	velocity at 0.2 of the total depth;
$\mathbf{V_{avg}}$	vertical average velocity;
$\mathbf{V_i}$	average velocity for each strip;
$\mathbf{V_{max}}$	maximum cross sectional velocity;
$\mathbf{V_b}$	velocity at 0.5 of the total depth;
$\mathbf{V_c}$	velocity at 0.8 of the total depth;
$\mathbf{V^*}$	shear velocity;
v	velocity of an elementary area of cross section;
$\mathbf{w_i}$	width of each strip; and Y_{max} : max water depth.

GREEK LETTERS:

$\boldsymbol{\alpha}$	energy coefficient;
Β	momentum coefficient;
$\boldsymbol{\epsilon}$	Ratio correlating cross section maximum and mean velocities;
$\boldsymbol{\upsilon}$	Kinematic viscosity.

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