A Proposed Model Describing The Flow Through A Flat Channel Containing A Packed-Sphere Bed

مقترح بنموذج واصف للسريان فى قناة ممتلئة بوسط مسامى

M. G. WASEL

Mechanical Power Eng. Dept., Faculty of Engineering,
Mansoura University, Mansoura, Egypt

خلاصة: فى هذه الدراسة أقترحت صورة لمعادلة الحركة للسريان فى ممر ذى سطح مستو ممتلئ بكرات محصورة. لكى تحقق هذه المعادلة سلوك السريان، تم تعريف للزوجة الافتراضية - خلاف لزوجة المائع- كدالة فى مسامية الوسط بالاضافة للزوجة المائع. تم عمل مقارنة بين توزيع السرعة الناتج من النموذج الحالى والناتج باستخدام النموذج السابق عمله فى دراسات سابقة.

Absract- In this paper, a equation of motion describing the flow through a packed bed of spheres, contained in a flat-walls passage, is suggested. To satisfy the fluid behaviour within the bed a virtual kinematic viscosity is defined as a function of both the porosity of the medium and the kinematic viscosity of the fluid.

Numerical solutions of both the suggested model and the classical model are presented here. The produced velocity profile according to the present model is compared with that according to the classical model.

1. Introduction

Several thermal engineering applications can benefit from a better understanding of convective flow through porous materials exemplified by geothermal systems, thermal insulations, grain storage, solid matrix heat exchangers, oil extraction, filtering devices and various applications of chemical engineering.

As mentioned in [1], the first mathematical model describing the flow through porous medium is after Darcy, which states that the volumetrically avereged velocity in any direction in space is proportional to the pressure gradient in that direction. In spite of the simplicity of Darcy's model, it has some important limitations. This model is not suitable for describing fast flows and no-slip condition on the solid boundary is not satisfied by it. Moreover; the change of porosity near the solid boundaries is not considered according to Darcy's model.

Brinkman model takes into consideration the effect of viscous force in the momentum equation. As has been reported in [1-2], The effects of inertia and variable porosity are introduced to improve the model of Brinkman. Vortmeyer [3] approximated the variable permeability by an exponential function. Many investigators as in [4-6] used the modified Brinkman model to study the convective flow through porous medium.

In present proposed model, the momentum equation is formulated in a manner similar to that of simplified Navier-Stokes equation of fully developed single component flow. In this model a virtual kinematic viscosity is assumed to satisfy the characteristics of flow through porous media. A definition of virtual kinematic viscosity as a function of porosity and kinematic viscosity of the fluid is proposed here. Numerical solutions of both modified Brinkman and the present models are presented here for the flow between two parallel flat plates. A comparison between the results of both models are made.

2. Modified Brinkman model and its solution

Fig.(1) shows schematic description of the flow and the system of chosen coordinates. u denotes the velocity in x-direction, τ_w is the shear stress at the walls. The passage height is taken as 2H.

According to modified Brinkman model [2], the momentum equation of the flow is given by :

$$-\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{d^2u}{dy^2} - \frac{\nu}{k}u - A\,u^2 = 0 \quad , \qquad (1)$$

where ρ, ν and p are the density, kinematic viscosity of fluid and the pressure; respectively. Both the permeability k and the coefficient A are functions of the sphere diameter and the matrix porosity as follows;

$$k = \frac{d^2\,\phi^3}{175\,(1-\phi)^2} \quad , \qquad A = \frac{1.75\,(1-\phi)}{\phi^3\,d} \quad . \qquad (2\text{-a})$$

where d is the diameter of the sphere and ϕ is the porosity of the medium. The porosity is defined as ;

$$\phi = \phi_\infty\,[\,1 + \lambda_1\,e^{-\lambda_2\,y/d}\,] \quad . \qquad (2\text{-b})$$

where ϕ_∞, λ_1 and λ_2 are constants, their values depending on the diameter of the sphere.

Equation (1), the govering equation, has the following boundary conditions;

$$u = 0 \qquad \text{at} \quad y = 0 \quad . \qquad (3\text{-a})$$

$$\frac{du}{dy} = 0 \qquad \text{at} \quad y = H \quad . \qquad (3\text{-b})$$

Defining the dimensionless variables u^* and y^* as:

$$u^* = \frac{u}{\nu H} \quad , \quad y^* = \frac{Y}{H} \quad . \qquad (4)$$

yields to the dimensionless form of the governing equation as:

$$u^* + C_1 u^{*2} = B\,C_2 + C_2 \frac{d^2 u^*}{dy^{*2}} \quad , \qquad (5)$$

with the boundary conditions :

$$u^* = 0 \quad \text{at} \quad y^* = 0 \quad ,$$
$$\frac{du^*}{dy^*} = 0 \quad \text{at} \quad y^* = 1 \quad , \qquad (6)$$

where C_1, C_2 and B are defined as:

$$C_1 = \frac{d^*}{100\,(1-\phi)} \quad , \quad C_2 = \frac{d^{*2}\,\phi^3}{175\,(1-\phi)^2} \quad , \qquad (7\text{-a})$$

$$B = -\frac{dp}{dx}\,\frac{H^3}{\rho\,\nu^2} \quad . \qquad (7\text{-b})$$

The governing equation and its boundary conditions are solved,numerically, by the finite difference technique. Figs.(2-3) show the effect of step size ratio and the total number of nodes on the accuracy of the solution. The details of the adapted finite difference technique are presented in the appendix.

3. Present model and its solution

From the point of view of the classical fluid mechanics, the acting forces on a fully developed flow, in present case, can be restricted to two main kinds of forces, namely, friction forces and pressure forces. Appling this concept to the present problem, the momentum equation can be written as follows:

$$-\frac{1}{\rho}\frac{dp}{dx} + \frac{d\tau}{dy} = 0 \quad , \qquad (8)$$

where τ is the shear stress defined as $\tau = \bar{\nu}\,\frac{du}{dy}$. $\bar{\nu}$ is a virtual kinematic viscosity. Comparing equation (1) and (8), it is clear that $\bar{\nu}$ is no longer constant, and must be dependent on the kind of fluid and on the porous medium characteristics; as well.

Introducing the definition of τ in equation (8), the momentum equation can be written as:

$$-\frac{1}{\rho}\frac{dp}{dx} + \frac{d}{dy}\left(\bar{\nu}\,\frac{du}{dy}\right) = 0 \quad . \qquad (9)$$

with the boundary conditions;

$$u = 0 \quad \text{at} \quad y = 0 \;,$$
$$\frac{du}{dy} = 0 \quad \text{at} \quad y = H \;. \qquad (10)$$

According to the definition of the dimensionless variables u^* and y^* [eqn. (4)], one can put eqns. (9-10) in dimensionless form as;

$$- \frac{H^3}{\rho \nu^2} \frac{dp}{dx} + \frac{d}{dy^*} \left(\frac{\bar{\nu}}{\nu} \frac{du^*}{dy^*} \right) = 0 \;, \qquad (11)$$

$$u^* = 0 \quad \text{at} \quad y^* = 0 \;.$$
$$\frac{du^*}{dy^*} = 0 \quad \text{at} \quad y^* = 1 \;. \qquad (12)$$

Equation (11) can be written in simpler form as;

$$B + \frac{d}{dy^*} \left(\frac{1}{\varepsilon} \frac{du^*}{dy^*} \right) = 0 \;, \qquad (13)$$

where B is the dimensionless pressure gradient defined as;

$$B = - \frac{H^3}{\rho \nu^2} \frac{dp}{dx} = \text{constant}.$$

ε is the reciprocal of dimensionless virtual kinematic viscosity ($\varepsilon = \nu/\bar{\nu}$).

Integration of eqn.(13) followed by introducing the boundary conditions gives;

$$B \, (1 - y^*) = \frac{1}{\varepsilon} \frac{du^*}{dy^*} \;, \qquad (14)$$

with the boundary condition;

$$u^* = 0 \quad \text{at} \quad y^* = 0 \;. \qquad (15)$$

Equations (14-15) represent the proposed momentum equation in case of fully developed flow through saturated packed porous medium. The dimensionless virtual kinematic viscosity is determined using eqn.(14) together with the available data from the previous studies. Knowing the formula defining ε; one can solve momentum eqn.(14), numerically, by Runge-Kutta method.

4. Results

To determine the proper definition of the reciprocal of dimensionless virtual kinematic viscosity (ε), some formulas in the form of $\varepsilon = a\, e^{b\phi} - \phi^c$ are tested as indicated in fig.(4).
The coefficient c is taken as 4,5 and 6. The coefficients a and b are determined by the best fit of the values of ε

obtained by substitution of the available data from the previous work in eqn.(14). Fig.(5) shows a comparison between the results obtained by the present model and those obtained by the modified Brinkman model. It is clear from this figure. that ε can be fairly defined as:

$$\varepsilon = 7.3631 \times 10^{-\cdot} \exp(12.2874\,\phi) - \phi^5 \quad ,$$

for case of $d^* = 0.1$ and $d = 3$ mm. Fig.(6) shows the effect of the value of pressure gradient (B) on the velocity profile obtained by both the modified Brinkman model and the present model.

5. Conclusion

The proposed model presented here gives a model simple in appearance and application. Moreover, it overcomes the paradox arises in Darcy's model and its improvements due to the presence of physically unexplained terms in these models. In spite of the advantages of the present model; the definition of the virtual kinematic viscosity still needs more studies.

6. Appendix

According to the finite difference technique, the first and second derivative of u^* at general position (i) can be approximated by :

$$\left.\frac{du^*}{dy^*}\right)_i = \frac{u^*_{i+1} - u^*_{i-1}}{(1-\gamma)\ h} \quad . \qquad (1A-a)$$

$$\left.\frac{d^2u^*}{dy^{*2}}\right)_i = \frac{\beta\ u^*_{i+1} - (1+\beta)\ u^*_i + u^*_{i-1}}{0.5(1+\gamma)\ h^2} \quad . \qquad (1A-b)$$

where i denotes the node number, γ is the ratio between two successive step sizes and β is the reciprocal of γ. The step size is taken variable because of the rapid change of the velocity near the wall and thus it is suitable to take the value of it near the wall ($y^*=0$) very small. compared with that far from it, to ensure good prediction of velocity through out the flow field. Now if h denotes the initial value of the step size (h), accordingly the step size value at any position (n) can be expressed as:

$$h_n = \gamma^{n-2}\ h_2 \quad , \text{ where } \quad h_2 = \frac{1-\gamma}{1-\gamma^{j-1}} \quad .$$

where j is the total number of nodes.

Substitution of equation (1A-b) in eqn. (5) yields to:

$$a_i\ u^*_{i+1} + b_i\ u^*_i + c_i\ u^*_{i-1} + d_i = 0 \quad . \qquad (2A)$$

where $a_i = C_2\ \beta$.

$$b_i = -C_2\ (\beta+1) - \sigma_i - \sigma_i\ C_1\ u_i^* \quad .$$
$$c_i = C_2 \quad , \qquad (3A)$$
$$d_i = \sigma_i\ B\ C_2 \quad ,$$
$$\sigma_i = 0.5\ (\gamma+1)\ h_i^2 \quad .$$

Equation (2A) represents a set of simultaneous linear algebric equations with (j-2) unkowns (u_2^*, u_3^*, - - - - , u_{j-1}). This equation can be written in detail for i = 2,3.4.----. .j-1 as;

$$a_2\ u_3^* + b_2\ u_2^* + c_2\ u_1^* + d_1 = 0 \qquad \text{for } i = 2 \quad ,$$
$$a_3\ u_4^* + b_3\ u_3^* + c_3\ u_2^* + d_3 = 0 \qquad \text{for } i = 3 \quad ,$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$a_n\ u_{n+1}^* + b_n\ u_n^* + c_n\ u_{n-1}^* + d_n = 0 \qquad \text{for } i = n \quad ,$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$a_{j-1}\ u_j^* + b_{j-1}\ u_{j-1}^* + c_{j-1}\ u_{j-2}^* + d_{j-1} = 0 \qquad \text{for } i = j-1 \quad .$$
$$- - - - - - - - - - \qquad (4A)$$

Equations (3A) of the type known as three diagonal matrix. which can be written such that; each value of u_i^* can be expressed as a function of u^* at the next position (u_{i+1}^*). From equations (4A);

$$u_2^* = z_2\ u_3^* + v_2 \quad ,$$
$$u_3^* = z_3\ u_4^* + v_3 \quad ,$$
$$\cdots\cdots\cdots\cdots$$
$$u_n^* = z_n\ u_{n+1}^* + v_n \quad , \qquad (5A)$$
$$\cdots\cdots\cdots\cdots$$
$$u_{j-1}^* = z_{j-1}\ u_j^* + v_{j-1} \quad ,$$

where;

$$z_2 = -\frac{a_2}{b_2} \quad , \quad v_2 = -\frac{d_2}{b_2} \quad .$$
$$z_3 = -\frac{a_3}{b_3 + c_3\ z_2} \quad , \quad v_3 = -\frac{c_3\ v_2 + d_3}{b_3 + c_3\ z_2} \quad ,$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$z_n = -\frac{a_n}{b_n + c_n\ z_{n-1}} \quad , \quad v_n = -\frac{c_n\ v_{n-1} + d_n}{b_n + c_n\ z_{n-1}} \quad , \qquad (6A)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$z_{j-1} = -\frac{a_{j-1}}{b_{j-1} + c_{j-1}\ z_{j-2}} \quad , \quad v_{j-1} = -\frac{c_{j-1}\ v_{j-2} + d_{j-1}}{b_{j-1} + c_{j-1}\ z_{j-2}} \quad .$$

According to the boundary condition at the center of the passage (n=j) eqn(6), one can. fairly, put $u_j^* = u_{j-1}^*$ and hence:

$$u^*_{j-1} = \frac{v_{j-1}}{1 - z_{j-1}} \quad . \qquad (7A)$$

Knowing the value of u^*_{j-1}, one can obtain, in recursive manner, the values of u^*_{j-2}, u^*_{j-3}, ..., u^*_n .....u^*_3 and u^*_2 .

7. Nomenclature

a_i, b_i, c_i & d_i	the coefficients of the difference eqn.(4A)
B	dimensionless pressure gradient, defined by eqn.(7)
2H	the passage height
p	pressure
u_*	velocity component in x-direction
u	dimensionless velocity in x-direction, $u/\nu H$
v_i, z_i	coefficients defined by eqns.(6A)
x	co-ordinate along the lower wall of the passage
y_*	co-ordinate normal to the lower wall of the passage
y	dimensionless co-ordinate, y/H
ε	reciprocal of the virtual kinematic viscosity, $\nu/\bar{\nu}$
ϕ	porosity of the medium, defined by eqn.(2-c)
γ	the ratio between two successive step sizes
ν	kinematic viscosity of the fluid
$\bar{\nu}$	virtual kinematic viscosity, defined in eqn.(14)
ρ	density of the fluid
τ	shear stress, $\tau = \bar{\nu}\, du/dy$
τ_w	shear stress at the wall

8. Refrences

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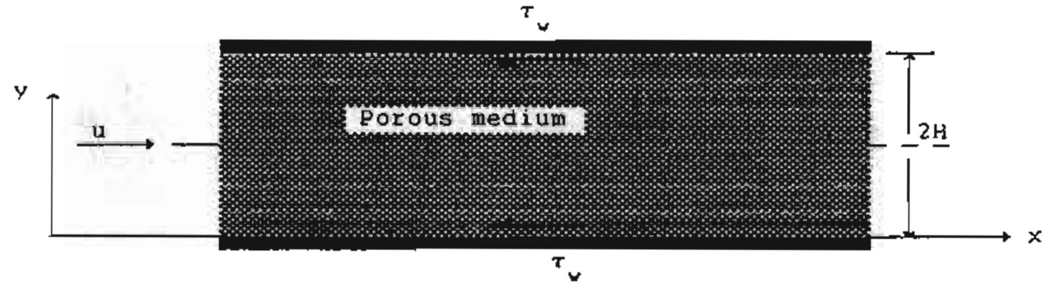

Fig.(1) Schematic description of the flow through a passage filled with fluid saturated porous medium

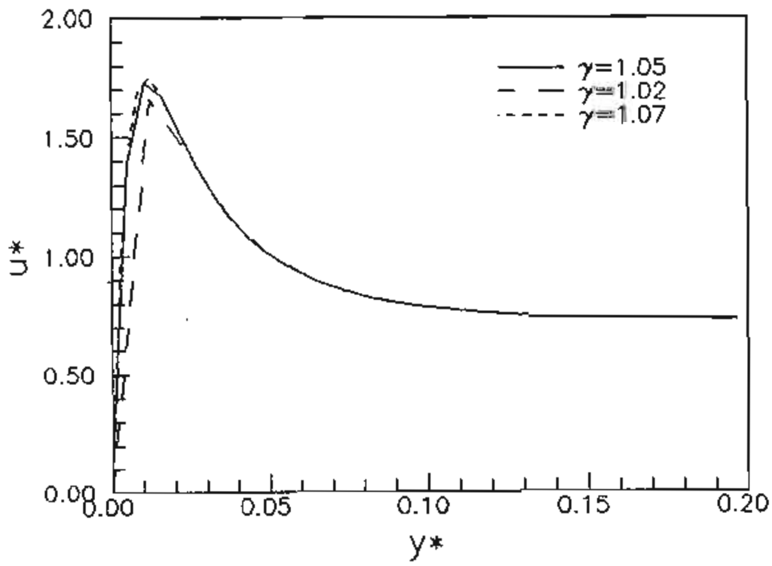

Fig.(2) The effect of the step size ratio (γ) on the dimensionless velocity profile.

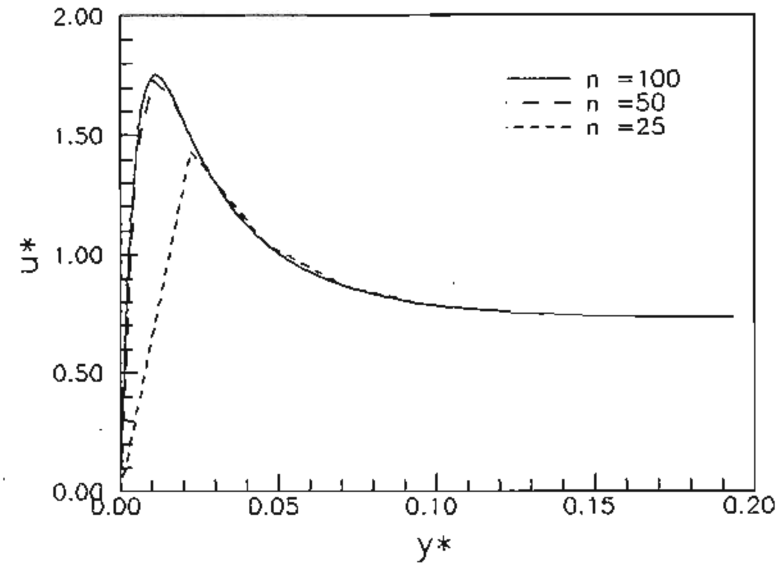

Fig.(3) The effect of the number of nodes used in numerical calculations.

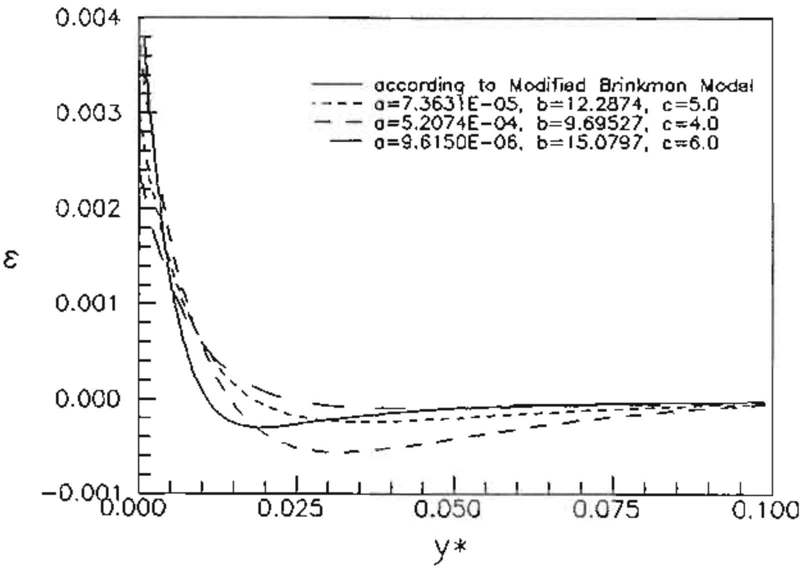

Fig.(4) The dimensionless virtual kinematic viscosity ($\varepsilon = a\ e^{b\varphi} - \varphi^{c}$) against the dimensionless distance y*

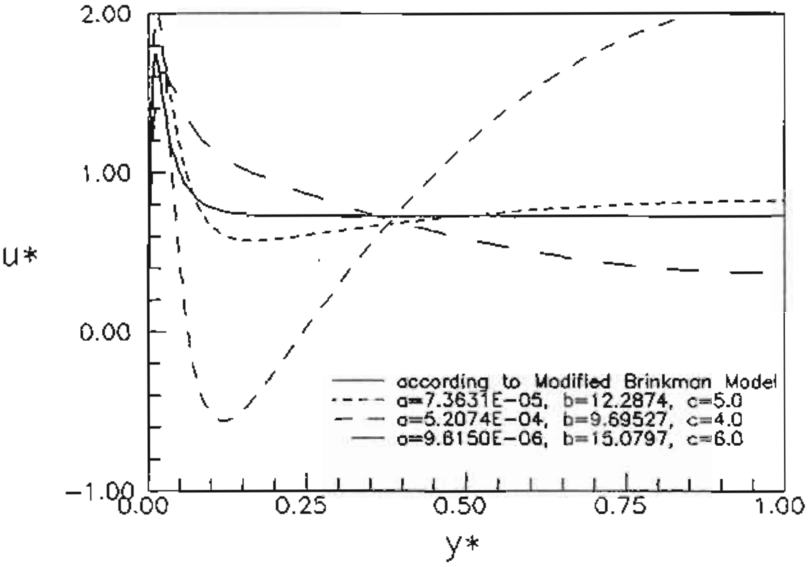

Fig.(5) The dimensionless velocity profile (u* versus y*) according to darcy model compared with that obtained using the proposed formulas of dimensionless virtual kinematic viscocity shown in fig.(4)

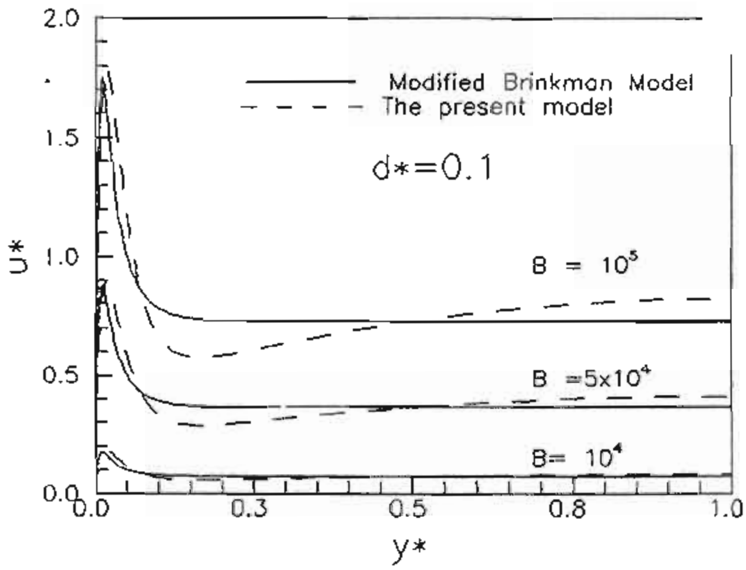

Fig.(6) Velocity profile across the passage at different values of dimensionless pressure gradient (B).