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APPLICATION OF LINEAR PROGRAMMING IN THE DESIGN OF GLASS MIX

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ABSTRACT

In our previous studies we first determined relation-ships between glass composition and physical properties and between dimensional stability and viscosity in the forming range.

In this paper we introduce for the first time an optimization model which minimize the glass cost and fulfill the required requirements both the physical properties stated be the specification of end product and also for the forming process requirements of dimensional stability and forming viscosity.

The technique applied is linear programming.

Key Words : Linear programming - Physical constrations Forming constrains - Cost objective.

INTRODUCTION

The reason of this research is to investigate the feasibility of applying optimization techniques in an integrated model that include glass mix cost and the physical constraints and the forming constrain.
We used the results of our previous researches of physical properties relation-ships and the stability of dimensions and viscosity in the forming range together which data available for compostions of raw materials and their cost to develop a linear programming model.

The results are very promising and indicate the feasibility of this new technique.

The problem of glass-making technology is the various factors affecting the process. The composition of glass affects the various properties of glass and the forming process also requires certain properties and the cost of product is very important.

To develop a criterion of choice and relation of these different parameters has always been a challange to the production engineer.

The proposed technique is leading the way for further extension and applications of optimixation techniques, and process control based on cost minimization. This is also a key issue in automated process control.

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I- THE MODEL :

The proposed approach is to develop an integrated model for the process engineer that involve all the control parameter affecting the manufacturing process in the glass making industry.

1- Furnace

2- Product

3- Machine

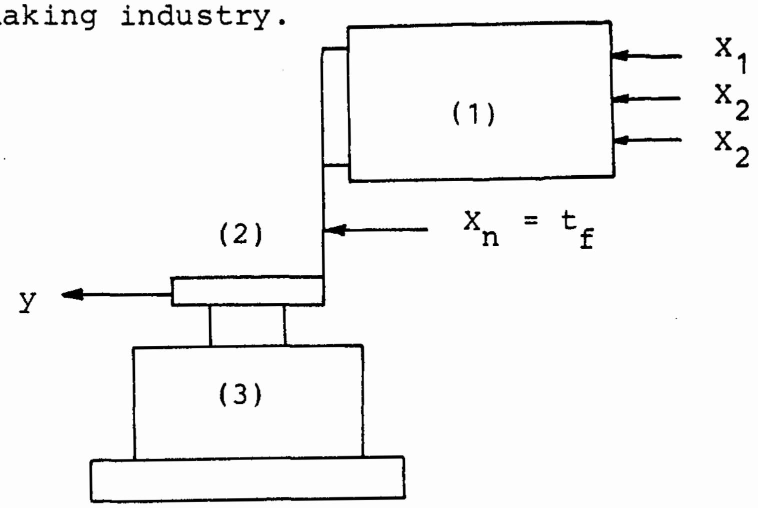


Fig.(I) Decision variables in glass process

To built up this model we have to consider the following facts :-

I.1. PHYSICAL CONSTRAINTS :

The product must conform to physical pre-specified properties which are determined according to the application of the product. .

The most widely used properties are :-

a- Density : Which reflects it self on the weight of final product and also on the strength of glass. In many applications control of this parameter is of prime importance specially in dense glasses such as Lead-glasses (Crystals).

This property, as shown in our previous paper[4] can be expressed as a linear function of the glass forming oxides, for if (X_j) is the % ge of forming oxide (j) and (a_{1j}) is parameter representing the effect of the % ge of oxide (j) on density, then

$$\Sigma\ a_{1j}\ X_j = b_1 \qquad (1)$$

where

b_1 = resulting density

according to the application of glass this constrain may manifest it self on different way :

i) Upper Bound Constraints : Where the density must not exceed upper limit (b_1^U)

$$\Sigma\ a_{1j}\ X_j \leq b_1^U \qquad (2)$$

ii) Lower Bound Constraints : Where the density should not be less than lower limit (b_1^L)

$$\Sigma\ a_{1j}\ X_j \geq b_1^L \qquad (3)$$

iii) Tolerance Limits : Where the density must fall within tolerance range :

$$b_1^U \geq \Sigma\ a_{1j}\ X_j \geq b_1^L \qquad (4)$$

b - Thermal Expansion : This is an important physical property which depends completely on the application of glass and affect the rate of cooling and the residual stresses (glass strain) and has a direct influence on the dimensions of product. It is also very important when the glasses are adhesive to metallic parts as the adjustment of coefficients of expansion is very important, if (a_{2j}) is the effect of the percentage (X_j) of oxide (j) on the thermal expansion (b_2) then we may have again the following conditions

$$\left.\begin{array}{r} \Sigma\ a_{2j}\ X_j \leq b_2^U \\ \Sigma\ a_{2j}\ X_j \geq b_2^L \\ b_2^U \geq \Sigma\ a_{2j}\ X_j \geq b_2^L \end{array}\right\} \quad \ldots\ldots(5)$$

c - <u>Viscosity</u> : viscosity is very important property for the forming process. The determination of viscosity range is governed by the nature of product and the forming process. Let (a_{3j}) be the effect on viscosity (b_3) of the forming oxide (j) of percentage (X_j) then :

$$b_3^U \geq \Sigma\, a_{3j}\, X_j \geq b_3^L$$

is the normal constraint in this case.

Generally speaking if (a_{ij}) is the effect of oxide (j) on property (i) of the percentage (X_j) then :

$$\Sigma a_{ij}\, X_j \geq b_i^L \qquad i = 1, 2, \ldots, k$$

$$\Sigma a_{ij}\, X_j \leq b_i^U \qquad i = k + 1, \ldots, l$$

$$b_i^U \geq \Sigma a_{ij}\, X_j \quad b_i^L \qquad i = l + 1, \ldots, m$$

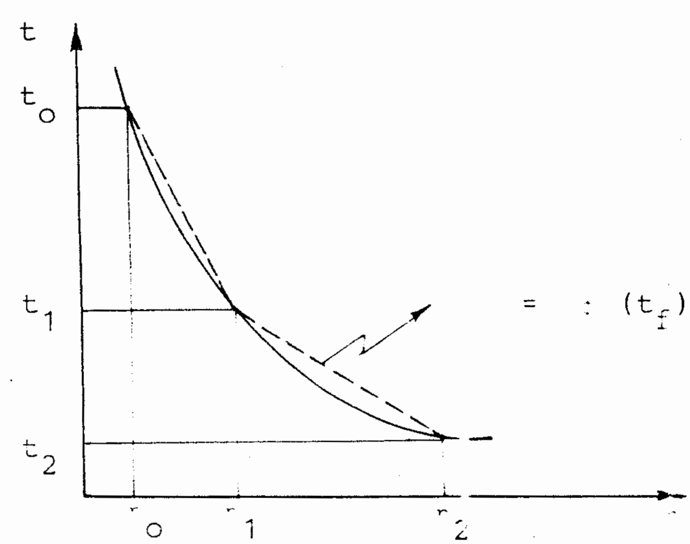
$$j = 1, 2, \ldots, n$$

I.2. FORMING CONSTRAINTS :

The forming process in glass as discussed in forming properties[8] depends on the viscosity of glass in the forming range, thus it depends mainly on the viscosity - temperature relation ship and this infact depend on the process type and set-up.

Let : $b_3 = \log \eta$ be the viscosity of glass then for a given forming temperature (t_f) the viscosity temperature relation is given by :

$b_3(t_f) = \log \eta\ (t_f)$ and is illustrated in Fig.(II).



The dotted lines are a linearization of the curve and are given by the expression :

$$\eta_f = \eta_o - \left(\frac{\eta_o - \eta_1}{t_o - t_1} \right) (t_f - t_o) \qquad \ldots(6)$$

$$t_o \leq t_f \leq t_1$$

$$\eta_f = \eta_1 - \left(\frac{\eta_1 - \eta_2}{t_1 - t_2} \right) (t_f - t_1) \qquad \ldots(7)$$

$$t_1 \leq t_f \leq t_2$$

However, the shape of the viscosity-temperature relation ship suggest the relation :

$$\eta = A\, e^{-ct} \qquad \ldots(8)$$

thus :

$$\log \eta = A - ct \qquad \ldots(9)$$

$$b_{tf} = A - ct_f \qquad \ldots(10)$$

if (b_3) is the cold viscosity of glass then $(b_3 = A)$

$$b_{tf} = b_3 - c\,.t_f \qquad \ldots(11)$$

In our case as we have proved that the dimensional stability of the forming process depend on the viscosity by the relation :

$$\tau = a_1 + B_1(\log \eta) + B_2(\log \eta)^2 \qquad \ldots(12)$$

For optimality

$$\frac{\partial \tau}{\partial \log \eta} = 0$$

Thus :

$$\log \eta = \frac{-B_1}{2\,B_2} \qquad \ldots(13)$$

will be our aimed at viscosity.

Thus the following type of constraint may be applicable

$$\text{i - } b^L_{tf} \leq A - ct_f \leq b^L_{tf} \qquad \ldots(14)$$

$$\text{ii- } b_3 - ct_f = \frac{-B_1}{2\,B_1} = B \qquad \ldots(15)$$

I.3. RAW MATERIALS :

The forming oxides of glass are formed from raw material added together with certain weight. if, 1, 2, ..., r, R represent the set of available raw materials, and if (y_{rj}) is the percentage of oxide (j) in raw material (r) and if (d_r) is amount used of raw material (r) then: in raw material (r) then the weight of oxide (j) is :

$$W_j = \sum_{r=1}^{R} . y_{rj} . d_r \qquad \text{...(16)}$$

it is apparent that, the % ge (X_j) is calculated as :

$$X_j = (W_j)/(\sum_{j=1}^{r} W_j) \qquad \text{...(17)}$$

$$X_j = (\sum_{r=1}^{R} .y_{rj}.d_r)/(\sum_j . \sum_r . y_{rj}.d_r) \qquad \text{...(18)}$$

$$\Sigma a_{ij} X_j \leq b_i$$

is equivalent to

$$[\Sigma a_{ij}(\sum_{r=1}^{R} .y_{rj}.d_r)]/(\sum_j . \sum_r .y_{rj}.d_r) \leq b_i \qquad \text{...(19)}$$

$$\Sigma a_{ij}(\sum_{r=1}^{R} .y_{rj}.d_r) \leq b_i(\sum_j . \sum_r .y_{rj}.d_r)$$

$$\Sigma(a_{ij} - b_i) \sum_{r=1}^{R} .y_{rj}.d_r \leq 0 \qquad \text{..(20)}$$

as (a_{ij}, b_i, y_{rj}) are known constant, the problem is to determine the weights of mix ingredients (d_r, r = 1, 2, ..., R).

I.4. COMPOSITION VARIATIONS IN RAW MATERIALS :

An important feature of the problem is the effect of variation of the composition of raw materials, this problem must be treated carefully.

If the standard deviation of composition of oxide (j) in raw material (r) is (σ_{rj}), then it is expected that the stated deviation of property (i) is given by :

$$\left.\begin{aligned} \sigma_i^2 &= \Sigma a'_{ij} \cdot \Sigma y_{rj}\, \sigma_{rj}^2 \\ a'_{ij} &= |a_{ij} - b_i| \end{aligned}\right\} \qquad \ldots(21)$$

However, it also possible to define (S_{rj}) as the range so that :-

$$S_i = \Sigma a'_{ij} \cdot \Sigma y_{rj} \cdot S_{rj} \qquad \ldots(22)$$

and the deviation allowed for property (i) is :

$$S_i = b_i^U - b_i^L$$

Thus it is possible to add constraint (22) to express the expected variation of properties due to variation in chemical composition.

II. EFFECTIVENESS MEASURE :

We are now in a position to study the objective of our model.

II.1. COST EFFECTIVENESS :

One of the direct objective would be the cost of the Mix.

for if (c_r) is the unit cost of raw material (r), then the total cost would be :

$$Z = \sum_{r=1}^{R} c_r\, d_r \qquad \ldots (23)$$

The production engineer may seek the minimization of (Z) given by (23) subject to constraints (15), (20) and (22).

II.2. PRODUCTION GOAL :

In many occasions it is required to adjust the viscosity and/or other properties to a very specified limit.

Any deviation from this must have a weighted penalty in this sense :

$$|\Sigma a_{ij} X_j - b_i| = E_i$$

will be considered as an errors that have a penalty (p_i) In this sense

$$Z_e = \Sigma p_i E_i \quad \ldots(24)$$

is considered the objective or goals and this to be minimized subjected to constraints (15), (20) and (22).

II.3. MULTI-OBJECTIVE :

It seems logic for our research to combine mix cost, and reject cost caused by dimensional variation of product. For if the design limits are m, which is caused by viscosity variation E_3

$$Z_m = \text{reject cost} = c_m \times E_3 \times G \quad \ldots(25)$$

$$E_3 = \Sigma a_{3j} X_i \cdot b_3 \quad \ldots(26)$$

where : G = Production rate

$$Z_R = \text{Material cost} = \Sigma c_r d_r \quad \ldots(27)$$

$$\text{Total cost} = Z = Z_m + Z_R \quad \ldots(28)$$

and then (Z) is to be minimized subject to constraints (15), (20) and (22).

III. SOLUTION :

To solve the above formulation we suggest the following procedure.

III.1. LINEAR PROGRAMMING (L.P.) :

For the linear programming formulation the problem can be easily solved by using the standard simplex Method programme. We recommend the SYSTEM 1360 FORTRAN Programme (Code-360 D-15.2.006).

In our case study we applied this programme sucessfully. Various extension can be adopted. One important extension is to check the senstivity of solution to viscosity change (due to composition or temperature change).

This is very important to automated process to determine control limits.

III.2. GOAL PROGRAMMING :

For the G.P. we recommend REVSIM ALGORITHM of IBM SYSTEM 1360 MODEL CODE 360 D-15.2.007

III.3. MULTI-OBJECTIVE D. PROGRAMMING (M.O.D.P.):

For Multi-objective Formulation we refer to Lee[5] and Ignizio[6]. The new interactive techniques of Geoffrin[7] proved to be very efficient.

V. CASE STUDY :

We will apply the proposed approach of the integrated model on a case study for the optimum design of mix and process control for glasses in the Electronic Industry.

For our study we will take the L.P. approach.

The Model will have the following form :

(1) Objective function based on total costs of glass mix and rejects, (described in section II.2).

(2) Properties constraints, (described in section I.1).

(3) Technical constraints based :-

a- Minimum requirement of raw materials for glass refining.

b- Maximum allowable % ge of some oxide.

c- Minimum % ge ratio of some material to avoid glass devetrification.

r =		Raw Material
1	d_1	Sand
2	d_2	Sod. Carb.
3	d_3	Sod. Nitrate
4	d_4	Potass. Carbonate
5	d_5	Calumite
6	d_6	Dolomite
7	d_7	Barium Carbonate
8	d_8	Potass. Feldspar
9	d_9	Antimony Oxide
10	d_{10}	Sod. Sulphate

Symbols used in the models

i	Properties
1	Density $= b_1$
2	Thermal Expansion $= b_2$
3	Viscosity $= b_3$

$$\text{Minimize } (Z) = 104\ d_1 + 28\ d_2 + 33\ d_3 + 90\ d_4 + 14\ d_5 + 7\ d_6 + 70\ d_7 + 18\ d_8 + 500\ d_9 + 22\ d_{10} + (120\ E_3 \times 0.2) \quad \text{Subject to :}$$

Properties Constraints:

$$410 \leq 454 + 0.0\ X_1 - 1.012\ X_2 - 1.83\ X_3 \leq 436$$

$$69 + 0.18\ X_1 + 3.82\ X_2 + 1.93\ X_3 + 34\ X_6 - 0.22\ X_9 \leq 101$$

$$817 + 9.2\ X_1 - 4.1\ X_2 + 11.20\ X_3 \leq 14.6$$

Technical Constraints:

$$X_8 \leq 0.3$$

$$0.9 \leq X_{10} \leq 1.2$$

$$X_3 \geq 1.4$$

$$X_4$$

$$d_3 + d_{10} \geq 0.5$$

$$d_9 \geq 0.7$$

$$X_1 = \frac{0.997\ d_1 + 0.3\ d_5 + 0.03\ d_6 + 0.665\ d_8}{d_1 + d_2 + \ldots\ldots\ldots + d_{10}}$$

$$X_2 = \frac{0.544\ d_2 + 0.362\ d_3 + 0.03\ d_8 + 0.43\ d_{10}}{d_1 + d_2 + \ldots\ldots\ldots\ldots + d_{10}}$$

$$X_3 = \frac{0.43\ d_5 + 0.407\ d_6 + 0.005\ d_8}{d_1 + d_2 + \ldots\ldots\ldots + d_{10}}$$

$$X_4 = \frac{0.08\ d_5 + 0.214\ d_6}{d_1 + d_2 + \ldots + d_{10}}$$

$$X_5 = \frac{0.77\ d_5}{d_1 + d_2 + \ldots + d_{10}}$$

$$X_6 = \frac{0.68\ d_4 + 0.11\ d_8}{d_1 + d_2 + \ldots + d_{10}}$$

$$X_7 = \frac{d_9}{d_1 + d_2 + \ldots + d_{10}}$$

$$X_8 = \frac{0.001\ d_1 + 0.002\ d_6 + 0.001\ d_8}{d_1 + d_2 + \ldots\ldots\ldots + d_{10}}$$

$$X_9 = \frac{0.002\ d_1 + 0.13\ d_5 + 0.001\ d_6 + 0.185\ d_8}{d_1 + d_2 + \ldots\ldots\ldots\ldots + d_{10}}$$

$$X_{10} = \frac{0.021\ d_5}{d_1 + d_2 + \ldots + d_{10}}$$

$$d_1, d_2, \ldots, d_{10} \geqq 0 \ \& \ X_1, X_2, \ldots, X_{10} \geqq 0$$

VI. RESULTS :

By applying (L.P) Algorithm we reached the following results

d_1	64.6	X_1	68.7
d_2	29.3	X_2	17.8
d_3	0.5	X_3	5.4
d_4	1.0	X_4	3.8
d_5	-	X_5	2.0
d_6	17.8	X_6	1.0
d_7	2.6	X_7	0.7
d_8	5.5	X_8	0.334
d_9	0.7	X_9	1.1
d_{10}	0.0	X_{10}	0.0
E_3	0.115		

With Total cost

Z_{min} = 20.55 Pound/100 Kg.

and Total rejects of = 2.3 %

CONCLUSION

By using linear programming for optimum production parameteris it was possible to achieve a production cost of 12% lower and to ensure the uniformity of the products and the end specification such as density and thermal expansion.

We strongly recommend the application of optimization techniques in the production and process set-up as most of processes the number of variables and constant is large enough so that trial and error technique and even statistical experiments fail to prove to be valuable.

APPENDIX

Table (1) : Contains the different raw materials used in glass industry.

Table (2) : The effect of different glass oxides on the properties.

Table (3) : Contain compositions of different glasses,

Table (4) : Raw material prices.

Table (1) : VALUES OF (y_{rj})

Raw Material	r \ j	1	2	3	4	5	6	7	8	9	10
		SiO_2	Na_2O	CaO	MgO	BaO	K_2O	Sb_2O_3	Fe_2O_3	AL_2O_3	TiO_2
Sand	1	0.997	-	-	-	-	-	-	0.001	0.002	-
Sod. Carb.	2	-	0.584	-	-	-	-	-	-	-	-
Sod. Nitrat.	3	-	0.362	-	-	-	-	-	-	-	-
Potass. Carb	4	-	-	-	-	-	0.680	-	-	-	-
Calumite	5	0.300	-	0.430	0.080	-	-	-	-	0.130	0.021
Dolomite	6	0.003	-	0.307	0.214	-	-	-	0.002	0.001	-
Bar. Carb.	7	-	-	-	-	0.770	-	-	-	-	-
Potassium Feldspar	8	0.665	0.030	0.005	-	-	0.110	-	0.001	0.185	-
Antimony Oxide	9	-	-	-	-	-	-	1.000	-	-	-
Sod. Sulph.	10	-	0.430	-	-	-	-	-	-	-	-

Table (2) : VALUES OF (a_{ij})

Propert.	i	D	1	2	3	4	5	6	7	8	9	10
			S_iO_2	Na_2O	CaO	MgO	BaO	K_2O	Sb_2O_3	Fe_2O_3	AL_2O_3	TiO_2
b_1	1	0.454	-	-1.012×10^{-3}	-1.83×10^{-3}	-	-	-	-	-	-	-
b_2	2	0.690	0.180	3.820	1.930	-	-	3.400	-	-	-0.22	-
b_3	3	8.170	-0.041	0.0112	-	-	-	-	-	-	-	-

b_1' = 2.293 gm/Cm³

b_1 = $1/b_1'$ = 0.4361 = Density

b_2 = 101 (10^{-7}) = C.O.E. (30° . 30' °C)

b_3 = log η_o = 14.6

Table(3) : Contain composition of different glasses.

Oxide	Sheet glass	Plate glass	Cast glass	Botlle Glass			Opal	Lamp Bulb
				French	Ovens	Feeder		
S_iO_2	71.5	72.0	70.3	57.4	64.8	73.3	74.43	69
AL_2O_3	0.5	0.3	0.95	10.6	6.0	1.5	4.84	1.2
Fe_2O_3	0.1	0.05	0.12	2.2	2.0	0.04	0.14	-
TiO_2	0.05	0.03	-	0.1	0.2	0.02	-	-
CaO	8.8	13.7	13.0	23.6	10.0	9.8	2.35	4.2
MgO	3.2	-	0.73	0.4	0.5	0.1	-	3.8
Na_2O	15.3	13.3	13.5	5.4	1.2	14.2	13.6	17.2
K_2O	-	-	-	-	2.1	0.6	2.74	0.6
SO_3	0.5	-	-	0.3	0.3	0.4	-	-
SiO_3	-	0.2	-	-	-	-	-	2.0
BaO	-	-	-	-	1.0	-	-	-
Mn_2O_3	-	-	-	-	-	-	-	-
F	-	-	-	-	-	-	3.97	-
PbO	-	-	-	-	-	-	-	-
AS_2O_3	-	-	-	-	-	-	-	-
ZnO	-	-	-	-	-	-	-	
B_2O_3	-	-	-	-	-	-	-	-
CaF_2	-	-	-	-	-	-	-	-
LiO_2	-	-	-	-	-	-	-	-
ZrO_2	-	-	-	-	-	-	-	-

Survey of Glass Compositions *

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Table (3): Contain composition of different glasses.

Oxide	Domestic And Lighting Ware					Technical glass		T.V	Opt. Fli. Glas.	Fiber Glass (E)	Ceramic.
	Boh.	Norm. L.G.	Heav. Crys.	Pres.	Ligh Ware	Pyrex	Vico.				
SiO_2	76	64.8	56	75.5	65	81	96	56	50	55.2	70.1
AL_2O_3	0.1	0.1	0.1	0.3	3.6	2.0	0.5	-	-	14.8	18.5
Fe_2O_3	0.01	0.01	0.01	0.01	0.1	0.15	-	-	-	0.3	-
TiO_2	0.01	0.01	0.01	0.01	0.05	0.05	-	-	-	-	1.85
CaO	6.7	2.0	-	6.5	9.0	0.3	-	-	-	18.7	-
MgO	-	-	-	-	-	0.2	-	-	-	3.3	1.55
Na_2O	2.3	-	-	14.8	10.7	4.5	0.5	4.0	5.0	0.3	0.15
K_2O	14.1	14.6	11.4	2.0	3.4	0.1	-	8.0	5.0	0.2	-
SO_3	0.3	-	-	0.7	0.2	-	-	-	-	-	-
SiO_3	-	-	-	-	-	-	-	-	-	-	-
BaO	-	-	-	-	-	-	-	13.0	-	-	0.80
Mn_2O_3	-	-	-	-	-	-	-	-	-	-	-
F	-	-	-	-	5.8	-	-	-	-	0.3	-
PbO	-	18.0	32.0	-	-	-	-	-	-	-	-
AS_2O_3	0.5	0.5	0.5	0.2	-	0.3	-	-	46.0	-	0.6
ZnO	-	-	-	-	4.0	-	-	8.0	-	-	1.0
B_2O_3	-	-	-	-	-	11.4	3.0	8.0	-	7.3	-
CaF_2	-	-	-	-	-	-	-	3.0	-	-	-
LiO_2	-	-	-	-	-	-	-	-	-	-	2.9
ZrO_2	-	-	-	-	-	-	-	-	-	-	1.95

Servey of Glass Compositions *

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Table (4): Prices of raw material (1982):
VALUES OF (C_r)

100 Kg	Pounds
Sand	1.4
Quartz Floor	7.0
Fusing Quartz	450.0
Sodium-Carbonate	28
Sodium-Nitrate	33
Potasium-Carbonate	90
Calcite	14
Dolomite	7
Barium Carbonate	70
Red Lead	280
Potasium Feldspar	18
Antimony Oxide	500
Arsenic Oxide	95
Cobalt Oxide	2200
Zinc White	220
Boric Acid	85
Petalitc	45
Sodium Sulphate	22
Portefer	30
Manganeze Oxide	320
Ceruse Oxide	600
Colour Powder	430

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