Menofia University Faculty of EngineeringShebien El-kom Basic Engineering Sci. Department. Academic Year : 2016-2017 Date : 31 / 12 / 2016	Minoufia University	Subject : Numerical Analysis Code: BES 601 Time Allowed : 3 hours Year:Master Total Marks: 100 Marks

Answer all the following questions: [100 Marks]

Q.1 [50]

1) Define briefly each of the following expressions:

   Interpolation- Chebyshev norm- Splines- Digital image.

2) Write a Matlab code to find the cubic spline for function $y = sinx$ in the interval $[-3:3]$.

3) Use the numbers $x_0 = 2. x_1 = 2.75. x_2 = 4$ to find the second Lagrange interpolating polynomial for the function $f(x) = \frac{1}{x}$, and use this polynomial to approximate $f(3)$ and $f(3.5)$.Then Write a Matlab code to find this polynomial.

4) Determine an approximate backward difference representation for $\frac{\partial^3 f}{\partial x^3}$ which is of order (Δx) , Given evenly spaced grid points $f_i. f_{i-1}. f_{i-2}. f_{i-3}$ by means of:

   a) Taylor series expansions.

   b) Backward difference recurrence formula.

   c) Third degree polynomial passing through the four points.

5) Derive a central difference approximation for $\frac{\partial^3 f}{\partial x^3}$ which is of order$(\Delta x)^2$.

6) Use the Matlab environment to generate the **Wilkinson's Polynomial** and find the following statements:

   i) State the command which calculates the root of this polynomial.

   ii) State the command which calculates the value of this polynomial at

$$x = [-1.2, i, NaN, inf]'.$$

Q.2 **(A)** Consider the following three-dimensional Helmholtz equation in the following form: [25]

$$a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^2 u}{\partial y^2}+\lambda u=F(x,\, y, z),$$

with initial conditions:

$$u(0,y)=f_1(y),\quad u_x(0,y)=f_2(y),$$

$$u(x,0)=f_3(x),\quad u_y(x,0)=f_4(x).$$

Where;

$F(x,y)$, $f_1(y)$, $f_2(y)$, $f_3(x)$, $f_4(x)$ and a, b, λ are given functions and given constant respectively.

Solve the two-dimensional Schrodinger equation using *the differential transform method* (*DTM*), in the following form:

$$F(x,y,z)=(12x^2-3x^4)\sin(y).$$

$$a=b=1,\quad \lambda=-2, \text{ and } f_1(y)=0,\quad f_2(y)=0.$$

(B) Consider the following three-dimensional Helmholtz equation in the following form:

$$a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^2 u}{\partial y^2}+c\frac{\partial^2 u}{\partial z^2}+\lambda u=F(x,\, y, z),$$

with initial conditions:

$$u(0,y,z)=f_1(y,z),\quad u_x(0,y,z)=f_2(y,z).$$

$$u(x,0,z)=f_3(x,z),\quad u_y(x,0,z)=f_4(x,z).$$

$$u(x,y,0)=f_5(x,y),\quad u_z(x,y,0)=f_6(x,y).$$

Where;

$f_1(y,z)$, $f_2(y,z)$, $f_3(y,z)$, $f_4(y,z)$, $f_5(y,z)$, $f_6(y,z)$ and a, b, c, λ are given functions and given constant respectively.

Solve the three-dimensional *Helmholtz equation* using *the differential transform method* (*DTM*),in the following form:

$$F(x,y,z) = (12x^2 - 4x^4)x\sin(y)\cos(x).$$

$$a = b = c = 1, \quad \lambda = -4, \text{ and } f_1(y,z) = 0, \quad f_2(y,z) = 0.$$

(C) Consider the nonlinear singular initial value problem:

$$y'' + \frac{2}{x}y' + 4(2e^{y} + e^{y/2}) = 0$$

with initial conditions:

$$y(0)=0, \quad y'(0)=0$$

Solve the nonlinear singular initial value problem using the adomian decomposition method (ADM).

(D) Consider the following Riccati equation

$$y'(t) = -(3 - y(t))^2,$$

with initial conditions:

$$y(0) = 1$$

Solve the Riccati equation problem using the adomian decomposition method (ADM).

Q.3 **(A)** Consider the following non-homogenous differential system: [25]

$$\frac{dx}{dt} = z - \cos(t),$$

$$\frac{dy}{dt} = z - e^{t},$$

$$\frac{dz}{dt} = x - y$$

with initial conditions:

$$x(0) = 0, \quad y(0) = 0, \quad z(0) = 2$$

Solve the non-homogenous differential system using the differential transform method (DTM).

(B) Consider the following systems of non-linear differential equations:

$$\frac{dx}{dt} + \frac{dy}{dt} + x + y = 1$$

$$\frac{dy}{dt} = 2x + y.$$

with initial conditions:

$$x(0) = 0, \quad y(0) = 1$$

Solve the non-linear differential systems using *the differential transform method* (*DTM*).

(C) The governing equation of a uniform Bernoulli–Euler beam under pure bending resting on fluid layer under axial force is:

$$EI\frac{\partial^4 \nu}{\partial x^4} + p\frac{\partial^2 \nu}{\partial x^2} + k_f \nu + F(x,t) = 0, 0 \le x \le L_e.$$

with boundary conditions (Clamped–Simply supported):

$$at \;\; x = 0, W(x) = \frac{dW(x)}{dx} = 0$$

$$at \;\; x = L_{e,} W(x) = \frac{d^2W(x)}{dx^2} = 0$$

Solve the Riccati equation problem using the adomian decomposition method (ADM). Then compared the results with exact solutions.

(D) Consider the following Initial value problem equation

$$\frac{dy}{dt} = t^3 y^2(t) - 2t^4 y(t) + t^5 + 1$$

with initial conditions:

$$y(0) = 0.$$

Solve the problem using the adomian decomposition method (ADM).

This exam measures the following ILOs								
Question Number	**Q1-a**	**Q1-b**	**Q3-b**	**Q4-a**	**Q1-c**	**Q2-a**	**Q3-a**	**Q4-c**
	Q4-b				**Q2-b**	**Q2-c**	**Q3-c**	
	Knowledge &understanding skills				**Intellectual Skills**		**Professional Skills**	

With our best wishes

Dr.Rizk Masoud   *Dr.Ramzy M. Abumandour*