Menofia University Faculty of Engineering Shebien El-kom First Semester Examination Academic Year : 2018-2019		Department : Civil Engineering Year : 1st Subject : Eng. Mathematics(2) Time Allowed : 3 hours Date : 21/1/2019

Allowed Tables and Charts : None — الامتحان في صفحتين

Answer all the following questions: [100 Marks]

Question 1 [50 Marks]

(A) Find the general solution of the differential equations [16 Marks]

(i) $\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$ *(ii)* $ysin2xdx - (y^2 + cos^2 x)dy = 0$

(iii) $\frac{1}{x}\frac{dy}{dx} - \frac{2y}{x^2} = xcosx,\ x > 0$ *(iv)* $y\frac{d^2y}{dx^2} + 1 = \left(\frac{dy}{dx}\right)^2$

(B) Find the solution of the ODE problem

$$\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$$ [4 Marks]

(C) *i)* Solve the differential equation $(x^2D^2 - xD + 2)y = xlnx$ [4 Marks]

ii) Calculate the volume of the body bounded by the surfaces:

$z = 4 - x,\quad x + y = 2,\quad x = y = z = 0$ [4 Marks]

(D) *(i)* Solve the following system of simultaneous ordinary differential equations. $\frac{d^2x}{dt^2} - 3x - 4y = 0$, $\frac{d^2y}{d^2t} + x + y = 0$ [8 Marks]

(ii) Solve the following ODEs [8 Marks]

1. $(D^3 - 5D^2 + 7D - 3)y = e^{2x}coshx$
2. $(D^2 + 9)y = cos2x + sin2x$

(E) Solve the differential equation $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4x = te^t$ **using Laplace transform** with initial conditions: $x(0) = 0\ and\ x'(0) = 0$. [6 Marks]

Question 2 [50 Marks]

(A) (i) By transforming into polar coordinates evaluate the integral (Jacobian)

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} x^2 + y^2 \, dy dx$$ [4 Marks]

i) Find the interval of convergence of the series $S_n = \sum_{n=1}^{\infty} \frac{(2x)^n}{n}$. [4 Marks]

ii) Calculate the double integral $\iint_D f(x,y)dxdy$ for $f(x,y) = 3 + x^2 + y^3$ and D is bounded by $0 \le x \le 1, \quad 0 \le y \le x$. [4 Marks]

(B) Find the inverse Laplace transform of the functions

(i) $F(s) = \ln \frac{s+1}{s-1}$ *(ii)* $F(s) = \frac{1}{(s)(s-2)^2\left(s^2+1\right)}$ [8 Marks]

(C) Find Laplace transform of the following functions

(i) $f(t) = \frac{1-\cos t}{t}$ *(ii)* $f(t) = \begin{cases} 0 & t < \frac{2\pi}{3} \\ \cos(t - \frac{2\pi}{3}) & t > \frac{2\pi}{3} \end{cases}$ [8 Marks]

(D) Test the convergence of the following series:

(i) $S_n = \sum_{n=1}^{\infty} \frac{2n-1}{2^n}$ *(ii)* $S_n = \sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$ [8 Marks]

(E) A periodic function $f(x)$ with period 2π is defined as follows:

$$f(x) = x \qquad -\pi \le x \le \pi$$

i) Plot the function. *ii)* Find the corresponding Fourier series. [8 Marks]

(F) Find coordinates of the center of the mass of a thin plate bounded by the parabola $y = 6x - x^2$ and the straight line $y = x$, given that it has a mass density $f(x,y) = \rho(x,y) = 1$. [6 Marks]

With my best wishes

Associate Prof. Dr. Islam M. Eldesoky