MULTI-LEVEL POWER FLOW FOR LARGE NETWORKS

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(طريقة متعددة المستويات لحل مشكلة أسياب القدرة فى الشبكات الكبيرة)

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الخلاصة:
يعرض هذا البحث طريقة جديدة لحل مشكلة أسياب القدرة فى الشبكات الكبيرة جدا . ولانقاص وقت الحسابات بشدة يتم تقسيم الشبكة الى شبكات منفصلة صغيرة ، كل تحل على حدة فى أن واحد . وهذه الحلول المنفصلة يتم دفعها للحل الشامل (كما لو لم تقسم الشبكة) عن طريق منسق علوى . وقد تم أستنباط نظرية لاثبات التقارب للحل الشامل لهذه الطريقة وكذلك أعطيت أمثلة توضيحية ..

ABSTRACT : The paper presents a new method for load flow solution for large electric networks. The computation time is reduced by tearing the network into a number of subsystems. Each of the decomposed sub network is a separate load flow problem to be solved. The solution of sub networks may be executed in parallel, resulting in a considerable time saving for on line control. The separate solutions are iteratively driven, via a hierarchical coordinator, into the original load flow solution employing the interaction prediction principle. A theorem to guarantee convergence of the algorithm and illuatrative examples are given.

1. INTRODUCTION

The Newton Raphson method is now widely adopted by power industry to solve the load flow problem[1,2,3]. For ill conditioned power systems, the problem is solved in[4,5] using optimization techniques.

For load flow solution of large power systems, decomposed or piece wise or diakoptical methods [6] are encouraged. In the such approach a number of smaller subproblems are coordinated by a master processor. Application of such methods within coupled computer networks has a number of advantages important in on line applications (low overall solution time, low storage requirements and low interprocessor data communication[7]).

In order to achieve the above objectives, the conventional load flow formulation as an optimization problem[8] has been modified and a new mathematical approach to the coordination process has been developed.

2- PROBLEM FORMULATION

The load -flow problem using the nonlinear programming formulation is outlined [8]. Given a power network of n+1 buses, the load-flow problem may be defined as that of seeking the solution (column) vectors

$$e = [\, e_1 \ldots e_n \,]^T \;,\; v = [\, v_1 \ldots v_n \,]^T \qquad (1)$$

to the set of nonlinear algebraic equations

$$\frac{p_i - jq_i}{e_i^2 + v_i^2}(e_i + jv_i) = \sum_{k=0}^{n} (g_{ki} + jb_{ki})(e_k + jv_k) \qquad (i=1,\ldots,n) \qquad (2)$$

Where $p_i + jq_i$ is the impressed power, $e_i + jv_i = V_i$ is the bus voltage, $g_{ki} + jb_{ki} = y_{ki}$ is the ki element of the admittance matrix and the superscipt T denotes transpose. (The last equation is abtained using the fact that

$$\text{conj}\ [(p_i + jq_i)/(e_i + jv_i)] = I_i = \sum_{k=0}^{n} Y_{ki} V_k \ )$$

Busbar 0 is the so called slack bus and its voltage is constant, usually 1+j0. Seperating the real and imaginary parts of (2) gives

$$\sum_{k=1}^{n} (g_{ki} e_k - b_{ki} v_k) + g_{oi} - \frac{p_i e_i + q_i v_i}{e_i^2 + v_i^2} = 0 \qquad (i=1,\ldots,n) \qquad (3)$$

$$\sum_{k=1}^{n} (g_{ki} v_k + b_{ki} e_k) + b_{oi} - \frac{p_i v_i - q_i e_i}{e_i^2 + v_i^2} = 0 \qquad (i=1,\ldots,n) \qquad (4)$$

Using matrix notation (3) and (4) may be written as

$$r = yx + y_o + f \qquad (5)$$

Where r = residual vector , $x = [e_1 \ldots e_n ; v_1 \ldots v_n]^T$

$$y = \begin{bmatrix} G & -B \\ B & G \end{bmatrix}$$

$$y_o = [\, g_{o1} \ldots g_{on} \,;\, b_{o1} \ldots b_{on} \,]^T$$

and

$$f = \left[\ldots - \frac{p_i e_i + q_i v_i}{e_i^2 + v_i^2} \ldots\ldots ; \ldots\ldots - \frac{p_i v_i - q_i e_i}{e_i^2 + v_i^2} \ldots \right]^T = [\, f^{*} \,;\, f^{\sim} \,]^T$$

Since the power mismatches are exactly the residual vector r, we are led to the following restatement of the load flow problem. Find the solution vector X, such that a performance index F,

$$F = r^T r \tag{6}$$

is minimized and convergence is assumed if a certain convergence criterion is met (e.g. $F \leq \epsilon$).

However, the large dimensionality of the problem may lead to numerical difficulties. Moreover, it may be more beneficial to solve this problem on a decentralized basis. Therefore, a procedure based on system decomposition will be proposed.

3-DECOMPOSITION-COORDINATION SOLUTION

Decomposing the large network into N subnetworks 1,2,...,I,J,...,N, we have.

$$X^T = (E_1^T \ldots E_N^T \;;\; V_1^T \ldots V_N^T)$$

where $E_I (V_I)$ is the vector of real (imaginary) parts of voltages of all the buses of subnetwork I.

For the true solution X, we assume an approximate one $U^T = [\; E_1^{`T} \ldots E_N^{`T} \;;\; V_1^{`T} \ldots V_N^{`T} \;]$. Hence the minimization problem becomes

$$\min \sum_{I=1}^{N} (\| E_I - E_I^{`} \|^2 + \| V_I - V_I^{`} \|^2) \tag{7}$$

Subject to the constraints [obtained from (3) & (4)]:

$$G_I E_I + \sum_{J \neq I} G_{IJ} E_J^{`} - B_I V_I - \sum_{J \neq I} B_{IJ} V_J^{`} + g_{OI} + f_I^{*`} = 0 \tag{8}$$

$$G_I V_I + \sum_{J \neq I} G_{IJ} V_J^{`} + B_I E_I + \sum_{J \neq I} B_{IJ} E_J^{`} + b_{OI} + f_I^{\sim`} = 0 \tag{9}$$

$$E_I - E_I^{`} = 0 \tag{10}$$

$$V_I - V_I^{`} = 0 \tag{11}$$

Writing the Lagrangian of the above problem, one gets

$$L=\sum_{I=1}^{N} L_I=\| E_I- E_I' \|^2 + \| V_I- V_I' \|^2$$

$$+\alpha_I^T(G_I E_I+ \Sigma G_{IJ}E_J' - B_I V_I- \Sigma B_{IJ}V_J'+ g_{oI}+\overset{*}{f}_I')$$

$$+\beta_I^T(G_I V_I+ \Sigma G_{IJ}V_J' + B_I E_I+ \Sigma B_{IJ}E_J'+ b_{oI}+\tilde{f}_I')$$

$$+\gamma_I^T(E_I-E_I') + \nu_I^T(V_I-V_I') \qquad (12)$$

Where the vectors $\overset{*}{f}_I'$ and $\tilde{f}_I'$ are those corresponding to subnetwork I with the substitution E_I' and V_I' for the buses voltages.

The necessary conditions of optimality are given by:

$$\frac{\partial L}{\partial E_I}= 2(E_I-E_I') + G_I^T\alpha_I+B_I^T\beta_I+\gamma_I=0 \qquad (13)$$

$$\frac{\partial L}{\partial E_I'}= -2(E_I-E_I') +\sum_{J\neq I} G_{JI}^T\alpha_J+D_I\alpha_I+\sum_{J\neq I} B_{JI}^T\beta_J+H_I\beta_I-\gamma_I=0 \qquad (14)$$

Where

$$D_I=\text{diag}[..(p_i(e_i'^2- v_i'^2)+2q_i e_i' v_i')/(e_i'^2+v_i'^2)^2...] \ \forall i\in I$$

$$H_I=\text{diag}[..(q_i(v_i'^2- e_i'^2)+2p_i v_i' e_i')/(e_i'^2+v_i'^2)^2...] \ \forall i\in I$$

$$\frac{\partial L}{\partial V_I}= 2(v_I-v_I') - B_I^T\alpha_I+G_I^T\beta_I+\nu_I=0 \qquad (15)$$

$$\frac{\partial L}{\partial V_I'}= -2(v_I-v_I') - \Sigma B_{JI}^T\alpha_J+H_I\alpha_I+\Sigma G_{JI}^T\beta_J+D_I\beta_I-\nu_I=0 \qquad (16)$$

$$\frac{\partial L}{\partial \alpha_I}=G_I E_I+ \Sigma G_{IJ}E_J' - B_I V_I- \Sigma B_{IJ}V_J' +g_{oI}+\overset{*}{f}_I'=0 \qquad (17)$$

$$\frac{\partial L}{\partial \beta_I}=G_I V_I+ \Sigma G_{IJ}V_J' + B_I E_I+ \Sigma B_{IJ}E_J' +b_{oI}+\tilde{f}_I'=0 \qquad (18)$$

$$\frac{\partial L}{\partial \gamma_I}= E_I-E_I' =0 \qquad (19)$$

$$\frac{\partial L}{\partial \nu_I}= V_I-V_I' =0 \qquad (20)$$

Based on the above derived necessary conditions of optimality, the following algorithm is proposed (Fig 1):

1) Given an initial guess to the coordinating variables E'_I, V'_I, γ_I & ν_I using eq^{ns} (19),(20),(14) and (16) respectively and send them to the lower level. Put the iteration index k=1

2) At the subsystem level, solve for V_I, E_I using (17) and (18). Also solve for β_I, α_I using (13) & (15). Send the result to the coordinator level .

3) Update E'_I, V'_I, γ_I and ν_I using (19),(20),(14) and (16) respectivly .Put the iteration index k=k+1. If the error is less than a specified tolerance ϵ, stop the iteration s and print the results . If not go to step (2) above .

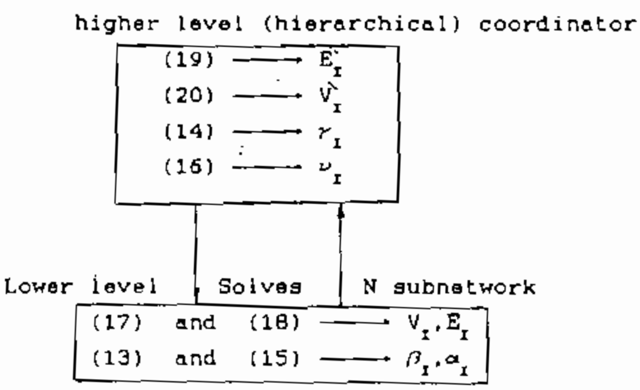


Fig (1)
Schematic representation of the two Level algorithm

4- CONVERGENCE OF THE ALGORITHM

Although the algorithm is general ,certain simplifying assumptions are put to derive a sufficient condition for the convergence as given by the following theorem.

Theorem

Under the following assumptions (valid in electric power networks):

i) When the iteration number is sufficiently large, the bus voltages $\grave{e}_I \approx 1$ and $\grave{v}_I \approx 0$

ii) Negligible conductance $G \approx 0$

the above load_flow algorithm converges to the solution if

$$\max \left\{ \rho\,(B_b^{-1} B_o)\ ,\ \rho \begin{bmatrix} -(H_b+B_o^T)B_b^{-T} & D_b B_b^{-T} \\ (H_b-B_o^T)B_b^{-T} & -D_b B_b^{-T} \end{bmatrix} \right\} < 1$$

Where $\rho(.)$ is the spectral radius of the matrix (.) ie the eigenvalue with largest absolute value.

Proof

Let us assume that we are at the k^{th} iteration .Then using (19) and (20),we have :

$$E^{\backslash k+1} = E^k \tag{21}$$

$$V^{\backslash k+1} = V^k \tag{22}$$

where $E^T = [E_1^T \ldots E_I^T \ldots E_N^T]$,and similarly for $E^\backslash$,V &$V^\backslash$

Also (14) and (16) give

$$\gamma^{k+1} = -2(E^k - E^{\backslash k}) + G_o^T \alpha^K + D_b \alpha^K + B_o^T \beta^K + H_b \beta^K \tag{23}$$

$$\nu^{K+1} = -2(V^K - V^{\backslash K}) - B_o^T \alpha^K + H_b \alpha^K + G_o^T \beta^K + D_b \beta^K \tag{24}$$

Where

$\gamma^T = [\gamma_1^T \ldots \gamma_N^T]$ & similarly for α, β, ν

$$G_o = \begin{bmatrix} 0 & G_{12} & \cdots & G_{1N} \\ & 0 & & \\ & & \ddots & \\ G_{N1} & & & 0 \end{bmatrix}$$ & similarly for B_o

D_b = bloc diag $[D_1 \ \ldots \ D_I \ \ldots \ D_N]$ & similarly for H_b

Under assumption (ii), substituting from (18)in (21) gives

$$E^{\backslash k+1} = -B_b^{-1}\,[B_o E^{\backslash k} + b_o + f^{\sim\backslash}] \tag{25}$$

Where $f^{\sim\backslash}$ reduces to $[..q..]^T$ under assumption (ii)

Substituting from (17) in (22),one obtains:

$$V^{\backslash k+1} = B_b^{-1}[\, -B_o V^{\backslash k} + g_o + f^{*\backslash}] \tag{26}$$

Where g_o & $f^{*\backslash}$ reduce respectively to the zero vector & $[..-p..]^T$ under assumption (ii).

Also from (13) under assumption (ii),

$$\beta_b^k = -B^{-T}[2(E^k - E^{\backslash k}) + \gamma^k] \tag{27}$$

Similarly from (15)

$$\alpha^k = B_b^{-T}\ [2(V^k - V^{\backslash k}) + \nu^k] \tag{28}$$

Substituting in (23) by (27) & (28)&with assumption (ii):

$$\therefore \gamma^{k+1} = -2(E^k - E^{\backslash k}) + D_b B_b^{-T}[2(V^k - V^{\backslash k}) + \nu^k] - [B_o^T + H_b]B_b^{-T}[2(E^k - E^{\backslash k}) + \gamma^k] \tag{29}$$

Substituting in (24) by (27) & (28)&with assumption (ii):

$$\therefore \nu^{k+1} = -2(V^k - V^{\backslash k}) + [H_b - B_o^T]B_b^{-T}[2(V^k - V^{\backslash k}) + \nu^k] - D_b B_b^{-T}[2(E^k - E^{\backslash k}) + \gamma^k] \tag{30}$$

Now let the error in $E^{\backslash}$ be defined as :

$$\epsilon_{E\backslash}^{k+1} \triangleq E^{k+1} - E^k$$

Similarly for $\epsilon_{V\backslash}^{k+1}$, ϵ_{γ}^{k+1} & ϵ_{ν}^{k+1}

Then from (25),(26),(29)&(30) we get :

$$\epsilon_{E\backslash}^{k+1} = -\ B_b^{-1}B_o\ \epsilon_{E\backslash}^k \tag{31}$$

$$\epsilon_{V\backslash}^{k+1} = -\ B_b^{-1}B_o\ \epsilon_{V\backslash}^k \tag{32}$$

$$\begin{aligned}\epsilon_{\gamma}^{k+1} = {} & 2(B_b^{-1}B_o + I)\epsilon_{E\backslash}^k + D_b B_b^{-T}\ [-2(B_b^{-1}B_o + I)\epsilon_{V\backslash}^k + \epsilon_{\nu}^k\ ] \\ & - (B_b^T + H_b)B_b^{-T}[-2(B_b^{-1}B_o + I)\epsilon_{E\backslash}^k + \epsilon_{\gamma}^k\ ]\end{aligned} \tag{33}$$

$$\begin{aligned}\epsilon_{\nu}^{k+1} = {} & 2(B_b^{-1}B_o + I)\epsilon_{V\backslash}^k + (H_b - B_o^T\ )B_b^{-T}[-2(B_b^{-1}B_o + I)\epsilon_{V\backslash}^k + \epsilon_{\nu}^k\ ] \\ & - D_b B_b^{-T}[-2(B_b^{-1}B_o + I)\epsilon_{E\backslash}^k + \epsilon_{\gamma}^k\ ]\end{aligned} \tag{34}$$

Or:

$$\begin{bmatrix} \epsilon_{E'} \\ \epsilon_{V'} \\ \epsilon_{\gamma} \\ \epsilon_{\nu} \end{bmatrix}^{K+1} = \begin{bmatrix} -B_b^{-1}B_o & 0 & 0 & 0 \\ 0 & -B_o^{-1}B_o & 0 & 0 \\ \times & \times & -(B_o^T+H_b)B_b^{-T} & D_bB_b^{-T} \\ \times & \times & -D_bB_b^{-T} & (H_b-B_o^T)B_b^{-T} \end{bmatrix} \begin{bmatrix} \epsilon_{E'} \\ \epsilon_{V'} \\ \epsilon_{\gamma} \\ \epsilon_{\nu} \end{bmatrix}^{K} \quad (35)$$

Where X is a matrix of no importance.
The error vector in the proposed alogarithm reduces to zero (convergence) if the spectral radius of the matrix in the R.H.S. of eqn.(35) is <1 ,or :

$$\max \left\{ \rho\,(B_b^{-1}B_o)\ ,\ \rho \begin{bmatrix} -(B_o^T+H_b)B_b^{-T} & D_bB_b^{-T} \\ -D_bB_b^{-T} & (H_b-B_o^T)B_b^{-T} \end{bmatrix} \right\} <1$$

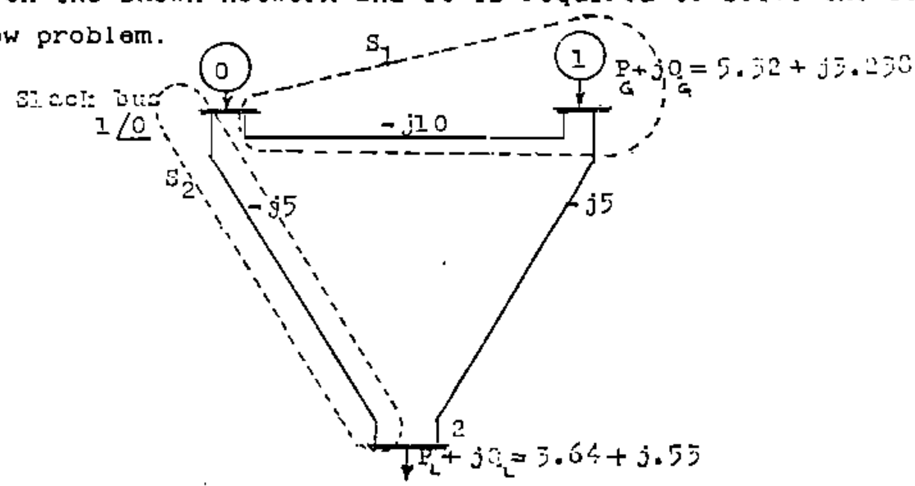
Which proves the assertion.

5- ILLUSTRATIVE EXAMPLES:

Here we give two examples to demonstrate the effectiveness of the proposed algorithm.

EXAMPLE 1:

Given the shown network and it is required to solve the load flow problem.



(all quantities are in per unit on a common base)

Fig (2)

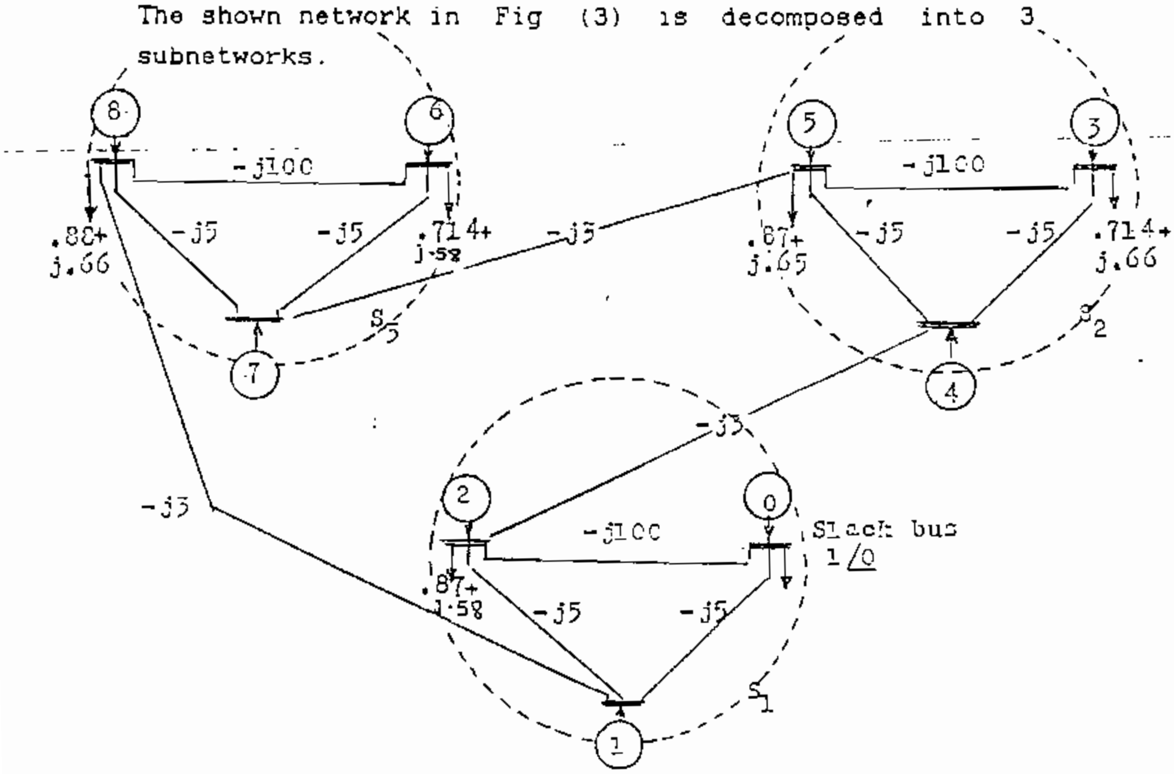
Applying the proposed algorthım , the solution :

$V_1 = 1.1 \angle 15^\circ$ and $V_2 = 0.9 \angle -15^\circ$

is obtained in 18 iterations.

EXAMPLE 2:

The shown network in Fig (3) is decomposed into 3 subnetworks.



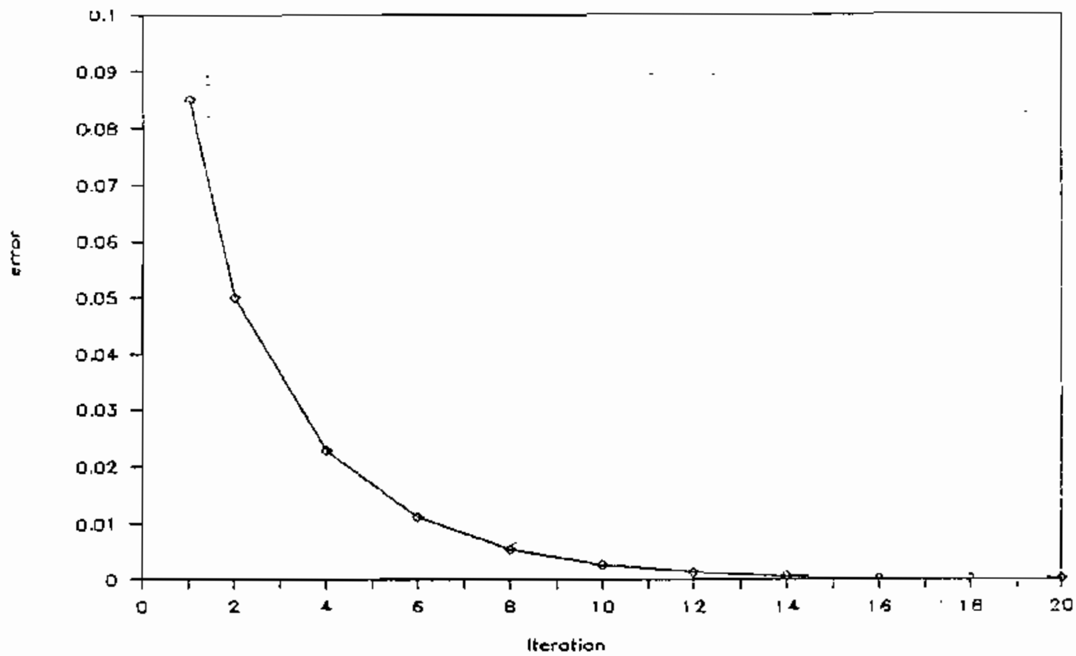
(all quantities are in per unit on a common base)

Fig (3)

For the following generation

Bus #	1	2	3	4	5	6	7	8
P	.92	.24	.24	.97	.4	.24	.92	.28
Q	0	.58	.66	0	.65	.58	0	.66

and starting the algorithm with $e_i^\cdot = 1$, $v_i^\cdot = 0$, $\gamma_i = 1$, $\nu_i = 1$ ∀buses, the solution is obtained in 20 iterations on Micronet computer using Microsoft Quick Basic 2.0. The calculation time is 30 sec to achieve an accuracy of 10^{-6} for the largest absolute error in bus voltages from iteration k to k+1, Fig(4). For the same accuracy, the example is recomputed using the solver owner's Handbook_Eureka and the compution time is 25 sec. Of course the computation time for the proposed algorithm will be much reduced if parallel processors are



0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
error
0
2
4
6
8
10
12
14
16
18
20
Iteration

Fig. (4)

computing simultaniously in the lower level and not computing serially as has been done in this example.

6-CONCLUSIONS

The paper presents a new method for large load flow proplems. The objective has been to reduce the computation time of a given system by tearing the network into a number of independent subsystems. The subsystem programs may be executed in parallel, resulting in a considerable time saving for on-line system control. Using a hierarchical coordination procedure, it has been shown that the decentralized solutions, converge iteratively to the overall global solution. Examples have been given to support the theoretical investigations.

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