هندسة الإنتاج

	Mansoura University Faculty of Engineerig First year Production	(اولى إنتاج) Time 3 hr. (Date: May, 2013)	Second semester: 2012-2013 Math. 4 - Code: BAS 5121 Final Semester Exam

Answer the following questions [Full Marks 110]

Question 1 [28 Marks]

(a) Prove that: $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$. **[8 marks]**

(b) Evaluate the integrals: (i) $\int_0^1 \sqrt{\ln\left(\frac{1}{x}\right)}\,dx$ **(ii)** $\int_0^{\frac{\pi}{2}} \sqrt{\tan x}\,dx$. **[8 marks]**

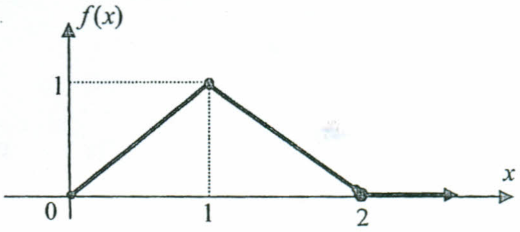
(c) Prove that: $\frac{d}{dx}\left[x^n J_n(x)\right]=x^n J_{n-1}(x)$, **where** $J_n(x)$ **is the Bessel function of order** ***n*****.**

[8 marks]

(d) Write the general solution of the Legendre D. E. $(1-x)^2 y''-2xy'+12y=0$. **[4 marks]**

Question 2 [27 Marks]

(a) Find the <u>Fourier sine series</u> to the function shown in Fig.



[8 marks]

(b) Use the Fourier cosine integral representation to the function $f(x)=e^{-3x}, x>0$**, prove that:**

$$\int_0^\infty \frac{\cos\omega x}{9+\omega^2}\,d\omega=\frac{\pi}{6}e^{-3x}, x>0.$$ **[6 marks]**

(c) (i) Use the separation of variables technique; find the solution of the heat equation:

PDE: $U_t(x,t)=kU_{xx}(x,t),\quad 0<x<L,\ t>0,$

BC: $U(0,t)=U(L,t)=0,$

IC: $U(x,0)=f(x).$

(ii) Find the solution if: $k=1,\ L=2$ **and** $f(x)$ **is the function shown in the Fig. of item (a).**

[13 marks]

Good luck (أنظر خلف الورقة)

Mansoura University,
Faculty of Engineering,
Mathematical & Physical Sciences Dept.

من الخارج

Second Term Examination, 2012-2013
1st year Eng. Students, Math. (4)
Time allowed: 3 Hours

Answer the following questions

1. (a) Prove that:

i. $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. [5 pts]

ii. $J_n'(x) - \frac{n}{x}J_n(x) = -J_{n+1}(x)$. [5 pts]

(b) Evaluate the following integrals: [12 pts]

$\int_0^\infty x^a a^{-x} dx$, $\int_0^1 \sqrt{x}\ \sqrt[3]{1-x^2} dx$,

$\int_{-1}^1 x\, P_n(x) P_{n-1}(x) dx$, $\int x^2 J_0(x) J_1(x) dx$.

(c) i. Expand the function $f(x) = 4x^3 + 6x^2 + 7x + 2$ in a series of Legendre polynomials. [6 pts]

ii. Prove that: [5 pts]

$nP_n(x) = (2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x)$.

2. (a) Expand $f(x) = \begin{cases} 2 & if \quad -2 \le x < 0 \\ x & if \quad 0 < x < 2 \end{cases}$, [10 pts]

in a Fourier series and prove that $\frac{\pi^2}{8} = \sum_{r=0}^{\infty} \frac{1}{(2r+1)^2}$.

(b) Using the Fourier integral, verify the identity [8 pts]

$\int_0^\infty \frac{\omega \sin \omega x}{\omega^2+1} d\omega = \frac{\pi}{2} e^{-x}, \quad x > 0$.

(c) Solve the boundary value problem [14 pts]

$u_{tt} = C^2 u_{xx}, \qquad 0 < x < L, \quad t > 0$

$u(0,t) = u(L,t) = 0, \qquad t > 0$

$u(x,0) = f(x),\ u_t(x,0) = g(x), \qquad 0 < x < L$

for a string of unit length, subject to the given conditions $f(x) = g(x) = x^2$ and $C = \frac{1}{\pi}$.

إقلب الورقة من فضلك

Mamdouh El-Saidy