ROBOT KINEMATIC CALIBRATION ACCURACY USING GENETIC ALGORITHM

Salem Samak
Prod. Eng. & Mech. Design Dept.
Faculty of Engineering
Menoufia University
Shebin El-Kom, EGYPT

ABSTRACT

Position and orientation accuracy of the end-effector is influenced by the precision of kinematic parameters elements of the robot. Thus, good precision requires good knowledge of robot physical parameters values. However, this condition can be difficult to meet in practice. Hence, calibration techniques can be devised in order to improve the robot accuracy through estimation of those particular parameters. In this paper, the Genetic Algorithm (GA) is used to calibrate the robot kinematic accuracy. A kinematic model is formulated and conducted as an optimization problem for serial robot manipulators. The objective is to analyze and evaluate the performance of the GA in optimizing such robot kinematic accuracy. In this algorithm, the errors in the robot parameters represent the parents and offspring population and the error matrix norms represent the cost functions. The convergence and effectiveness of the presented model are demonstrated by a numerical example.

KEYWORDS

Manipulators, Robotics, Calibration, Accuracy, Genetic Algorithm.

1. INTRODUCTION

Genetic Algorithm (GA) [1] is a rather new search tool in robotics, which exhibits high efficiency in certain multi-modal and multi-dimensional domains. The algorithm has been implemented to handle the optimization problems for a number of different application areas, such as robotics [2], composite materials

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design, scheduling [3] and also salesman problems [4]. Recently, it is a gate for the automatic design, which is the best way to develop artificial intelligence. It can be easily adapted to the planning problems for many types of assembly machines [5]. The GA uses population as parents to represent possible solutions. The fitness of all of the individuals in the population is evaluated by its cost. The genetic operators (such as crossover, inversion, rotation and mutation) are applied to generate new population called offspring in an iterative procedure to obtain nearly optimal solutions.

The position and orientation of the end-effector would have no errors other than those caused by imperfection of the repeatability and dynamic effects. However, in more sophisticated applications, errors in the position and orientation of the end–effector result from the kinematic parameters errors as well, which are mostly due to manufacturing and/or measurement errors.

The calibration of robot manipulators has attracted many researchers. Veitschegger and Wu [6] have presented a result comparison between two models. The first model has ignored the higher order terms and did not address the special case of two consecutive parallel joints, while the second model has considered both cases. Wu [7] has used a new technique to correct the kinematic errors of robot manipulator. Vukobratović and Borovac [8] have investigated the influence of the deviations of the links nominal measures (due to manufacturing tolerances) on the accuracy of positioning the manipulator tip for various mechanism configurations. Bruyninckx et al. [9] have developed a systematic and fully general model-based approach to compliant robot motion, taking into account uncertainties in the geometry of the manipulated object and the environment with which it is in contact. Samak et al. [10] have studied the effect of kinematic perturbations on robot precision. Kazerounian and Qian [11] have presented a kinematic calibration model for position and orientation of serial manipulator end-effector errors due to repeatability imperfections.

The present paper introduces a new perspective proposal in order to analysis, implement and evaluate the performance of the GA in optimizing the robot kinematic accuracy. The kinematic relationships have been described by using the zero-reference-position (ZRP) method. The prescribed analysis showed improvement in the robot accuracy precision.

2. ZERO POSITION ANALYSIS METHOD

The zero position analysis method was introduced by Gupta [12]. It has the advantages that it is not prone to the discontinuity difficulties as those in the Denavit Hartenberg notation. Due to the nature of this method, small changes in the structure inherently correspond to small changes in the structure parameters. It has also proven its effectiveness and versatility in many works on both kinematic and dynamic analysis of robot manipulators [13]. The joint coordinate systems in this method are not used. Instead, a convenient reference position of the robot is chosen and the following vectors are defined in the world coordinate system,

$\boldsymbol{u}_{0i} \Rightarrow$ a unit vector along joint axis i.

$\boldsymbol{b}_{0i} \Rightarrow$ a body vector which connects a point on joint (i-1) to a point on joint i.

$\boldsymbol{u}_{0a}$ and $\boldsymbol{u}_{0t} \Rightarrow$ two perpendicular vectors fixed on the end-effector.

All the above-mentioned parameters are given in their ZRP (with zero subscript). They are converted to the current position as the manipulator moves to new positions. The current vector are derived from their ZRP vectors as follows,

$$\boldsymbol{u}_i = \boldsymbol{R}_i \boldsymbol{u}_{0i} \tag{1}$$

$$\boldsymbol{b}_{i+1} = \boldsymbol{R}_i \boldsymbol{b}_{0,i+1} \tag{2}$$

$$\boldsymbol{u}_a = \boldsymbol{R}_h \boldsymbol{u}_{0a} \quad \text{and} \quad \boldsymbol{u}_t = \boldsymbol{R}_h \boldsymbol{u}_{0t} \tag{3}$$

Where i = 1, 2, ..., n; the 3 by 3 rotation matrix $\mathbf{R}_i$, for a revolute joint, is defined as

$$\boldsymbol{R}_i = \boldsymbol{R}(q_1, \boldsymbol{u}_{o1}) \boldsymbol{R}(q_2, \boldsymbol{u}_{o2}) \quad \cdots \quad \boldsymbol{R}(q_i, \boldsymbol{u}_{oi}) = \prod_{j=1}^{i} \boldsymbol{R}(q_j, \boldsymbol{u}_{oj}) \tag{4}$$

Hence, for n-revolute joints manipulators, the above equation represents the hand orientations $\boldsymbol{R}_h$ as

$$\boldsymbol{R}_h = \prod_{i=1}^{n} \boldsymbol{R}(q_i, \boldsymbol{u}_{0i}) \tag{5}$$

The matrix $\boldsymbol{R}(q_i, \boldsymbol{u}_{oi})$ represents a rotation by q_i about a screw axis $\boldsymbol{u}_{oi}$. It can be written as [14].

$$\boldsymbol{R}(q_i, \boldsymbol{u}_{oi}) = \begin{pmatrix} (u_x^2 - 1)V_i + 1 & u_x u_y V_i - u_z S_i & u_x u_z V_i + u_y S_i \\ u_x u_y V_i + u_z S_i & (u_y^2 - 1)V_i + 1 & u_y u_z V_i - u_x S_i \\ u_x u_z V_i - u_y S_i & u_y u_z V_i + u_x S_i & (u_z^2 - 1)V_i + 1 \end{pmatrix} \tag{6}$$

where, $V_i = 1 - cos(q_i)$ and $S_i = sin(q_i)$, and u_x, u_y and u_z are components of the unit vector $\boldsymbol{u}_{0i}$. If the i[th] joint is prismatic, then $\boldsymbol{R}(q_i, \boldsymbol{u}_{oi})$ is replaced with a 3 by 3 identity matrix.

Equation (6) can be decomposed as follows,

$$\boldsymbol{R}(q_i, \boldsymbol{u}_{oi}) = V_i \, \boldsymbol{A} + S_i \, \boldsymbol{B} + \boldsymbol{I} \tag{7}$$

where

$$A=\begin{pmatrix} u_x^2-1 & u_xu_y & u_xu_z \\ u_xu_y & u_y^2-1 & u_yu_z \\ u_xu_z & u_yu_z & u_z^2-1 \end{pmatrix} \quad \text{and} \quad B=\begin{pmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{pmatrix} \tag{8}$$

3. ROTATIONAL ERROR MODEL

For small changes in u_x, u_y and u_z, the corresponding small change in the rotation matrix $R(q_i, u_{oi})$ can be shown to be

$$\delta R(q_i, u_{oi}) = R_{xi}\delta u_{0i,x} + R_{yi}\delta u_{0i,y} + R_{zi}\delta u_{0i,z} \tag{9}$$

where

$$R_{xi}=\begin{pmatrix} 2X_i & Y_i & Z_i \\ Y_i & 0 & -S_i \\ Z_i & S_i & 0 \end{pmatrix}, \; R_{yi}=\begin{pmatrix} 0 & X_i & S_i \\ X_i & 2Y_i & Z_i \\ -S_i & Z_i & 0 \end{pmatrix} \; \text{and} \; R_{zi}=\begin{pmatrix} 0 & -S_i & X_i \\ S_i & 0 & Y_i \\ X_i & Y_i & 2Z_i \end{pmatrix} \tag{10}$$

and $X_i = u_{0ix}V_i$, $Y_i = u_{0iy}V_i$ and $Z_i = u_{0iz}V_i$

For small changes in $R(q_i, u_{0i})$, the hand orientation, Eq. (5), becomes

$$R_h + \delta R_h = \prod_{i=1}^{n} [R(q_i, u_{0i}) + \delta R(q_i, u_{0i})] \tag{11}$$

Ignoring second and higher orders of variations, the above equation leads to

$$\delta R_h R_h^T = \sum_{i=1}^{n} [R_{i-1} \delta R(q_i, u_{0i}) R_i^T] \tag{12}$$

The left-hand side of Eq. (12) is a skew symmetric matrix. It has only three significant elements namely (3,2), (1,3) and (2,1) or δr_1, δr_2 and δr_3. Therefore, $\delta R_h R_h^T$ could be converted to a vector δr_h, which shows the errors in the end-effector orientation. Substituting $\delta R(q_i, u_{0i})$ from Eq. (9) into Eq. (12) yields

$$\delta r_h = \sum_{i=1}^{n} \left\{ R_{i-1} \left(R_{xi}\delta u_{0i,x} + R_{yi}\delta u_{0i,y} + R_{zi}\delta u_{0i,z} \right) R_i^T \right\} \tag{13}$$

4. POSITIONAL ERROR MODEL

The position vector P_p of a reference point p at the hand

$$P_p = \sum_{i=1}^{n} b_{i+1} = \sum_{i=1}^{n} R_i b_{0,i+1} = \sum_{i=1}^{n} \left\{ \left(\prod_{j=1}^{i} R(q_j, u_{0j}) \right) b_{0,i+1} \right\} \tag{14}$$

For small changes in P_p, the position vector becomes

$$P_p + \delta P_p = \sum_{i=1}^{n} \left\{ \left(\prod_{j=1}^{i} \left(R(q_j, u_{0j}) + \delta R(q_j, u_{0j}) \right) \right) \left[b_{0,i+1} + \delta b_{0,i+1} \right] \right\} \tag{15}$$

Ignoring second and higher orders of variations, the above equation leads to

$$\delta P_p = \sum_{i=1}^{n} \left\{ R_{i-1} \delta R(q_i, u_{0i}) \left\{ b_{0,i+1} + \sum_{j=i+1}^{n} \left[\left(\prod_{k=i+1}^{j} R(q_k, u_{0k}) \right) b_{0,j+1} \right] \right\} \right\} + \sum_{i=1}^{n} R_i \delta b_{0,i+1} \tag{16}$$

The above equation shows the errors in the end-effector position. Substituting $\delta R(q_i, u_{0i})$ from Eq. (9), the above equation becomes

$$\delta P_p = \sum_{i=1}^{n} \left\{ R_{i-1} \left(R_{xi} \delta u_{0i,x} + R_{yi} \delta u_{0i,y} + R_{zi} \delta u_{0i,z} \right) E \right\} + \sum_{i=1}^{n} R_i \delta b_{0,i+1} \tag{17}$$

where

$$E = b_{0,i+1} + \sum_{j=i+1}^{n} \left(R(q_j, u_{0j})\, b_{0,j+1} \right) \tag{18}$$

In order to insure the length constraint of the unit vectors, the following constraint is to be satisfied,

$$\| u_{0i} \| = \sqrt{u_{0i,x}^2 + u_{0i,y}^2 + u_{0i,z}^2} = 1 \tag{19}$$

For small changes,

$$u_{0i,x} \delta u_{0i,x} + u_{0i,y} \delta u_{0i,y} + u_{0i,z} \delta u_{0i,z} = 0 \quad \text{or} \quad u_{0i}^T \delta u_{0i} = 0 \tag{20}$$

The above constraint is also implemented as an extension to the error Jacobian matrix.

5. ERROR JACOBIAN MATRIX

From Eqs. (13,17,20), the following equation can be constructed,

$$J \begin{Bmatrix} \delta u_0 \\ \delta b_0 \end{Bmatrix} = \begin{Bmatrix} \delta r_h \\ \delta p_p \end{Bmatrix} \tag{21}$$

The above equation describes a linear relationship between the errors in the robot kinematic parameter elements ($\delta \boldsymbol{u}_{0i}$ and $\delta \boldsymbol{b}_{0i}$), which can be defined as follows

$$\delta \boldsymbol{u}_0 = (\delta u_{01x} \quad \delta u_{01y} \quad \delta u_{01z} \quad \cdots \quad \delta u_{0nz})^T \tag{22}$$

$$\delta \boldsymbol{b}_0 = (\delta b_{01x} \quad \delta b_{01y} \quad \delta b_{01z} \quad \cdots \quad \delta b_{0nz})^T \tag{23}$$

and the error in the position and orientation of the end-effector ($\delta \boldsymbol{r}_h$ and $\delta \boldsymbol{p}_p$). The matrix $\boldsymbol{J}$ represents the error Jacobian matrix, which can be expressed as

$$\boldsymbol{J} = \begin{pmatrix} \boldsymbol{J1} & \boldsymbol{J2} \\ \boldsymbol{J3} & \boldsymbol{J4} \\ \boldsymbol{J5} & \boldsymbol{J6} \end{pmatrix} \tag{24}$$

The i[th] elements of the submatrix $\boldsymbol{J1}$ are defined as

$$\boldsymbol{J1}_i = \left[\boldsymbol{R}_{i-1} \boldsymbol{R}_{xi} \boldsymbol{R}_i^T \quad \boldsymbol{R}_{i-1} \boldsymbol{R}_{yi} \boldsymbol{R}_i^T \quad \boldsymbol{R}_{i-1} \boldsymbol{R}_{zi} \boldsymbol{R}_i^T \right] \tag{25}$$

while $\boldsymbol{J2}_i = \boldsymbol{0}$; and the i[th] elements of $\boldsymbol{J3}$ are

$$\boldsymbol{J3}_i = \left[\boldsymbol{R}_{i-1} \boldsymbol{R}_{xi} \boldsymbol{E} \quad \boldsymbol{R}_{i-1} \boldsymbol{R}_{yi} \boldsymbol{E} \quad \boldsymbol{R}_{i-1} \boldsymbol{R}_{zi} \boldsymbol{E} \right] \tag{26}$$

where $\boldsymbol{E}$ is given by Eq. (18); and

$$\boldsymbol{J4}_i = \boldsymbol{R}_i, \qquad \boldsymbol{J5}_i = \boldsymbol{u}_i \qquad \text{and} \qquad \boldsymbol{J6}_i = \boldsymbol{0} \tag{27}$$

$\boldsymbol{J1}$ to $\boldsymbol{J4}$ are $3 \times 6\text{n}$; while $\boldsymbol{J5}$ and $\boldsymbol{J6}$ are $\text{n} \times 6\text{n}$. Therefore, The error matrix $\boldsymbol{J}$ is then $(6+\text{n}) \times 6\text{n}$. The errors in the kinematic parameter elements ($\delta \boldsymbol{u}_0$ and $\delta \boldsymbol{b}_0$).

6. CALIBRATION ALGORITHM

In the calibration algorithm, the kinematic parameter errors are used to represent the GA's population and their Spectral norms represent the GA's cost functions. The algorithm could be proposed as follows:

1. The nominal link parameters ($\boldsymbol{u}_{0i}$ and $\boldsymbol{b}_{0i}$) and the joint variables $\boldsymbol{q}$ are used for an arbitrary configuration.
2. The nominal hand orientation and position $\boldsymbol{R}_h$ and $\boldsymbol{P}_p$ are computed by using Eqs. (5,14).
3. The error Jacobian matrix $\boldsymbol{J}$ is constructed by using Eqs. (25 – 27).

4. The actual joint variables q_a are calculated by adding a range of random error values.
5. The GA iterations is started by generating some initial populations for the link parameter errors δu_{0i} and δb_{0i}. These populations are used as parents from which the genetic operators are applied to produce new offspring population.
6. The actual hand orientation and position R_k^a and P_p^a are computed as in step 2 by utilizing the actual joint variables.
7. The right hand side of Eq. (21) is obtained as

$$\delta r = \delta R_k R_k^T \quad \text{and} \quad \delta P_p = P_p^a - P_p \tag{28}$$

where

$$\delta R_h = R_k^a - R_k \tag{29}$$

8. The offspring populations are computed from the following relationship (the Pseudo inverse is used).

$$\begin{Bmatrix} \delta u_0 \\ \delta b_0 \end{Bmatrix} = J^{-1} \begin{Bmatrix} \delta r_k \\ \delta p_p \end{Bmatrix} \tag{30}$$

Then, their population costs are evaluated by computing their Spectral norms.

9. The offspring together with their parents are evaluated by their costs. The most fit population are those with the lowest costs.
10. This process is iterated until a certain criterion (such as a certain number of iterations) is met.

7. A CASE STUDY

A Numerical example is presented in an arbitrary configuration for a six-degrees of freedom PUMA-type manipulator (Fig. 1). Its joint variables are chosen as

$$q = (-2.741 \quad 4.501 \quad 2.609 \quad 2.044 \quad 0.389 \quad 2.285)^T$$

The nominal kinematic parameters are listed in Table 1. The joint value errors are randomly taken from a range of ± 0.001 radians; while the kinematic parameter errors are randomly taken from a range of ± 0.01 for screw axes u's and ± 0.5 for body vectors b's.

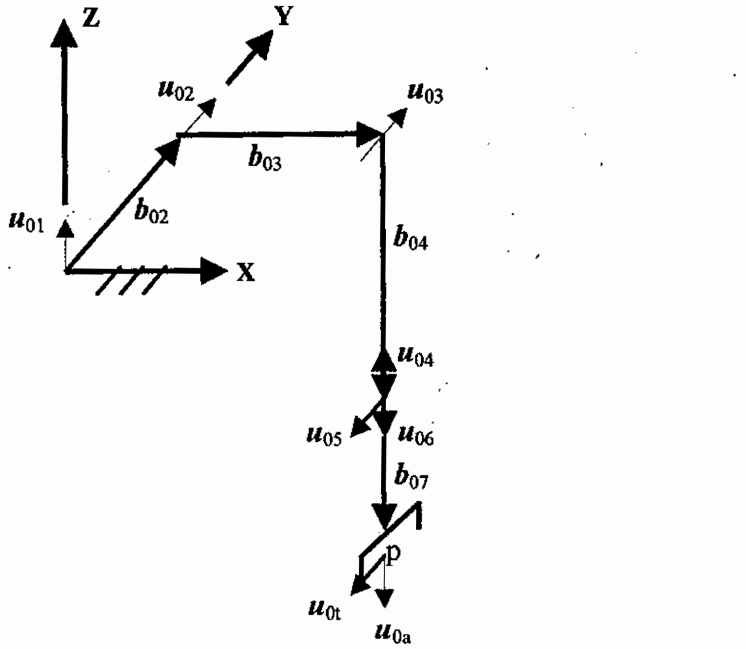

Fig. 1. PUMA-type robot in its ZRP configuration

Table 1. Nominal kinematic parameters

u_{01} : u_{06}						b_{02} : b_{07} (mm)					
0	0	0	0	0	0	0	500	0	0	0	0
0	1	1	0	-1	0	200	0	0	0	0	0
1	0	0	1	0	-1	0	0	-500	0	0	-100

Initial populations are generated to represent the parents from which an initial set of errors along with their initial cost are computed and listed in Table 2. The GA'S operators are then applied to generate the offspring. This algorithm is converged after 11 iterations. The optimal kinematic parameter errors and their cost are given in Table 3. Whereas, the algorithm convergence are shown in Fig. 2.

Table 2. Initial population with its cost

δu_{01} : δu_{06} ($\times 10^{-2}$)						δb_{02} : δb_{07} (mm)						Cost
-0.29	0.64	0.57	-0.12	0.95	-0.29	0.15	0.09	-0.26	0.23	0.31	-0.48	
0.76	-0.51	0.53	0.00	-0.99	0.77	-0.29	-0.07	-0.37	0.27	-0.47	-0.08	0.0112
0.01	0.63	0.25	0.92	0.77	-0.26	-0.09	-0.47	-0.07	-0.36	0.16	-0.00	

Table 3. Final optimal population with its cost

δu_{01} : δu_{06} ($\times 10^{-2}$)						δb_{02} : δb_{07} (mm)						Cost
-0.29	0.64	0.77	-0.99	0.00	0.53	-0.30	-0.32	-0.07	-0.44	-0.33	-0.30	
0.76	-0.51	-0.29	0.95	-0.12	0.57	-0.27	0.20	-0.38	-0.47	-0.38	-0.49	0.0027
0.01	0.92	0.77	0.63	0.25	-0.26	-0.46	0.11	-0.19	-0.40	-0.27	-0.48	

From the foregoing example, it has been shown that the initial cost is 0.0112 and the final optimal cost is 0.0027. Therefore, the optimal kinematic parameter errors are reduced by 24.1%.

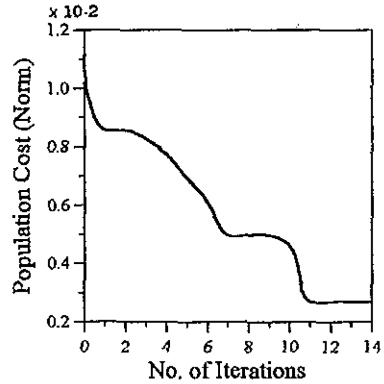

Fig. 2. Convergence of the algorithm

8. CONCLUSIONS

The Genetic Algorithm procedure is used to calibrate the robot kinematic errors based on the zero position analysis method. The effectiveness of the algorithm and its convergence in the presence of small joint errors and measurement errors is demonstrated through a numerical experiment. The kinematic parameter errors are reduced the by 24.1%.

REFERENCES

[1] Goldberg, D.E., "Genetic Algorithms in Search, Optimization, and Machine Learning", Addison-Wesley, 1989

[2] Zhuang, H., Wu, J. and Huang. W., "Optimal planning of Robot Calibration Experiments by Genetic Algorithms", Journal of Robotic Systems, Vol. 14 No. 10, John Wiley & Sons Inc, 1997.

[3] Maturana, F., Gu, P., Naumann, A. and Norrie, D.H., "Object-Oriented Job-Shop Scheduling Using Genetic Algorithms", Computers in Industry, Vol. 32, No. 3, pp. 281-294, 1997.

[4] Bozer, Y.A., Schorn, E.C. and Sharp, G.P., "Geometric Aproaches to solve the Chebyshev Traveling Salesman Problem", IIE Transactions, Vol. 22, pp. 238-254, 1990.

[5] Samak, S., "Optimal Component Assembly Planning of Printed Circuit Using Genetic Algorithm", 2nd Assiut University Inter. Conf. on Mechanical Engineering Advanced Technology for Industrial Production, pp. 377-380, Assiut, Egypt, 1999.

[6] Veitschegger, W.K. and Wu, C., "Robot Accuracy Analysis Based on Kinematics", IEEE Journal of Robotics and Automation, Vol. RA-2, No. 3, pp. 171-179, 1986.

[7] Wu, C., "The Kinematic Error Model for the Design of Robot Manipulators", American Control Conference, pp.497-502, San Francisco, 1983.

[8] Vukobratović, M. and Borovac, B., "Accuracy of the Robot Positioning and Orientation Assessed Via its Manufacturing Tolerances", Mechanism and Machine Theory, Vol. 30, No. 1, pp. 11-32, 1995.

[9] Bruyninckx, H., Demey, S., Dutré, S. and Schutter, J.D., "Kinematic Models for Model-Based Compliant Motion in the Presence of Uncertainty", The International Journal of Robotics Research, Vol. 14, No. 5, pp. 465-482, 1995.

[10] Samak, S., El-Sayed, S. El-Shakery, S. and El-Dardierie, M., Kinematic Perturbations on Robot Precision", 7^{th} IFToMM International Symposium on the Theory of Machines and Mechanisms, pp. 327-332, Bucharest, Romania, 1997.

[11] Kazerounian, K. and Qian, G.Z., "Kinematic Calibration of Robotic Manipulators", ASME Journal of Mechanisms, Transmissions, and Automation in Design, Vol. 111, pp.482-487, 1989.

[12] Gupta, K.C., "A Note on Position Analysis of Manipulators", Mechanism and Machine Theory, Vol. 19, pp. 5-8, 1984.

[13] Samak, S. and Gupta, K.C., "Parametric Uncertainty on Manipulators Dynamics", Mechanism and Machine Theory, Vol. 33, No. 7, pp. 945-956, 1998.

[14] Gupta, K.C., "Mechanics and Control of Robots", Springer, New York, 1997.

دقة المعايرة الكينماتيكية للإنسان الآلى باستخدام طريقة الجينات الوراثية

د. ســالم ســمك

قسم هندسة الانتاج والتصميم الميكانيكى

كلية الهندسة – جامعة المنوفية

ملخص البحث :

تتأثر دقة وضع ودوران النهاية الطرفية للإنسان الآلى بدقة قيم عناصره الكينماتيكية، وهذا الشرط منَ الصعب توافره فى الحياة العملية، لذا فلابد من إستنباط تقنيات للمعايرة (calibration techniques) لتحسين دقة الإنسان الآلى من خلال تقييم هذه القيم.

ومن ثم فإن هذا البحث يقدم طريقة جديدة لمعايرة الدقة الكينماتيكية للإنسان الآلى بتطبيق طريقة الجينات الوراثية (Genetic Algorithm) التى أظهرت حديثاً كفاءة عالية فى تطبيقات عديدة، وتم ذلك بتكوين نموذج كينماتيكى، عبارة عن علاقة خطية تربط بين قيم العناصر الكينماتيكية وأخطاء وضع ودوران النهاية الطرفية من خلال مصفوفة "جاكوبيان" للأخطاء (Error Jacobian matrix). وتم معاملة هذا النموذج كمشكلة مثالية (optimization problem) لمناولات الإنسان الآلى.

والهدف من ذلك هو تحليل وتقييم أداء طريقة الجينات. الوراثية فى معايرة أخطاء الإنسان الآلى، ففى هذه الطريقة أخطاء بارامترات الإنسان الآلى تمثل مجتمع الآباء والأبناء، ومعيار مصفوفة الأخطاء تمثل دالة التكاليف (cost function).

تقارب (convergence) النّموذجِ المُقَدَّمِ وفاعليته موضح بمثال عددى، عبارة عن وضع عشوائى لمناول من نوع PUMA ذو ستة درجات حرية، والنتائج المستخلصة بينت أن الطريقة المقدمة قد قللت أخطاء قيم البارامترات الكينماتيكية بنسبة ٢٤,١%.